AEM 4321 / EE 4231: HW#6

1. **DC Motor Control**

DC motors are used in many electro-mechanical systems. A model for a DC motor is:

$$J\dot{y} + by = cV \tag{1}$$

where y is the angular velocity of the motor shaft (deg/sec), and V is the input voltage (Volts). The model parameters are: J=Rotational Inertia (N m sec^2/deg^2), b=rotational damping (Nm sec/deg), and c=gain from input voltage to applied torque (Nm/Volts). For this problem we'll use the values $J = 3$, $b = 5$ and $c = 12$.

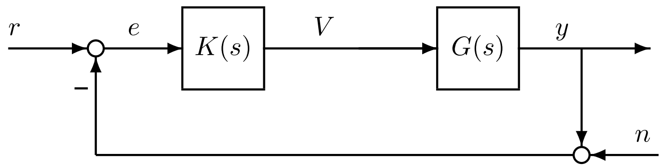


Figure 1: DC Motor Feedback System

Consider the feedback loop in the figure above where $G(s)$ is the transfer function for the DC motor and $K(s)$ is the controller transfer function. The specifications are to design a controller so that the closed loop:

i. is stable,

ii. has a loop crossover frequency near 20 rad/sec

iii. has steady state error $|e_{ss}| < 0.01$ deg/sec when r is a unit step command, i.e. when $r(t) = 1$ deg/sec and $n(t) = 0$ deg/sec for all $t \geq 0$,

iv. has gain less than 0.04 from input n to output y for frequencies above 200 rad/sec.

Perform the following steps:

(a) Translate specifications iii. and iv. into requirements on the loop gain $|L(j\omega)|$.

(b) Design a proportional control law $K(s) = K_p$ to satisfy requirements i. and iii. only. Plot $G(s)$ and $L(s) = G(s)K(s)$ on the same Bode plot using the `Matlab` command `bode`. Also, simulate the closed-loop system with $r(t) = 1$ deg/sec and $n(t) = 0.1\sin(200t)$. Plot the motor speed response $y(t)$ and the reference command $r(t)$ on the same plot. Hand in both your Bode plot and your step response plot. For your choice of K_p, what is the loop cross-over frequency and what is the value of $|L(j200)|$?

Note: An m-file and `Simulink` diagram have been posted with this homework. You only need to enter your proportional gain and the m-file will generate both the Bode plot and the step response plot.

(c) Next, design a proportional control law $K(s) = K_p$ to satisfy requirements i. and iv. only. Generate and hand-in the Bode plot and step response plots as described in the part (b). For your choice of K_p, what is the loop cross-over frequency and what is the value of $|L(j0)|$?

(d) At this point is should be clear that proportional control will not be able to satisfy the design objectives. Use the loop-shaping procedure described in class to design a controller that satisfies objectives ii., iii., and iv. Verify that the closed-loop is stable with your final control design. Generate and hand-in the Bode plot and step response plots as described in the part (b). Your controller will be specified as a transfer function and you'll need to modify the m-file and `Simulink` diagram.

Recommendation: Use a proportional gain to set the loop cross-over frequency (requirement ii.). Then use a low frequency boost and roll-off to satisfy requirements iii. and iv., respectively. This may require some iteration to get values that meet all design specifications.

(e) What is the ordinary differential equation that models the input-output dynamics of the control law designed in part (d)?

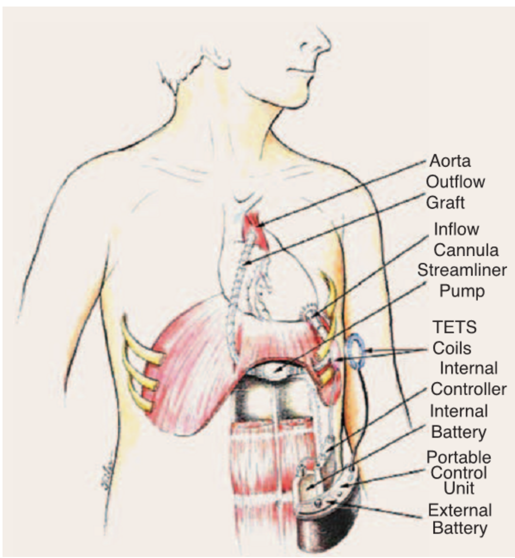
2. **Control of An Artificial Heart**

Heart transplants could save thousands of lives but there is a limited supply of donor hearts. There have been significant efforts to design mechanical artificial hearts as an alternative. In this problem you'll analyze a controller for a left ventricular assist device (LVAD) described in the references:

(1) "Streamliner Artificial Heart," by Antaki and Paden, IEEE Control Systems Magazine, 2002, vol. 2, no. 6, p 8 - 12.

(2) "Maglev Apparatus for Power Minimization and Control of Artificial Hearts," by Samiappan, Mirnateghi, Paden, and Antaki, IEEE Trans. on Control Systems Technology, 2008, vol. 16, no. 1, p.13 - 18.

The Streamliner system, shown in the left figure below, is an early generation LVAD. The pump, shown in the right figure below, draws blood from the left ventricle and pumps blood into the aorta. The Streamliner has an internal, rotating impeller that drives the fluid. One key issue is that damage of the red blood cells during pumping can cause clotting which may lead to a stroke or organ damage. The Streamliner pump magnetically levitates the internal impeller of the pump to ensure sufficient gaps for the blood cells to flow without damage.



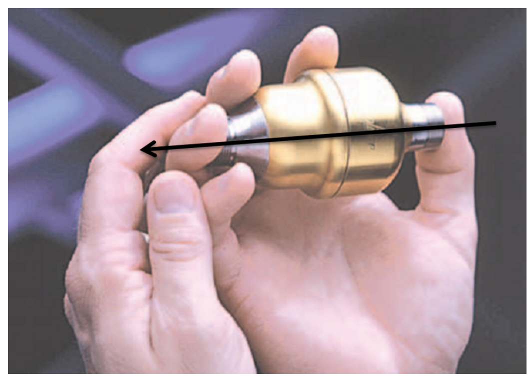


Figure 2: Streamliner system (left) and pump (right) [Pictures from Reference (1)]

This problem will focus on the axial motion of the pump impeller. Specifically, the levitated impeller can move along the axial direction denoted by the black arrow in the right figure above. A voice coil actuator is used to control the axial motion. The transfer function from the actuator voltage u (Volt) to axial position of the impeller x (m) is:

$$G(s) = \frac{5.737}{0.3573s^2 - 30200} \tag{2}$$

The model parameters are for an experimental testbed to study the axial control of levitated impellers. The model is a linearization of a nonlinear model as described in Reference (2). Notice that the dynamics are unstable in the axial direction. The feedback system to stabilize the impeller is shown in Figure 3. The reference command is set to $r = 0$ and the remainder of the problem will focus on the axial disturbance d.

(a) It is important to design a controller that minimizes power consumption. This leads to the following design constraint: If $d(t) = 1$ (V) for all $t \geq 0$ then the steady state control input should satisfy $|u_{ss}| \leq b$. Convert this into a design specification on one of the closed-loop transfer functions. Next, translate this into a design specification on the loop transfer function $L(s) = G(s)K(s)$.

(b) What constraint is placed on the controller $K(s)$ in order to satisfy the design specification for b=0.1, 0.01, and 0.001? What can you say about the tracking error due to the unit step disturbance as specification bound b gets smaller?

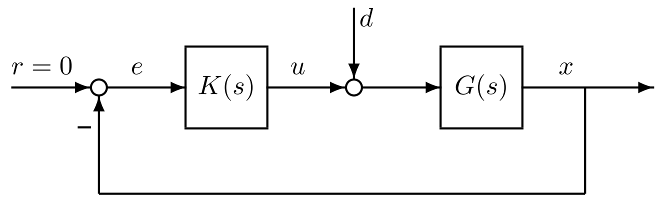


Figure 3: Impeller Control System

(c) Based on part b), what property should $K(s)$ have to ensure zero steady-state control input due to a step input disturbance? The property you will derive leads to a "Virtual Zero Power" (VZP) control patented by J. Lyman in 1975 for levitated bearings.

(d) Reference 2) proposes the following control law:

$$K(s) = \frac{100s^2 + 15000s}{s - 20} \tag{3}$$

Notice that both the plant $G(s)$ and this controller $K(s)$ are unstable. Is the closed-loop stable?

(e) $G(s)$ is a model of the pump and the actual dynamics of the real system will be slightly different, e.g. variations in the impeller mass. We need the controller to stabilize the true system even if the dynamics are slightly different than the model $G(s)$. As a simple analysis, we'll study the effect of variations in the numerator coefficient of $G(s)$. Vary the numerator coefficient from its nominal value of $n_0 = 5.737$ and simulate the closed-loop system with a step disturbance. From your simulations estimate the largest interval $(\underline{n}, \bar{n})$ containing n_0 such that the closed-loop remains stable.

(f) Another important effect is that the control law is implemented on an embededded processor. This is the "Portable Control Unit" located on the human hip in the Streamliner system diagram. The embedded processor takes a certain amount of time to perform the control law calculations. This computation time can be modeled as a pure time delay τ_d at the output of the controller $K(s)$. Simulate the closed-loop system with a step disturbance. From your simulations estimate the largest interval $(0, \bar{\tau}_d)$ such that the closed-loop remains stable.

Hint: If you are using `Simulink` to perform your simulations then you can model the delay with the "Transport Delay" block in the "Continuous" folder If you are using the `step` command to perform your simulations then you can model the delay with the `OutputDelay` property of the control law transfer function.