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MODELING SATELLITE DRAG COEFFICIENTS WITH RESPONSE SURFACES

Piyush M. Mehta*, Andrew Walker†, Earl Lawrence‡, Richard Linares§, David Higdon**, and Josef Koller††

Satellite drag coefficients are a major source of uncertainty in predicting the drag force on satellites in low Earth orbit. Among other things, accurately predicting the orbit requires detailed knowledge of the satellite drag coefficient. Computational methods are an important tool in computing the drag coefficient but are too intensive for real-time and predictive applications. Therefore, analytic or empirical models that can accurately predict drag coefficients are desired. This work uses response surfaces to model drag coefficients. The response surface methodology is validated by developing a response surface model for the drag coefficient of a sphere where the closed-form solution is known. The response surface model performs well in predicting the drag coefficient of a sphere with a root mean square percentage error less than 0.3% over the entire parameter space. For more complex geometries, such as the GRACE satellite, the Hubble Space Telescope, and the International Space Station, the model errors are only slightly larger at about 0.9%, 0.6%, and 1.0%, respectively.

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Introduction

The Committee for the Assessment of the U.S. Air Force’s Astrodynamics Standards established by the National Research Council (NRC) recently released a report highlighting the issues with current algorithms, models, and operational standards of the Air Force Space Command (AFSPC). The report cites atmospheric drag as the largest source of uncertainty for low-perigee objects due to inaccurate knowledge of atmospheric density and improper modeling of the interaction between the atmosphere and the object (Aeronautics and Space Engineering Board, 2012).

The theoretical drag model is given by

$$\ddot{\mathbf{a}}_{\text{drag}} = \frac{1}{2} \rho \frac{C_D A}{m} \left( \mathbf{v}_{\text{rel}} \cdot \frac{\mathbf{v}_{\text{rel}}}{|\mathbf{v}_{\text{rel}}|} \right)$$  \hspace{1cm} (1)

where $\ddot{\mathbf{a}}_{\text{drag}}$ is the drag acceleration, $\rho$ is the atmospheric mass density, $C_D$ is the drag coefficient, $A$ is the cross-sectional area, $m$ is the satellite mass, and $\mathbf{v}_{\text{rel}}$ is the bulk velocity of the atmospheric gas particles relative to the satellite.

Accurate satellite drag coefficient values are important for reducing biases in densities derived from satellite drag measurements as well as explicitly reducing orbit prediction errors. Numerical simulations produce accurate drag coefficient estimates subject to uncertainties in atmospheric and gas-surface interaction (GSI) models, but are too slow for predictive conjunction assessment applications. Therefore, accurately and efficiently modeling the drag coefficient is very important. In this work, we present a technique for modeling drag coefficients with response surfaces that replicates numerical simulations. The response surface models (RSMs) can be evaluated quickly while maintaining a high degree of accuracy. Current work uses the state-of-the-art atmospheric and GSI models (Walker et al., 2013, 2014).
In the realm of spacecraft dynamics and orbit determination, the drag coefficient is defined in three distinct ways: (i) a fixed drag coefficient, (ii) a fitted drag coefficient, and (iii) a physical drag coefficient. Fitted drag coefficients are estimated as part of an orbit determination process and fixed drag coefficients simply use a constant value for the drag coefficient. A drag coefficient value of 2.2 is an approximation for the physical drag coefficient of satellites with compact shapes and has been commonly used in the past. Errors from the use of fixed drag coefficients arise because of the application of the value of 2.2 derived for compact satellites to satellites with complex geometries or geometries with high aspect ratios such as a rocket bodies (Jacchia 1963, Slowey 1964, and Cook 1965). For high aspect ratio objects, shear can drastically increase the drag coefficient. Meanwhile, multiple reflections for complex geometries can also lead to divergence from the commonly used value of 2.2. The drag coefficient also changes with altitude and solar conditions since the atmospheric properties that affect the drag coefficient are heavily dependent on the solar flux and geomagnetic conditions (Moe, 1998). Fitted drag coefficients are specific to the atmospheric model used and therefore carry along the limitations of the atmospheric model and also frequently absorb other model errors. In addition, fitted drag coefficients are also dependent on the mass and cross-sectional area of the object used in the drag model. Physical drag coefficients are determined by the energy and momentum exchange of freestream atmospheric particles with the spacecraft surface (Schaaf and Chambre, 1961). Throughout this work, the term drag coefficients will refer to physical drag coefficients, unless stated otherwise.

The drag coefficient, characterized by the interaction between the atmosphere and the object, is an independent source of error whereas the errors in atmospheric mass density often stem from the use of fixed and/or fitted drag coefficients in its derivation from orbital drag measurements. Accurately deriving densities from drag measurements requires, in addition to accurate and high temporal resolution data (as in the case of an accelerometer), accurate modeling of the drag coefficient along the orbit. In addition, if the fixed drag coefficient is significantly different than the
true physical drag coefficient, or if the conditions (in terms of dynamic model error) for which the fitted drag coefficient is estimated do not apply to the conditions for the orbit prediction, the use of fixed and/or fitted drag coefficients can by itself induce large orbit prediction errors.

Closed-form solutions for the drag coefficients of satellites with simple convex geometries like a sphere, cylinder, and cube in free molecular flow (FMF) were developed early in the Space Age (Schaaf and Chambre, 1961 and Sentman, 1961); however, most satellites have complex shapes with concave geometries and require numerical modeling of the drag coefficient. The need for numerical modeling arises from multiple surface reflections and flow shadowing that changes the incident velocity distribution that is assumed to be Maxwellian for the analytic solutions. The drag coefficient in FMF is a function of the atmospheric translational temperature, $T_\infty$, surface temperature, $T_s$, spacecraft relative velocity, $v_{rel}$, chemical composition of the atmosphere, GSI model (Walker et al., 2014), as well as the mass, geometry, and orientation of the object.

A comparison of drag coefficients computed with the Direct Simulation Monte Carlo (DSMC) method using the diffuse reflection with incomplete accommodation (DRIA) and the quasi-specular Cercignani-Lampis-Lord (CLL) GSI models shows the highly sensitive nature of drag coefficients to GSIs (Mehta et al., 2013). The present work uses the CLL GSI model because it is able to reproduce the quasi-specular reflection observed in molecular beam experiments (Cercignani and Lampis, 1971). The CLL model uses the normal energy accommodation coefficient, $\alpha_n$, and the tangential momentum accommodation coefficient, $\sigma_t$, to describe the exchange of energy and momentum between the gas and surface (Lord, 1991). The value of $\sigma_t$ is assumed to be unity for free molecular flows (Walker et al., 2014). An empirical model linking $\sigma_n$ and $\alpha_n$ was recently developed for use with the CLL GSI model (Walker et al., 2014). Drag coefficients computed using the DRIA and CLL GSI models are within 2-3% of each other at altitudes up to ~500 km (Mehta et al., 2013; Walker et al., 2014).
A technique for creating parameterized drag coefficient models for satellites with complex geometries was recently developed (Mehta et al., 2013). The technique was applied to the Gravity Recovery and Climate Experiment (GRACE) satellite (Tapley et al., 2004) by developing parameterized relations between drag coefficient and sensitive input parameters based on a local sensitivity analysis. The model was developed for use with the DRIA GSI model (Mehta et al., 2013).

This work presents and validates a state-of-the-art technique for modeling drag coefficients with a response surface. The developed model takes into account all the relevant parameters that affect the drag coefficient and uses the CLL GSI model. The technique is validated using a sphere (simple geometry), where the closed-form solution is known, and then extended and applied to the more complex cases of the GRACE satellite, a simplified model of the Hubble Space Telescope (HST) with articulating solar panels, and the International Space Station (ISS).

The GRACE mission uses two identical satellites, GRACE-A and GRACE-B. The two satellites GRACE-A and GRACE-B are separated by an along track distance of approximately 200 km. The leading satellite is flipped 180 degrees about the sideslip axis in order to maintain communication with the trailing satellite. The use of the model developed in this work is only valid and recommended for the trailing satellite, which is more specifically, GRACE-A before Dec 2005 maneuver and GRACE-B post-maneuver (Tapley et al., 2004).

A simplified HST and the ISS are used to examine the applicability of the technique to articulating satellites and satellites with highly complex model geometries, respectively. The 3D mesh models for GRACE and the simplified HST are generated as part of this work. The model for the
ISS was obtained from the National Aeronautics and Space Administration (NASA) website*. The developed RSMs are available to the community and can be downloaded† at our website.

**METHODOLOGY**

A response surface models how a dependent state parameter responds to variations in one or more of the independent design parameters. The idea behind a response surface model is to use a series of designed experiments to characterize the optimal response of a system. Response surface modeling is a form of empirical modeling that uses a sample of system responses, typically obtained from experiments, to estimate or predict the response or state of the system at any given time in the future given the design input parameters.

In the current work, experiments are substituted with numerical simulations for cost and time efficiency. The Test Particle Monte Carlo (TPMC) method is used in simulating the environment encountered by satellites in low Earth orbit (LEO) for drag coefficient calculations. All simulations performed in the current study fall in the FMF regime. TPMC was used over DSMC because it is computationally inexpensive and equally accurate in the FMF regime. Previous work using DSMC for drag coefficient computation has been performed by, among others, Pilinski et al. (2011) and Mehta et al. (2013). IMPACT-TPMC is validated using the DSMC Analysis Code (DAC) developed by the NASA.

In this case, the dependent state parameter is the satellite drag coefficient and the independent design parameters are the atmospheric properties ($T_\infty$, and the chemical composition of the atmosphere), satellite characteristics ($v_{rel}$ and $T_w$), and the GSI parameters ($\alpha_n$ and $\sigma_t$) for a sphere. In addition, the independent parameters for GRACE, HST, and ISS include the orientation angles

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* http://www.nasa.gov/multimedia/3d_resources/assets/iss_c2.html
† https://sourceforge.net/projects/responsesurfacemodels/
(β, Φ). The orientation angle of the articulating solar panels is also included for the HST. At LEO altitudes, the atmosphere is primarily composed of atomic oxygen (O), atomic nitrogen (N), molecular oxygen (O₂), molecular nitrogen (N₂), helium (He), hydrogen (H), argon (Ar), and anomalous oxygen (AO) (Picone et al., 2002). However, the mole fraction of Ar is always so low that neglecting it results in drag coefficient errors less than 0.1%; therefore, Ar is justifiably ignored for the current study. The mole fractions of AO are also always very low below ~500 km but can reach values of ~25% percent at higher altitudes. However, AO is also neglected in this work since all space objects modeled orbit close to or below 500 km.

Insert Figure 1

Insert Figure 2

Insert Figure 3

Figures 1-3 show the attitude orientation definitions for GRACE, HST and the ISS, respectively. Drag coefficient simulations are performed for each of the six chemical species individually. Therefore, a response surface is developed for each individual species and the total drag coefficient, \( C_D \), is calculated using the relation:

\[
C_D = \sum_{i=1}^{6} C_{D_i} \chi_i \cdot m_i
\]

where \( C_{D_i} \) is the drag coefficient, \( \chi_i \) is the mole fraction, and \( m_i \) is the particle mass for the \( i^{th} \) species. The mole fractions \( \chi_i \) are computed using the following equation:

\[
\chi_i = \frac{n_i}{\sum_{i=1}^{6} n_i}
\]
where \( n_i \) is the number density of the \( i^{th} \) species.

**Gaussian Process Modeling**

Several important assumptions guide the choice for a response surface model. First, the data is assumed to have extremely low variance, i.e., repeated runs of the simulator for a given set of inputs produce essentially identical outputs (in this case, there is a small amount of Monte Carlo noise). Thus, the response surface needs to be able to nearly interpolate the training data. Additionally, it is assumed that the surface is continuous and fairly smooth. Finally, it is desired to be able evaluate the fitted model extremely quickly at new input settings. For these reasons, we will use a Gaussian process (GP) model, which has long been used to build approximations for complex computer simulators (Sacks et al., 1989).

**Formulation**

To develop the model, consider the results for a single chemical species. Let \( y_i \) be the simulated drag coefficient for a given input vector \( x_i \). Let \( \mathbf{y} = [y_1, \ldots, y_n]^{\top} \) be the vector containing all of the simulation results and assume that it has been standardized to have mean zero and variance one. Let \( X \) be an \( m \times p \) matrix with rows given by \( x_i \) and assume that it has been standardized to the \( p \)-dimensional unit hypercube (all \( X_{ij} \) are between zero and one). A zero-mean Gaussian process model says that \( \mathbf{y} \) has a multivariate Gaussian distribution with a particular structure for the covariance matrix that is a function of \( X \).

\[
y \sim N \left( \mathbf{0}, \frac{1}{\lambda_0} R(X) \right)
\] (4)

where \( \lambda_0 \) is the marginal precision (inverse of the variance) and \( R(X) \) is the correlation matrix given as a function of the inputs. There are many choices for \( R(X) \), but we choose the form described in Gattiker et al. (2006).

\[
R_{i,j} = \prod_{k=1}^{p} \theta_k^{4(x_{i,k} - x_{j,k})^2}
\] (5)
Consider $R_{i,j}$ as inputs $x_i$ and $x_j$ approach each other where $\theta$ is the spatial correlation parameter. As the distance between these inputs goes to zero, $R_{i,j}$ goes to unity and the difference between realizations $y_i$ and $y_j$ at these inputs also goes to zero. Thus, this correlation structure ensures that a draw of $\mathbf{\bar{y}}$ for some set of inputs $X$ will be a set of points on a continuous surface.

In practice, simulation codes are not perfectly smooth. In this case, the simulation is actually a Monte Carlo code, so the results do have some variation around a presumably smooth underlying function. For this reason, we modify the specification slightly

$$y \sim N\left(\mathbf{0}, \sum (X) = \frac{1}{\lambda_0} R(X) + \frac{1}{\lambda_e} I\right)$$

where $I$ is the identity matrix. This formulation assumes that the vector $\mathbf{\bar{y}}$ is a draw of a smooth function with a small amount of uncorrelated Gaussian noise, with variance $1/\lambda_e$, at each observed location.

**Prediction**

Prediction for new points is based on the properties of the conditional multivariate Gaussian. Consider the joint distribution of a set of observed $(y_i, x_i)$ pairs and a set of new points $(y_i^*, x_i^*)$, where the $x_i^*$ are known, but the $y_i^*$ are as yet unobserved,

$$\begin{pmatrix} \mathbf{\bar{y}}^* \\ \mathbf{\bar{y}} \end{pmatrix} \sim N\left(\mathbf{0}, \begin{bmatrix} \sum (X^*) & \sum (X^*, X) \\ \sum (X^*, X)' & \sum (X) \end{bmatrix}\right)$$

where $\Sigma(X)$, $\Sigma(X^*)$, and $\Sigma(X^*, X)$ are computed using the formula for the covariance given in Eq. (6). The matrices $\Sigma(X)$ and $\Sigma(X^*)$ are the covariance matrices for the observations associated with $X$ and $X^*$, respectively. The matrix $\Sigma(X^*, X)$ contains the covariances between $X$ and $X^*$.

We can make predictions about $\mathbf{\bar{y}}^*$ using the conditional distribution for $\mathbf{\bar{y}}^* | \mathbf{\bar{y}}$, which is simply Gaussian $\mathbf{\bar{y}}^* | \mathbf{\bar{y}} \sim N(\mu, \Psi)$ where
\[ \mu = \sum (X^*, X) \sum (X)^{-1} \tilde{y} \]  
\[ \Psi = \sum (X^*) - \sum (X^*, X) \sum (X)^{-1} \sum (X^*, X)' \]  

The top panel of Figure 4 shows a set of random vectors drawn from a GP with a single input dimension. The center panel shows three observations along a function of interest. The bottom panel shows 10 draws from the Gaussian process conditioned on those three points. The conditional draws pass through the observations and points nearby have nearly the same value as the observations. Uncertainty near the observations is very low and grows with distance from the observations. Most importantly, the conditional draws track the unobserved parts of the function of interest very well. It should be noted that Figure 4 is intended to be a cartoon to show what a Gaussian process is (a distribution on functions) and to show how it can be conditioned on observed data. Therefore, qualitative analysis of this data is not intended.

Training Set Design with Latin Hypercube Sampling
Sampling of the design parameter space for the response surface is performed using the Latin hypercube sampling (LHS) technique in order to fill the space of possible inputs (McKay et al., 1979, and Santner et al., 2003). This ensures that any new point will be close to the training data and reduce the uncertainty of the prediction as given by in Eq. (9). Assume that we want a training set of size \( m \) over \( p \) dimensions. The technique divides the expected range of each independent into \( m \) equiprobable intervals. In this case, it is assumed that the inputs are uniformly distributed; so equiprobable is equivalent to equal widths. In LHS, the training points are chosen such that each interval for each dimension has a single point. Thus, when the inputs are projected to any single dimension, the inputs are perfectly stratified for that dimension. Figure 3 shows an ex-
ample of LHS with 10 samples in two dimensions. Note that each column has only one point and each row has only one point.

Insert Figure 5

Drag coefficients for the cumulative sample of the design parameters were obtained using TPMC simulations. The calculated drag coefficients along the with cumulative design parameter sample were used to model the response surface.

Estimation

**Bayesian Formulation and Prior Distributions**

The formulation described above has $p+2$ unknown parameters: a correlation parameter for each predictor, $\rho_k$, and two precision parameters, $\lambda_0$ and $\lambda_\varepsilon$. We will follow the Bayesian approach to this estimation problem described in Gattiker et al. (2006). Thus, we need prior distributions on these parameters to complete the specification and proceed with estimation.

For each $\theta_k$ in Eq. (5), we use a beta prior,

$$\pi(\theta_k) \propto \theta_k^{\tau-1}(1-\theta_k)^{\tau-1}$$  \hspace{1cm} (10)

with $\tau = 1$ and $\tau = 0.1$. These parameters put a substantial portion of the prior distribution near one. Values of $\theta_k$ near one indicate a function that is very flat in that dimension (similar to a regression coefficient near zero). The data easily move this parameter lower when necessary.

For both $\lambda_0$ and $\lambda_\varepsilon$, we assume a gamma prior,

$$\pi(\lambda) \propto \lambda^\eta \exp\{-\delta \lambda\}$$  \hspace{1cm} (11)

For $\lambda_0$, we assume $\eta_0 = \delta_0 = 5$. Because we standardize our data, we expect $\lambda_0$ to be somewhat close to one. These parameters give a prior expectation equal to one, but allow the value to vary substantially around this value. For $\lambda_\varepsilon$, we assume $\eta_\varepsilon = 1$ and $\delta_\varepsilon = 0$. This prior is actually impro-
per, but encourages large values for $\lambda_c$. Large values of this precision mean that the resulting conditional surfaces will be close to interpolating the observed data. We want this parameter to be as large as the data will allow.

The posterior distribution for $\tilde{\theta}$, $\lambda_0$, and $\lambda_c$ is found by combining Eqs. (6), (10), and (11) and plugging in the observed simulation inputs and outputs.

$$p(\tilde{\theta}, \lambda_0, \lambda_c | \tilde{y}, X) \propto \prod_{k=1}^{n} \pi(\theta_k) \pi(\lambda_0) \pi(\lambda_c) \left( \sum (X) \right)^{-\frac{1}{2}} \exp \{ \tilde{y}' \sum (X)^{-1} \tilde{y} \}$$  \hspace{1cm} (12)

Because this distribution has no closed form, we use Markov chain Monte Carlo (MCMC) (Tierney, 1994) to make inferences about the unknown parameters.

**Markov Chain Monte Carlo**

Markov Chain Monte Carlo (MCMC) is a technique for sampling from complicated probability distributions. MCMC methods construct a Markov chain whose stationary distribution is the desired distribution. We will use the Metropolis-Hastings (MH) algorithm (Chib and Greenberg, 1995) to construct our chain.

Assume that we have some distribution $g(\cdot)$ for a parameter $\omega$. The goal is to obtain a sample from this distribution $\omega^1, \cdots, \omega^M$ that we can use to make inferences, e.g. compute the mean and several quantiles. The MH algorithm generates the sample sequentially. Starting with sample $s$, the $s+1$ sample is generated according to the following rules.

1. Generate a candidate sample parameter conditional on the $s^{th}$ sample parameter from a proposal distribution $h(\omega' | \omega^s)$.

2. Compute the acceptance ratio

$$\kappa = \frac{g(\omega') h(\omega' | \omega^s)}{g(\omega^s) h(\omega^s | \omega')}$$ \hspace{1cm} (13)

3. With probability $\kappa$, set $\omega^{s+1} = \omega'$, otherwise set $\omega^{s+1} = \omega^s$. 


The theoretical basis for this algorithm is discussed in Tierney (1994). The samples produced from this algorithm converge to a sample from \( g(\cdot) \). Specifically, the mean and quantiles computed from this sample will converge to the mean and quantiles of \( g(\cdot) \) as the sample size \( M \) goes to infinity. The sample is correlated, so the convergence of the estimated mean and quantiles to the true values is not as fast as would be the case for an uncorrelated sample. In practice, the initial draws are discarded to avoid any bias introduced by a poor starting value. One notable feature of this algorithm is that \( g(\cdot) \) can be unnormalized. When there is a vector of parameters \( \omega \), each \( \omega_i \) is updated individually within the \( s \)th iteration using the full conditional density \( g(\omega_i | \tilde{\omega}_{-i}) \), which is just proportional to the joint density \( g(\tilde{\omega}) \).

In our problem, we replace \( g(\tilde{\omega}) \) with \( p(\tilde{\theta}, \tilde{\lambda}_y, \tilde{\lambda}_x | \tilde{y}, \tilde{X}) \). To complete the algorithm, we need to specify proposal densities. As long as the acceptance ratio is calculated correctly, almost any proposal density will be acceptable. The efficiency of the algorithm is determined by the autocorrelation of the sample (independent is best) and the choice of proposal distribution determines this. For the spatial correlation parameters \( \omega \), we use a symmetric uniform proposal distribution around the previous sample,

\[
\theta_k' \sim \text{Unif}(\theta_k' - \gamma_k, \theta_k' + \gamma_k)
\]  

For the precision parameters, we again use a uniform distribution centered at the current value, but with a width partially determined by the current value,

\[
\lambda_j' \sim \text{Unif}([1 - \tau_j] \lambda_j, [1 + \tau_j] \lambda_j)
\]  

Increasing the width with the current draw of the parameter allows the algorithm to propose bigger steps when the parameter is larger, which allows the algorithm to explore the space better. The parameters \( \gamma_k \) and \( \tau_j \) are selected after some short, initial runs to reduce the autocorrelation of the resulting Markov chain.

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RESULTS AND DISCUSSION

Response Surface Models (RSMs) are developed for estimating the drag coefficient of a sphere (a simple convex geometry); the GRACE satellite and ISS (both with complex concave geometries); and the simplified HST (with articulating solar panels). A cumulative LHS sample of 1,000 ensemble members and the associated drag coefficients calculated using TPMC are used to train and develop the RSMs for sphere, GRACE, and HST. The ISS uses a sample of 400 ensemble members to examine the effect of training sample size on the performance of a RSM. RSMs are developed for each of the six species that comprise the atmosphere. The developed RSMs for each species are validated using a separate cumulative LHS sample test set with 100 ensemble members for sphere, GRACE, and the ISS while HST uses 1,000 members. An additional cumulative LHS sample with 1,000 ensemble members using all species simultaneously, along with the associated drag coefficients, is also used to validate the RSM for GRACE.

Insert Figure 6

Figure 6 shows the validation of the RSM developed for a sphere in O. Figure 6a shows good agreement between the drag coefficients predicted using the RSM and those calculated using TPMC. The performance of the RSMs developed using individual species for the sphere, GRACE, HST, and ISS are tabulated in Table 1. All residuals are tightly packed close to zero and are on the order of $2\times10^{-3}$ with some outliers and exhibit no undesired trends. The histogram for the test set residuals in Figure 6b shows that the model is nearly unbiased (histogram centered near zero) and there are no regions of the input space that produce large outliers.

Insert Table 1
Insert Figure 7

Figure 7a shows the validation of the RSM developed for a sphere in He. The RSM for He has the worst performance characteristics of all the species tested for the sphere RSM based on the summary presented in Table 1. The He residuals are three times larger than O at $6 \times 10^{-3}$; however, since He is much lighter than O, the drag coefficient caused by He is much higher than O and therefore, the residual root mean square percentage error is only double that of O. For GRACE, the He RSME is higher than for H. We attribute this to Monte Carlo noise in the simulations. For both the ISS and HST RSMs, the RSME and RSMPE are higher for H than for He, as expected. The histogram for the test set residuals in Figure 7b again shows that the model is unbiased and there are no regions of the input space that produce large outliers.

Similar RSM performance is observed for all the species as documented in Table 1. Plots for the performance of the RSM for O and He are shown, because O and He dominate the atmosphere at low and high altitudes, respectively.

Insert Figure 8

Figure 8 shows the validation of the RSM developed for GRACE in O. Good agreement is observed between the drag coefficients predicted using the RSM and TPMC, albeit with more noise than for the sphere. The residuals in O for GRACE are almost an order of magnitude higher than for a sphere at $2.1 \times 10^{-2}$. The increased noise is attributed to the complex and concave geometry that results in multiple reflections and flow shadowing. The residuals have a much wider spread than the sphere but still do not exhibit any undesired trends. The test set residual quantiles suggest a near-Gaussian distribution with a heavy tail at the higher end.
Figure 9 shows the validation of the RSM developed for GRACE in He. The RSM for He again has the worst performance characteristics off all GRACE RSMs based on the combination of the root mean square error (RMSE) and the root mean square percentage error (RMSPE) given in Table 1. The residuals at $2.9 \times 10^{-2}$ are $\sim 38\%$ higher compared to O and root mean square percentage error is $\sim 22\%$ higher than for O. The distribution of the residuals is similar to O except that the distribution is tail heavy at both ends. Similar performance is observed for the GRACE RSM of all species as documented in Table 1.

In addition to the validation of the GRACE RSM for each individual species, a multi-species validation test set is generated with TPMC using all the species simultaneously. As shown in Figure 10 the RSMs developed for GRACE are able to successfully estimate the drag coefficient by combining drag coefficients for each species using Eq. (2). The RMSPE for the multi-species estimation is well below $1\%$ as shown in Table 1.
Figure 11 and Figure 12 show the validation of the RSM developed for HST in O and He respectively. Good agreement is again observed between the drag coefficients predicted using the RSM and TPMC. The residuals in O and He for HST are almost an order of magnitude higher than for a sphere at $1.0 \times 10^{-2}$ and $1.3 \times 10^{-2}$ respectively, but are roughly 50% smaller than for GRACE. The residuals resemble a Gaussian distribution more closely because of the larger sample used and do not exhibit any undesired trends.

Insert Figure 13

Insert Figure 14

Figure 13 and Figure 14 show the validation of the RSM developed for ISS in O and He respectively. Good agreement is again observed between the drag coefficients predicted using the RSM and TPMC; however, the test exhibits a larger scatter due to the smaller size of the training set used to develop the RSM model. The larger noise is also partly attributed to the highly complex geometry of the ISS. Even for the very complex geometry of the ISS, the worst-case RSM error is ~1%. The residuals in O and He for ISS are almost an order of magnitude higher than for a sphere at $2.8 \times 10^{-2}$ and $2.8 \times 10^{-2}$ respectively, are roughly the same as for GRACE. The residuals again do not exhibit any undesired trends.

**Drag Coefficient Sensitivity Analysis**

The sensitivity of the response surface models for the $C_D$ of GRACE, HST, and the ISS to the independent parameters is summarized in Figures 15 – 17. The sensitivity plots are generated using response surface models, as the number of independent parameters would require a large
number of data point simulations for a global view. The nominal values for independent parameters are provided in Table 2.

Insert Table 2

Figure 15 shows the sensitivity of GRACE’s $C_D$ to a variety of selected parameters. Figure 15(a) shows GRACE’s $C_D$ as a function of the pitch, $\Phi$, and yaw, $\beta$, angles that control GRACE’s orientation. For low yaw angles, $C_D$ is very sensitive to changes in the pitch angle. Conversely, $C_D$ is most sensitive to the yaw angle when the pitch angle is near zero. $C_D$ is more sensitive to the pitch angle than the yaw angle because GRACE’s geometry is wider than it is tall, leading to larger area variations as the pitch angle changes. Figure 15(b) shows the sensitivity of GRACE’s $C_D$ to the relative velocity and the normal energy accommodation. Over the expected range of parameter values, $C_D$ is far more sensitive to $\alpha_n$ than $v_{rel}$ for near complete normal energy accommodation. Previous studies have shown that $C_D$ has a nonlinear dependence on $\alpha_n$ that is significantly stronger near complete accommodation (Walker et al., 2014). For near zero normal energy accommodation, the sensitivity to $v_{rel}$ increases; however, $C_D$ is still more sensitive to $\alpha_n$. GRACE’s $C_D$ is increasingly sensitive to $v_{rel}$ as it decreases because of the increased contribution of shear to the drag coefficient. For large $v_{rel}$, the flow can be approximated as hyperthermal where $C_D$ is relatively insensitive to changes in the speed ratio. Figure 15(c) compares the sensitivity of GRACE’s $C_D$ to the atmospheric translational temperature and yaw angle. Over the expected range of parameter values, $C_D$ is equally sensitive to the two investigated parameters for high $T_{\infty}$ and low $\beta$. As $T_{\infty}$ decreases and $\beta$ increases, $C_D$ becomes more sensitive to the yaw angle because a lower $T_{\infty}$ decreases the contribution of shear to the drag coefficient. Furthermore, Figure 15(a) shows that the sensitivity of $C_D$ to $\beta$ increases for larger $\beta$. Figure 15(d) compares the sensitivity of GRACE’s $C_D$ to the normal energy accommodation coefficient and the pitch angle.
As expected from analysis of Figure 15(b), over the expected range of parameter values, $C_D$ is more sensitive to $\alpha_n$ near complete accommodation. Near zero accommodation, $C_D$ becomes more sensitive to the pitch angle due to the large area variations caused by the change in GRACE’s orientation.

Insert Figure 15

Figure 16 shows the sensitivities of a simplified version of HST to a variety of selected parameters. Figure 16(a) shows the sensitivity of HST’s $C_D$ to the pitch and yaw angles similar to Figure 15(a) for GRACE. Unlike GRACE, HST’s $C_D$ is dominantly controlled by the pitch angle over the entire space. This is due to large changes in the orientation of the cylinder as a function of pitch. The cylinder is insensitive to changes in the yaw angle due to its symmetry; only the attached solar panels change orientation when the yaw angle changes. The sensitivity of $C_D$ increases to the yaw angle for negative pitch angles. This is because the flat plate solar panels present a larger projected area at this orientation, and hence a larger drag coefficient. When the yaw angle changes, there is a larger change in the drag coefficient due to the solar panels. Figure 16(b) shows the sensitivity of HST’s $C_D$ to the relative velocity and the normal energy accommodation coefficient. The results are similar to those for GRACE in Figure 15(b) with the drag coefficient being more sensitive to changes in $\alpha_n$ as $\alpha_n$ approaches unity. For HST, $C_D$ is more sensitive to $\alpha_n$ than to $v_{rel}$. This is because bodies with a higher ratio of area tangential to the flow will have more shear which is more sensitive to $v_{rel}$. GRACE has a much higher tangential-to-normal area ratio than HST and therefore, it is more sensitive to $v_{rel}$. Figure 16(c) shows the sensitivity of HST’s $C_D$ to the atmospheric translational temperature and yaw angle. HST is much more sensitive to changes in $T_x$ than to changes in $\beta$. This is again due to the cylindrical symmetry of HST.
as a function of the yaw angle. Only the drag coefficient contribution of the solar panels changes as a function of the yaw angle. Figure 16(d) shows the sensitivity of HST’s $C_D$ to the normal energy accommodation coefficient and the pitch angle. The drag coefficient is much more sensitive to changes in $\alpha_n$ than $\Phi$. $C_D$ becomes more sensitive to $\Phi$ as $\alpha_n$ decreases and $\Phi$ increases. The weak dependence of HST’s $C_D$ on $\Phi$ compared to GRACE is due to the relatively small projected area change as $\Phi$ varies for HST. GRACE’s high tangential-to-normal area ratio leads to large changes in the projected area and drag coefficient as $\Phi$ varies. Figure 16(e) shows the sensitivity of HST’s $C_D$ to the solar panel orientation angle and the normal energy accommodation coefficient. The solar panel orientation angle is defined by the angle between the solar panel normal and the cylinder axis. The complicated morphology of the sensitivity is due to the superposition of the drag coefficients of the cylinder and the solar panel flat plates. For very low solar panel orientation angles (solar panel normal nearly aligned with the cylinder axis), $\zeta$ is the more sensitive indicator of $C_D$. This is because changes orientation angle result in large changes in the projected area of the flat plate sonar panels which have a much larger drag coefficient due to normal forces than due to shear forces. As the solar panel orientation angle increases, $C_D$ becomes increasingly sensitive to $\alpha_n$. At $\zeta = 25^\circ$, the drag coefficient becomes insensitive to changes in the solar panel orientation angle. This same behavior can be seen from a single flat plate when calculating the drag coefficient based on its changing projected area. For angles between 30° and 80°, $C_D$ is very sensitive to the solar panel orientation angle. As the orientation angle approaches 90°, $C_D$ becomes dominantly controlled by the normal energy accommodation coefficient.

Figure 17

Figure 17 shows the sensitivities of the ISS’s $C_D$ to a variety of selected parameters. Figure 17(a) shows the sensitivity of the ISS’s $C_D$ to the pitch and yaw angles similar to Figure 15(a) for
GRACE and Figure 16(a) for HST. The ISS is not symmetric in pitch angle and therefore the point at which there is little to no sensitivity to the pitch angle occurs at \(-0.5^\circ - 1.0^\circ\). As the pitch angle departs from this angle, \(C_D\) becomes increasingly sensitive to the pitch angle. Generally, the ISS’s \(C_D\) is more sensitive to the yaw angle. Figure 17(b) shows the sensitivity of the ISS’s \(C_D\) to the relative velocity and the normal energy accommodation coefficient. The ISS sensitivity to \(\alpha_n\) and \(v_{rel}\) are very similar to GRACE. For \(\alpha_n\) near unity, \(C_D\) is increasingly sensitive to \(\alpha_n\). As \(v_{rel}\) decreases, \(C_D\) is more sensitive to \(v_{rel}\) because of the increased shear. Figure 17(c) shows the sensitivity of the ISS’s \(C_D\) to the yaw angle and the atmospheric translational temperature. The results are very similar to that of GRACE. Figure 17(d) shows the sensitivity of the ISS’s \(C_D\) to the pitch angle and the normal energy accommodation coefficient. The structure is generally similar to that of GRACE; however, there is not such a strong peak about a pitch angle of zero in the ISS data as there is in the GRACE data. As mentioned earlier, the ISS is asymmetric as a function of the pitch angle meaning that the symmetry in the drag coefficient occurs offset from zero. As the pitch angle departs from \(-0.5^\circ - 1.0^\circ\), the sensitivity to the pitch angle increases. Generally, the drag coefficient is more sensitive to the normal energy accommodation coefficient, especially when the normal energy accommodation coefficient is near unity.

Insert Figure 17

**Application of the Response Surface Model to Orbit Prediction**

Drag coefficient estimates from the developed response surface model for the GRACE satellite are compared with drag and ballistic coefficient estimates from previous work in the field. Comparisons are made to drag coefficient estimates by Sutton (2008) and the average fitted ballistic coefficient deduced by Bowman et al. (2008). Sutton (2008) uses a flat plate model for the GRACE satellite geometry and assumes a diffuse reflection with incomplete accommodation
model with a constant energy accommodation coefficient ($\alpha = 0.93$). However, for the purposes of this comparison, Sutton’s model uses the RSM with $\alpha = 0.93$. Bowman et al.’s. (2008) fitted ballistic coefficient for GRACE ($BC = 0.00687 \text{ m}^2/\text{kg}$) is derived using the High Accuracy Satellite Drag Model (HASDM) while averaging over a long time period (Storz et al., 2002). A fitted drag coefficient ($C_D = 3.52$) is derived from the Bowman et al. (2008) ballistic coefficient using a mass of 487 kg and a frontal area of 0.942 m$^2$. A flat plate model cuboid with the same normal-to-tangential area ratio as GRACE is also included in the comparison to test high-fidelity geometry effects.

Precision orbit ephemerides (POE) from the University Corporation for Atmospheric Research (UCAR) derived through orbit determination from GRACE GPS data are used to compare different drag coefficient models. GRACE is propagated between Aug. 28th – Aug. 31st, 2009 which is a period of low solar activity but active geomagnetic activity. The orbits are propagated for 96 hours and the norm of the total position error is calculated every 60 seconds.

Figure 18 compares the drag coefficients computed by the different models throughout GRACE’s orbit. Both the RSM and Sutton’s model have a periodic variation along each orbit due to the changing atmospheric properties. The drag coefficient derived from Bowman et al.’s. (2008) ballistic coefficient is constant and consistently smaller than the $C_D$ for both the RSM and Sutton’s (2008) model. The RSM drag coefficients are ~3% higher than those predicted by Sutton (2008). The difference in the profile per orbit is mainly due to the variation of the accommodation coefficient as a function of atomic oxygen as GRACE moves along its orbit compared with Sutton’s data that uses a constant value of accommodation. As a result, the densities derived using
the two drag coefficient models will not only be different on average by ~3% but will also have variations similar to $C_D$ along the orbit.

Figure 19 shows the norm of the total position error for orbits propagated using drag coefficients from the RSM, Sutton (2008), Bowman et al. (2008), and a flat plate cuboid model with respect to the POE data. The GRACE precision orbit data (POD) has been calculated based on the on-ground post-processing of GPS data that has an accuracy of a few centimeters and is considered truth for this work. The orbit of GRACE is propagated with an initial state vector using different $C_D$ models, HASDM for density and NRLMSISE-00 for mole fractions. The developed response surface models are agnostic to the atmospheric models. A comparison of mole fractions between the empirical NRLMSISE-00 (Picone et al., 2002) and physics based Global Ionosphere and Thermosphere Model (Ridley et al., 2006) and its effects on the drag coefficient has been previously performed by the authors in Walker et al., (2014). The propagated orbits are then compared with the POD orbit for position error.

The largest position errors over 96 hours are ~3.5 km (not shown) using Bowman et al.’s constant 3.52 drag coefficient. The next best model is the cuboid model which has a position error of ~1.3 km after 96 hours. Finally, the RSM and Sutton’s model perform the best. The RSM slightly outperforms Sutton’s assumption of a constant accommodation coefficient. The Sutton (2008) model position errors are ~50 m larger than the RSM after 96 hours. RSM errors are ~450 m and the Sutton (2008) model errors are ~500 m.

The errors on the order of 50 m may seem negligible; however, when the goal is high-fidelity modeling and when collision probabilities are assessed on the order of tens of meters, errors on the order of 50 m can make a significant difference. Also, the size of the relative errors is decreased due to the altitude of GRACE’s orbit (~500 km).
Performance Analysis

Performance for the model is measured in terms of computational intensity of developing and evaluating the RSM as well the accuracy of the estimated parameters compared with other methods. In the case of the GRACE satellite, a single TPMC solution takes approximately 4 minutes and 16 seconds to reach a solution with Monte Carlo noise on the order of 0.1%, while evaluating the RSM model takes 320 microseconds. The computational time for TPMC increases with complexity of the geometry (based on the number of mesh facets), while the RSM is insensitive to complexity of the geometry or higher dimensionality. Therefore, predictive applications require an empirical model.

Computational requirements for model development of RSMs are comparable, if not lower than developing other empirical models. As shown in this work, developing a RSM to within 1% accuracy requires approximately 500 TPMC simulations for each species for a highly complex geometry like the International Space Station. A gridded lookup table or a lookup table with linear interpolation would require 78,125 simulations for 5 intervals in 7 dimensions which is about ~25 times the number of simulations required for a RSM. For the above resolution, the accuracy of the gridded lookup tables is about 6% when using the nearest grid point and reduces to ~2% using linear interpolation. Achieving the same accuracy as the RSM would require far more intervals in each dimension and as the lookup table also suffers with higher dimensionality problems, the required number of TPMC simulations grows rapidly. In addition, a RSM requires only 500 kilobytes of memory storage whereas a lookup with 5 intervals in 7 dimensions requires about 8 megabytes. The storage also grows exponentially with number of intervals and dimensions.

Although building the Gaussian process can be computationally intensive, predictions
from the Gaussian process are competitive with almost any algorithm. For accuracy, Gaussian process prediction has been found to consistently outperform more conventional methods like high-dimensional interpolators for several data sets of varying complexity (Rasmussen, 1996; Ben-Air et al., 2007). In terms of computational complexity, Gaussian process predictions are also competitive. Equation (8) gives the predictor for a new point with inputs $X^*$. Assuming that we use plug-in estimators for all of the GP parameters (e.g. the mean of the MCMC samples), real-time predictions have the same complexity as any other interpolator, requiring a set of distance calculations and a simple dot product. The vector $\Sigma(X^*, X)$ requires the computation of a weighted distance between the new input and the training inputs, a computation that is linear in the size of the training set. Because the vector $\Sigma(X)^{-1} \tilde{y}$ is based on training data, it can be pre-computed and stored. The prediction is just the dot product of these two vectors, also linear in the size of training set.

**CONCLUSIONS**

A technique for modeling satellite drag coefficients in free molecular flow using response surface models has been successfully developed. Response surface models are developed and validated for the simple convex geometry of a sphere, where the closed-form solution is known, and then extended to the more complex and concave geometry of the GRACE satellite and the International Space Station as well as the Hubble Space Telescope with articulating satellites. Response surface models are developed for each of the six individual chemical species that comprise the majority of the Earth’s lower atmosphere. The individual response surface models are developed using Latin-Hypercube samples with 1000 ensemble members and the associated drag coefficients computed for sphere, GRACE, and Hubble Space Telescope and with 400 members for International Space Station using the Test Particle Monte Carlo method. GSIs are modeled using the Cercignani-Lampis-Lord Model. For the individual species response surface models developed for the sphere, the independent parameters are the atmospheric translational temperature, the
spacecraft relative velocity, the spacecraft surface temperature, the normal energy accommodation coefficient, and the tangential momentum accommodation coefficient. The independent parameters for the GRACE satellite and the International Space Station include, in addition to the parameters for the sphere, the attitude orientation angles of pitch and yaw. The independent parameter for the Hubble Space Telescope also includes the orientation angle of the articulating solar panels.

The individual response surface models for the sphere, GRACE, and the ISS are validated using test data sets composed of 100 ensemble members generated using the Latin-Hypercube sampling technique. A test data set with 1,000 samples was used for the Hubble Space Telescope. The response surface models for GRACE are also validated using a multi-species TPMC test data set and the associated total drag coefficients. Results show that the technique of modeling drag coefficients using response surfaces performs well with the worst-case root mean square percentage error of 1.029% for the International Space Station in pure hydrogen. The root mean square percentage error in the total drag coefficient for GRACE is less than 0.7.

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REFERENCES


Figure 1: Attitude orientation definition for the GRACE satellite.

Figure 2: Attitude orientation definition for the simplified Hubble Space Telescope.
Figure 3: Attitude orientation definition for the International Space Station.

Figure 4: Top Panel: Ten draws from a Gaussian process with a single input dimension. Each curve is realized at 100 evenly spaced points over the domain. Center Panel: Three observations along one draw of a Gaussian Process. Bottom Panel: Ten draws from a Gaussian process conditioned on the three observations shown in the Center Panel.
Figure 5: An example of Latin Hypercube sampling with 10 samples in 2 dimensions.

Figure 6: (a) A comparison of drag coefficients for a sphere in pure atomic oxygen estimated using the RSM (test set predictions) and computed with TPMC (test set). (b) A histogram of the residuals obtained for the O test sample.
Figure 7: (a) A comparison of drag coefficients for a sphere in pure helium, He, between the RSM (test set prediction) and TPMC (test set). (b) A histogram of the residuals obtained for the He test sample.

Figure 8: (a) A comparison of drag coefficients for GRACE in pure O between the RSM (test set predictions) and TPMC (test set). (b) A histogram of the residuals obtained for the O test sample.
Figure 9: (a) A comparison of drag coefficients estimated for GRACE in pure He between the RSM (test set predictions) and TPMC (test set). (b) A histogram of the residuals obtained for the He test sample.

Figure 10: A comparison of drag coefficients estimated for GRACE using the RSM developed for all species and those calculated using TPMC for the multi-species test set.
Figure 11: (a) A comparison of drag coefficients estimated for the simplified HST in pure O between the RSM (test set predictions) and TPMC (test set). (b) A histogram of the residuals obtained for the O test sample.

Figure 12: (a) A comparison of drag coefficients estimated for simplified HST in pure He between the RSM (test set predictions) and TPMC (test set). (b) A histogram of the residuals obtained for the He test sample.
Figure 13: (a) A comparison of drag coefficients estimated for ISS in pure O between the RSM (test set predictions) and TPMC (test set). (b) A histogram of the residuals obtained for the O test sample.

Figure 14: (a) A comparison of drag coefficients estimated for ISS in pure He between the RSM (test set predictions) and TPMC (test set). (b) A histogram of the residuals obtained for the He test sample.
Figure 15: Line contours of GRACE’s $C_D$ in He as a function of (a) the pitch and yaw angles, (b) the normal energy accommodation coefficient and the velocity of the spacecraft relative to the atmosphere, (c) the atmospheric translational temperature and the yaw angle, and (d) the normal energy accommodation coefficient and the pitch angle.
Figure 16: Line contours of HST’s $C_D$ in He as a function of (a) the pitch and yaw angles, (b) the normal energy accommodation coefficient and the velocity of the spacecraft relative to the atmosphere, (c) the atmospheric translational temperature and the yaw angle, (d) the normal energy accommodation coefficient and the pitch angle, and (e) the solar panel orientation angle and the normal energy accommodation coefficient.
Figure 17: Line contours of ISS’s $C_D$ in He as a function of (a) the pitch and yaw angles, (b) the normal energy accommodation coefficient and the velocity of the spacecraft relative to the atmosphere, (c) the atmospheric translational temperature and the yaw angle, and (d) the normal energy accommodation coefficient and the pitch angle.

Figure 18: Drag Coefficient Variation along GRACE’s Orbit
Figure 19: Propagated orbit total position errors for different drag and ballistic coefficient models.
Table 1: The Root Mean Square Error (RMSE) and Root Mean Square Percentage Error (RMSPE) for the Sphere, GRACE, HST, and ISS Response Surface Models

<table>
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<th>Sphere</th>
<th>GRACE</th>
<th>HST</th>
<th>ISS</th>
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<td>RMSPE</td>
<td>RMSE</td>
<td>RMSPE</td>
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Table 2: Nominal values for the different parameters.

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