Dynamic Data-Driven Uncertainty Quantification via Generalized Polynomial Chaos

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Abstract

The U.S. Air Force collects and maintains knowledge of all space objects in orbit and the space environment. This task is becoming more difficult as the number of objects currently tracked by the U.S. increases due to breakup events and improvements in sensor detection capabilities. The Space Surveillance Network, managed by the United States Air Force, is tasked with maintaining information on over 22,000 objects, 1,100 of which are active. In particular, low-Earth orbiting satellites are heavily influenced by atmospheric drag which is difficult to model due to fluctuations in the upper atmospheric density. These fluctuations are caused by variations in the Solar energy flux which heats Earth’s atmosphere causing it to expand. This research uses probabilistic models to characterize and account for the fluctuations in the Earth’s atmosphere. By correctly estimating the fluctuations, our work contributes to improving the ability to determine the likelihood of satellite collisions in space. Improving this calculation is expected to reduce the cost of space missions and improve safety of manned space missions.

The main focus of this work is the application of a new Polynomial Chaos based Uncertainty Quantification (UQ) approach. UQ is important for many science and engineering applications. The challenge that this work seeks to address is the long-term integration problem, where simulations are used to forecast physics over long temporal and/or spatial extrapolation intervals. This work applies a generalized Polynomial Chaos (gPC) expansion and Gaussian Mixture Models (GMMs) in a hybrid fashion for UQ. This work uses GMM-gPC approach for orbital UQ, while the gPC approach is applied to atmospheric density UQ. These approaches are applied to satellite orbital UQ which is important for conjunction assessments.

Introduction

Recent events in space, including the collision of Russia’s Cosmos 2251 satellite with Iridium 33 and China’s Feng Yun 1C anti-satellite demonstration, have stressed the capabilities of the Space Surveillance Network (SSN) and its ability to provide accurate and actionable impact probability estimates. The SSN network has the unique challenge of tracking more than 22,000 Space Objects...
(SOs) and providing critical collision avoidance warnings to military, NASA, and commercial operators. However, due to the large number of SOs and the limited number of sensors available to track them, it is impossible to maintain persistent surveillance resulting in large observation gaps. This inherent latency in the catalog information results in sparse observations and large propagation intervals between measurements and close approaches. The large propagation intervals coupled with nonlinear SO dynamics results in highly non-Gaussian probability distribution functions (pdfs). In particular, satellites in Low-Earth Orbit (LEO) are heavily influenced by atmospheric drag which is difficult to model. Uncertainties in atmospheric drag must be folded into estimation models to accurately represent the position uncertainties for calculating impact probabilities or conjunction assessments (CA). This process then separates naturally into a prediction and correction cycle, where estimates are used to predict the orbital position at a future time and observations are used to improve or correct these predictions while decreasing uncertainty. The difficulty in this process lies in representing the non-Gaussian uncertainty and accurately propagating it. Accurate assessment of confidence in position knowledge will be a significant improvement, particularly for the space situational awareness (SSA) community.

A number of upper atmospheric models exist which can be classified as either empirical or physics-based models. The current Air Force standard is the High Accuracy Satellite Drag Model (HASDM) [1], which is an empirical model based on observations of calibration satellites. These satellite observations are used to determine atmospheric model parameters based on their orbit determination solutions. Although the HASDM model is accurate for determining the current state of the upper atmospheric environment, it has no forecasting capability which limits its effectiveness for CA calculations. A number of physics-based models exist, two of which are the Global Ionosphere-Thermosphere Model (GITM) [2] and the Thermosphere-Ionosphere-Electrodynamics General Circulation Model (TIE-GCM) [3, 4]. These are physics-based models that solve the full Navier-Stokes equations for density, velocity, and temperature for a number of neutral and charged chemical species components. The improved modeling and prediction capabilities of these models come at a high computational cost. The models are very high-dimensional, solving Navier-Stokes equations over a discretized spatial grid involving 2000-10000 state variables and 12-20 inputs and internal parameters. Satellite CA calculations usually involves long propagation intervals (3-8 days) resulting in nonlinear transformation and non-Gaussian errors.

This nonlinearity and high-dimensionality results in the so-called curse of dimensionality [5], where the combination of increasing problem dimension and order of nonlinearity, causes the number of required evaluations to grow in a super-linear manner. The curse of dimensionality as related to atmospheric models is a difficulty that this work addresses. Additionally, CA requires full knowledge of the pdf to calculate the impact probability as opposed to traditional state estimation and data assimilation approaches which only require the first two moments (mean and covariance). Our current work presents a new approach that solves for the full pdf.

A common but computationally intensive method of propagating uncertainty is the use of Monte Carlo (MC) simulations [6, 7]. Randomly generated samples from the initial uncertainty distribution are propagated through the function of interest. MC approaches require on the order of millions of propagations\(^1\) to generate statistically valid UQ solutions. Parallelizing the computations on multiprocessor CPUs or on Graphics Processing Units (GPUs) reduces the runtime of the simulations significantly [8, 9, 10] at the cost of increasing the difficulty of implementation [11]. Reducing the number of sample points required for a result with satisfactory confidence bounds is possible through importance sampling. Although the computational cost can be prohibitive for most applications due to the slow convergence, the generality of MC techniques makes them an ideal benchmark to

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\(^1\)this number is problem dependent
compare other methods.

A spectrum of techniques exists that propagate the state and uncertainty of an initially Gaussian distribution through a nonlinear function, such as orbit propagation [12]. Computational cost is traded for accuracy of the pdf. Using the first order Taylor series expansion of the dynamics to linearly propagate the covariance matrix lies on one extreme; while the MC simulation lies on the other extreme of computational cost. Two techniques that occupy a range within this spectrum of computational cost are Gaussian Mixture Models (GMMs) and generalized Polynomial Chaos (gPC) expansion.

GMMs can approximate any pdf using a weighted sum of Gaussian distributions with the approximation improving in an $L_1$-norm sense with increasing number of elements [13]. When the initial distribution is Gaussian, the approximate GMM for this case has spatially distributed means and each element has smaller variance than that of the initial Gaussian distribution (i.e. differential entropy). Using the GMM approximation, each Gaussian component is propagated through the nonlinear function using State Transition Tensors (STTs) [14], sigma-point based methods [15, 16], quadrature, or cubature$^2$ [17]. Each element has a smaller uncertainty than the initial Gaussian distribution at epoch and therefore, the Gaussian assumption for each element should hold for propagation times that are at least as long, or longer than the original distribution. The weighted sum of the Gaussian elements after propagation approximates the resulting forecast pdf while having the ability to approximate non-Gaussian distributions. GMMs have been successfully used in many uncertainty propagation applications such as orbit estimation [18, 19], orbit determination [20, 21], and conjunction assessment [22, 23].

The gPC [24] approach uses orthogonal polynomial (OP) expansions as a surrogate model for quantifying uncertainty. The most suited polynomial is chosen using the Wiener-Askey scheme and depends on the initial uncertainty distribution [25]. It is also possible to compute optimal orthogonal polynomials for arbitrary pdfs that are not part of the Wiener-Askey scheme using arbitrary gPC (agPC) [26]. For Gaussian distributions, Hermite polynomials are the corresponding OPs [24, 25]. For the multidimensional case, the coefficients of the multivariate polynomials are computed such that a mapping of the random variable from the initial time to the final time is approximated. Once the polynomial coefficients are computed, sampling from the gPC polynomial approximation generally has a lower computational cost than a full-blown MC run. The gPC approach has been used in many fields for uncertainty quantification of computationally intensive models [27, 28, 29, 30]. In orbital mechanics, gPC has been previously used for uncertainty propagation [31, 32] and conjunction assessment [33, 34].

Reference [35] used gPC and GMM in a hybrid fashion to quantify state uncertainty for spacecraft. Including a GMM with the gPC (GMM-gPC) was shown to reduce the overall order required to achieve a desired accuracy. Reference [35] converted the initial distribution into a GMM, and gPC was used to propagate each of the elements. Splitting the initial distribution into a GMM reduces the size of the covariance associated with each element and therefore, lower order polynomials can be used. The GMM-gPC effectively reduces the function evaluations required for accurately describing a non-Gaussian distribution that results from the propagation of a state with an initial Gaussian distribution through a nonlinear function. The current paper uses the GMM-gPC method for the satellite orbital UQ with atmospheric drag.

The organization of this paper is as follows. First, the GMMs are discussed. Next, the gPC approach is outlined and discussed. Following this the GMM-gPC approach is discussed. Additionally, results are shown for simulated examples for both orbital and atmospheric UQ. Finally, discussions and conclusions are provided.

$^2$multidimensional quadratures
1 Gaussian Mixture Models

A GMM approximates any PDF in an \( L_2 \)-distance sense by using a weighted sum of Gaussian probability distribution functions [13].

\[
p(x) = \sum_{i=1}^{N} \alpha_i p_g(x; \mu_i, P_i)
\]

where \( N \) is the number of Gaussian probability distribution functions, and \( \alpha_i \) is a positive non-zero weight, which satisfies the following constraint:

\[
\sum_{i=1}^{N} \alpha_i = 1
\]

where \( \forall \alpha_i > 0 \). For uncertainty propagation, the initial Gaussian distribution is split into a GMM and each element is propagated through the nonlinear function. Standard Gaussian propagation techniques such as STTs [14] or sigma-point methods [15, 16] are commonly used to approximate the Gaussian elements post propagation. Although each element remains Gaussian, the weighted sum forms a non-Gaussian approximation of the true distribution. Modifications of this procedure exist, where the weights can be updated post-propagation [36] or the elements can be further split into more elements or merged mid-propagation [18]. However, these modifications are not considered for this work.

Instead of forming a GMM approximation of the initial multivariate Gaussian distribution, a univariate GMM library of the standard normal distribution is formed [18, 20, 23, 37]. The univariate library is applied along a column of the square-root factor of the covariance matrix in order to form a GMM approximation of a multivariate Gaussian. The univariate splitting library has to be computed only once and is stored in the form of a lookup table. Finding the univariate library is converted to an optimization problem where the distance between the GMM and the standard normal distribution is minimized. The \( L_2 \) distance is used instead of \( L_1 \) because a closed-form solution exists for the \( L_2 \) distance between a GMM and a Gaussian distribution. A library where all the standard deviations in the split are the same (homoscedastic), \( \sigma = \sqrt{1/N} \), and odd \( N \) up to 39 elements is used in this work [38, 39]. With increasing \( N \), \( \sigma \) decreases and therefore, the differential entropy of each element decreases as seen in Figure 1.

To apply the univariate splitting library to a multivariate Gaussian distribution \( p_G \sim \mathcal{N}(\mu, P) \), the univariate splitting library is applied along a column of the square-root \( S \) of the covariance matrix:

\[
P = SS^T
\]

For an \( n \)-dimensional state, the covariance matrix of each element after the split is:

\[
P_i = [s_1 \ldots \sigma s_k \ldots s_n][s_1 \ldots \sigma s_k \ldots s_n]^T
\]

where \( s_k \) is the desired column of \( S \) that the split is along. The means of the multivariate GMM are:

\[
\mu_i = \mu + \mu_s s_k
\]

If Cholesky or spectral decomposition is used to generate \( S \), the possible splitting options are limited to \( 2n \) directions. However, it is possible to apply the univariate splitting direction along any desired direction by generating a square-root matrix with one column parallel to the input direction [40]. For extremely non-linear problems, splitting along a single direction may not account
for the entire non-linearity of the problem. Therefore, splitting the initial multivariate distribution in multiple directions is required in order to better approximate the non-Gaussian behavior post-propagation [39, 41]. In such cases the splitting library can be applied recursively as a tensor product to split along multiple directions.

**Polynomial Chaos**

The idea of gPC originates from a paper from Norbert Wiener [24], where the term chaos is used to refer to uncertainty. This theory has been used frequently for UQ and is now also being used in the Aerospace field [42, 31, 43, 44, 45]. In the gPC, the uncertainty in variables through a transformation is represented by a series of orthogonal polynomials.

\[
    u(\xi, t) = \sum_{i=0}^{\infty} c_i(t)\Psi_i(\xi)
\]

(6)

In Eq. (6) \(\xi\) is a random variable. The orthogonal polynomials \(\Psi_i\) are defined by the following inner product in a Hilbert space:

\[
    \int_{-\infty}^{\infty} \Psi_m(\xi)\Psi_n(\xi)w(\xi) = 0
\]

(7)

Based on the distribution of the random variable, the orthogonal polynomial type and weighing function, \(w(\xi)\) from Eq. (7), are chosen from the Weiner-Askey [25] scheme found in Table 1. Since most applications assume the initial distribution to be Gaussian, Hermite polynomials are chosen according to the Wiener-Askey scheme. We, however, use normalized probabilists Hermite polynomials where the weight function is changed to:

\[
    w(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}
\]

(8)

The new weight function assumes that the distribution has a mean of 0 and a standard deviation of 1, which effectively normalizes and improves the numerical properties. The normalized Hermite polynomials can be found by using the following recursive relation:

\[
    (n + 1)! \times \Psi_{n+1}(\xi) = \xi \Psi_n(\xi) - n\Psi_{n-1}(\xi)
\]

(9)
Distribution Type | Density | Polynomial | Weight | Range
--- | --- | --- | --- | ---
Normal | $\frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}$ | Hermite | $e^{-\frac{x^2}{2}}$ | $[-\infty, \infty]$ 
Uniform | $\frac{1}{\pi}$ | Legendre | 1 | $[-1, 1]$ 
Beta | $\frac{(1-x)^{\alpha}(1+x)^{\beta}}{2^{\alpha+\beta+1}B(\alpha+1,\beta+1)}$ | Jacobi | $(1 - x)^\alpha(1 + x)^\beta$ | $[-1, 1]$ 
Exponential | $e^{-x}$ | Laguerre | $e^{-x}$ | $[0, \infty]$ 
Gamma | $\frac{x^\alpha e^{-x}}{\Gamma(\alpha+1)}$ | Generalized Laguerre | $x^\alpha e^{-x}$ | $[0, \infty]$ 

Table 1: The Wiener-Askey scheme

Figure 2: Normalized probabilists Hermite polynomials

where

$$\Psi_0 = 1 \quad \Psi_1 = \xi$$

(10)

In reality, the infinite series from Eq. (6) is truncated at some order. The orthogonal univariate Hermite polynomials up to order 5 can be seen in Figure 2. The conjunction problem is a multivariate problem and therefore, requires orthogonal multivariate polynomials. Multivariate polynomials can be created using the multi-index notation. Two-dimensional multivariate polynomials up to order 2 can be seen in Table 2. The multivariate polynomial can then be written as:

$$u(\xi, t) = \sum_{i=0}^{L} c_i(t)\Psi_{\alpha i}(\xi)$$

(11)

where $L$ is given by

$$L = \frac{(n + l)!}{n!!l!!}$$

(12)

where $n$ is the dimension of $\xi$ and $l$ is the maximum order of the truncated univariate polynomial. A given order $\bar{L}$ of the multivariate polynomial equals the sum of the elements of the multi-index vector. If the output is also a vector function of dimension $n$, $u(\xi, t)$, $n \times L$ coefficients $c_i(t)$ have to be computed.

The final challenge is to determine the coefficients $c_i(t)$. The two major methods used to determine these coefficients are the Intrusive method and the Non-intrusive method. The intrusive method requires knowledge of the propagation function that determines the evolution of the random
Table 2: Two-dimensional multivariate polynomials up to order 2

<table>
<thead>
<tr>
<th>Order</th>
<th>Multi-index</th>
<th>Multivariate Polynomial</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>(\alpha_0 = [0, 0])</td>
<td>(\Psi_{\alpha_0}(\xi) = 1)</td>
</tr>
<tr>
<td>1</td>
<td>(\alpha_1 = [1, 0])</td>
<td>(\Psi_{\alpha_1}(\xi) = \Psi_1(\xi_1))</td>
</tr>
<tr>
<td>1</td>
<td>(\alpha_2 = [0, 1])</td>
<td>(\Psi_{\alpha_2}(\xi) = \Psi_1(\xi_2))</td>
</tr>
<tr>
<td>2</td>
<td>(\alpha_3 = [2, 0])</td>
<td>(\Psi_{\alpha_3}(\xi) = \Psi_2(\xi_1))</td>
</tr>
<tr>
<td>2</td>
<td>(\alpha_4 = [0, 2])</td>
<td>(\Psi_{\alpha_4}(\xi) = \Psi_2(\xi_2))</td>
</tr>
<tr>
<td>2</td>
<td>(\alpha_5 = [1, 1])</td>
<td>(\Psi_{\alpha_5}(\xi) = \Psi_1(\xi_1)\Psi_2(\xi_1))</td>
</tr>
</tbody>
</table>

vector of inputs. This then results in a system of equations that need to be solved for \(c_i(t)\). The intrusive method cannot be used with black-box dynamics, and therefore is not considered in this work. The non-intrusive method does not require any knowledge of the propagation function. Given that we can solve the system for a specified initial condition, we use the projection property (Galerkin Projection) for approximating Eq. (11):

\[
c_i(t) = \int u(\xi, t)\Psi_i(\xi)p(\xi)d\xi
\]  

(13)

where \(p(\xi)\) is the pdf of \(\xi\).

The coefficients in the non-intrusive method can be solved using either Least Squares (LS), or a quadrature method. When LS is implemented, the initial states are randomly sampled. If the quadrature method is used, the initial states are chosen based on the node locations of the quadrature rule. The number of initial states to be used can be vastly reduced by using Compressive Sampling (CS) when using LS, and by using Sparse Grids (SG) when using the quadrature method. In this work, the quadrature method is used with a Smolyak SG (SSG) [46]. The SSG uses fewer grid points than a full tensor product quadrature as can be seen in Figure 3. In the quadrature method, a grid is generated with \(N_q\) node points, where each node has a location \(\xi_n\) and weight \(q_n\) associated with them. The coefficients \(c_i(t)\) are then found using the following summation:

\[
c_i(t) = \sum_{n=1}^{N_q} q_n u(\xi_n, t)\Psi_{\alpha_i}(\xi_n)
\]  

(14)

It should be noted that the node points are generated from a zero mean and identity covariance matrix multivariate distribution for numerical accuracy. The initial points are simply scaled to the actual mean and covariance inside the transformation function \(u(\xi, t)\).

**Polynomial Chaos with Gaussian Mixture Models**

Both gPC and GMMs can represent non-Gaussian distributions with lower computational cost than that of a full blown MC simulation. However, they both have their limitations. The biggest problem with gPC is the curse of dimensionality. The number of coefficients required with increasing order and increasing dimension for multivariate polynomials can be computed from Eq. (12) and seen in Figure 4(a). The number of nodes where computation has to be carried out also increases rapidly with increasing order and dimension as seen in Figure 4(b). When GMMs are used for multivariate applications, the univariate library is applied along one specified direction. Thus, the spectral direction along which the splitting is carried out can play a very important role in the quality of the resulting non-Gaussian distribution after a nonlinear transformation [23].

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Figure 3: Difference between a full (red) and sparse (blue) two-dimensional quadrature grid

Figure 4: Curse of Dimensionality with Polynomial Chaos

A combination of GMMs with gPC results in a theory that can outperform each of the separate theories due to them complementing each other [35]. In this method, each of the mixture elements is represented by a gPC expansion. The GMMs splitting reduces the size of the distribution that each gPC expansion has to account for. This is analogous to reducing the range for Taylor series expansion (TSE), or the Finite Element Method (FEM). Therefore, we use more simple elements (lower order gPC expansions) over smaller subdomains (a GMM) to approximate the final non-Gaussian distribution over a larger domain. The benefit can be seen in a very simple test case where an initial Gaussian distribution of a state in polar coordinates is converted to Cartesian coordinates. Since this transformation is non-linear, the resulting distribution becomes non-Gaussian. The true (MC) and approximated distributions can be seen in Figure 5. The gPC approximation is much better than the strictly Gaussian approximation as can be seen in Figures 5(a) and 5(b). Combining gPC and GMM, however, results in a much lower discrepancy between the MC and approximated distributions.

Ionosphere-Thermosphere Models

The novel methods in this work are implemented for the problem of SSA (orbit estimation and propagation). The major source of uncertainty in orbital propagation is the ionosphere-thermosphere envi-
Figure 5: True distribution (blue) and approximated distribution (red) after conversion from Polar coordinates to Cartesian coordinates.
Figure 6: $F_{10.7}$

environment. Therefore, this work accurately characterize the uncertainty in the ionosphere-thermosphere through the gPC approach. For this purpose, this work uses a physics-based model, the Global Ionosphere-Thermosphere Model (GITM).

**Global Ionosphere-Thermosphere Model:**
The Global Ionosphere-Thermosphere Model (GITM) [2] is a physics based model that solves the full Navier-Stokes equations for density, velocity, and temperature for a number of neutral and charged components. The model explicitly solves for the neutral densities of $O$, $O_2$, $N(^2D)$, $N(^2T)$, $N(^4S)$, $N_2$, $NO$, $H$, and $He$; and the ion species $O^+(^4S)$, $O^+(^2D)$, $O^+(^2P)$, $O_2^+$, $N^+$, $N_2^+$, $NO^+$, $H^+$, and $He^+$. It also contains chemistry between species of ions and neutrals, ions and electrons, and neutral and neutrals. In addition, GITM self-consistently solves for the neutral, ion, and electron temperature; the bulk horizontal neutral winds; the vertical velocity of the individual species; and the ion and electron velocities. To account for solar activity GITM can use $F_{10.7}$ as a proxy EUV spectrum measurements.

Some of the more important features of GITM are: adjustable resolution; non-uniform grid in the altitude and latitude coordinates; the dynamics equations are solved without the assumption of hydrostatic equilibrium; the advection is solved for explicitly, so the time-step in GITM is approximately 2–4 seconds; the chemistry is solved for explicitly, so there are no approximations of local chemical equilibrium; the ability to choose different models of electric fields and particle precipitation patterns; the ability to start from MSIS [3, 4] and IRI [47] solutions; and the ability to use a realistic (or ideal) magnetic field determined at the time of the model run. The main parameter of interest is $F_{10.7}$, which is a measure of the solar radio flux at 10.7 cm wavelength and is used as a proxy in GITM for solar activity. Figure 6 shows the $F_{10.7}$ solar radio flux index from 1980 up to approximately 2011, where the 11-year solar cycle is clearly visible in the high and low activity peaks.

**Results**

Two simulation studies are conducted for this work, the first case investigates the orbital position UQ problem, while the second case investigates the atmospheric density UQ problem. The first case uses the GMM-gPC approach for the orbital position UQ problem and the second case uses
Orbital Uncertainty Quantification

In this section, a test simulation is carried out to investigate the validity of the GMM-gPC method developed by Ref. [35] for an orbital application. The non-linearity of the orbital equations combined with the presence of perturbation such as the atmosphere, make the orbital pdf non-Gaussian with increasing flight time. Thus, this test case propagates a satellite in an almost circular LEO orbit at an altitude of approximately 450 km, under the influence of atmospheric drag simulated using the Jacchia-Bowman 2008 (JB2008) Empirical Thermospheric Density Model [48].

A Gaussian distribution was generated about an initial condition of the orbit. A MC and a GMM-gPC simulation was then carried out for 1 day (Figure 7(a)) and for 5 days (Figure 7(b)). The simulation was only carried out as a planar 2-dimensional trajectory for simplicity, but can easily be extended to a full 3-dimensional simulation in the future. As can be seen in the results found in Figure 7, the final distribution is highly non-Gaussian. However, the GMM-gPC simulation with orders of magnitude fewer runs is able to represent the final distribution well.

Initial Results for Atmospheric Density Forecasting

Low-Earth orbiting (LEO) satellites are heavily influenced by atmospheric drag, which is very difficult to model accurately. One of the main sources of uncertainty is input parameter uncertainty. These input parameters include F10.7, AP, and solar wind parameters. These parameters are measured constantly and these measurements are used to predict what these parameters will be in the future. The predicted values are then used in the physics-based models to predict future atmospheric conditions. Therefore, for the forward prediction of orbital uncertainty, the uncertainty of the atmospheric density due to these parameters must be characterized.

These simulation examples focus on using the gPC technique for UQ of physics-based atmospheric models. Unlike the last case this case just studies the use of gPC for UQ of the atmospheric density. The gPC approach is used to quantify the forecast uncertainty due to uncertainty in F10.7, AP, and solar wind parameters. The gPC approach is used to perform UQ on future atmospheric conditions. As part of this CA process, accurate and consistent UQ is required for the atmospheric models used.

In this section, initial results for the gPC UQ applied to the GITM model is discussed. The goal...
here is to use a physics based atmospheric density model for obtaining accurate density forecast to be used in conjunction assessments. The GITM model has a number of input parameters that can be derived from observations but the model also needs forecasts of its inputs and these forecasted values may be highly uncertain. Therefore, we look at the uncertainty in the forecasted density based on the uncertainty of these inputs. The main input parameter that drives the main dynamics in the GITM model is $F_{10.7}$ (see Figure 6). Two simulation cases are considered here, the first case uses quiet solar condition model input parameters and the second case uses active solar condition model input parameters. The first case only considers $F_{10.7}$ as an input parameter. While the second case considers uncertainty in $F_{10.7}$, Interplanetary Magnetic Field (IMF) in GSM coordinates (nT) ($B_x$, $B_y$, $B_z$), Solar Wind (km/s) $V_x$, and Hemispheric power $HPI$. The result for this simulation are shown in Figure 8. The time period for the simulations shown is Oct 21-26, 2002.

For these simulation, parameters are modeled as constant during forecast but random. In the first case, $F_{10.7}$ is assumed to have a normal distribution $\mathcal{N}(165.98, 8.34^2)$. For the first case, one dimensional quadrature points are used as simulation ensembles and the gPC model is fit using one dimensional Hermite polynomials. The parameters for the second case are modeled as constant during forecast but random. The random variables have the following distribution $\mathcal{N}(\mu, P)$, with $\mu = [165.98, -1.45, 0.06, -0.5, -551.79, 38.07]^T$ and the covariance given by

$$P = \text{diag}(8.33^2, 4.84^2, 4.10^2, 2.15^2, 105.1^2, 38.87^2)$$  \hspace{1cm} (15)$$

For the second case, the Smolyak Sparse Cubature are used as simulation ensembles and fit to multi-dimensional Hermite polynomials. From the figure it is clear that the uncertainty has a complex
behavior across geographic locations. Moreover, the difference in the test cases highlight the fact the Solar conditions can drastically effect the model’s accuracy. From the figures it is seen that during storm conditions (Figure 8(d)) the uncertainty can be as large as 30% but only 5% during quiet times (Figure 8(b)).

Conclusion

The combination of Polynomial Chaos (gPC) expansion with Gaussian Mixture Models (GMMs) results in a framework than can efficiently capture the evolution of an initially Gaussian distribution into a highly non-Gaussian distribution through a non-linear transformation. Using an initial GMM reduces the domain covered by the gPC and thus, lower order polynomials can be used to get accurate results. Increasing the order of the polynomials increases the computational load in an exponential manner, while increasing the number of elements may result in a near linear increase in the computational load. Increasing the polynomial order only marginally increases the accuracy after a certain order.

This work applies the GMM-gPC approach to the orbital Uncertainty Quantification (UQ) problem. It was shown that the GMM-gPC approach outperformed the gPC approach for the cases considered in this work. Additionally, the gPC approach was applied to physics-based atmospheric models. It was shown that the uncertainty in atmospheric density models have a complex behavior across geographic locations. The test cases shown in this work highlight the fact the Solar conditions can drastically effect the model accuracy. The test cases showed that during Solar storm conditions the uncertainty can be as large as 30% but only 5% during quiet times. This work provides initial results of the GMM-gPC applied to orbital propagation of uncertainty and the gPC approach applied to atmospheric density.

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