AEM 4321 / EE 4231: HW#5

1. **Pole/Zero Cancellations**

   We used a linearized model for the attitude dynamics of a rocket in the previous homework. The dynamics are approximately given by:

$$\ddot{\theta}(t) - 0.09\ \theta(t) = 6\ \delta(t) \tag{1}$$

   The coefficients have been slightly modified to simplify the numerical results in this problem. Again, note that $\delta(t)$ denotes the thrust angle control input and this is not an impulse function.

   (a) What is the transfer function $G(s)$ for the rocket dynamics in Equation 1?

   (b) Consider the feedback system shown below in Figure 1. d denotes a disturbance acting on the rocket. Assume the control law is defined by $K(s) = \frac{s-0.3}{s+4}$. Notice that this control law perfectly cancels the unstable pole of the plant. Derive the closed-loop transfer function from r to θ. Remove any pole/zero cancellations. Is this transfer function stable?

   (c) Derive the closed-loop transfer function from d to θ. Remove any pole/zero cancellations. Is this transfer function stable?

   (d) Simulate the system with $r(t) = 0$ and $d(t) = 0.1$ for all $t \geq 0$. The `Transfer Fcn` block in the `Continuous` folder can be used to quickly construct the `Simulink` diagram of this feedback system. Hand in plots of δ vs. time and θ vs. time.

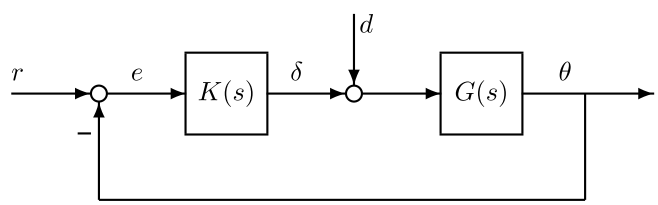

Figure 1: Feedback Loop

2. **Disk Drive Control**

   Figure 2 shows the basic components of a disk drive. The data is stored in concencentric circles on circular stacked disks (also called platters). The disks are coated with a magnetic material that allows data to be stored and retrieved by a read/write head. The read/write head is moved to a specific location in memory through a combination of spinning the disks about the spindle and moving the actuator arm of the read/write head about the actuator axis. θ denotes the angle of rotation of the acutator arm about the actuator axis (in rads). This actuator arm is driven by a motor located under the metal plate in the diagram.

   Disk drive memory is increased by closely spacing the memory locations on the disks. The width of the memory location on modern drives can be on the order of $10^{-7}m$. This requires very precise control of the actuator arm. This problem will focus on the effect of model uncertainty on the stability of the closed-loop system. The following survey paper has additional details on disk drive control:

   - D. Abramovitch and G. Franklin, "A brief history of disk drive control," IEEE Control Systems Magazine, p.28-42, June 2002.

   A simple model for the motion of the actuator arm is

$$G(s) = \frac{10^6}{s^2 + 12.5s} \tag{2}$$

   This is the transfer function from the drive motor voltage u (input) to the actuator arm angle θ (output). This model assumes the actuator arm and drive motor shaft are rigid.

Consider the feedback system shown in Figure 1 where r is the desired arm angle and u is the input voltage to the drive motor. The dynamics of the control law are described by the following transfer function:

$$K(s) = \frac{s + 1000}{0.1s + 1000} \tag{3}$$

This control law is known as a "lead" controller. It is similar to PD control and we will learn how to design this type of controller later in the course.

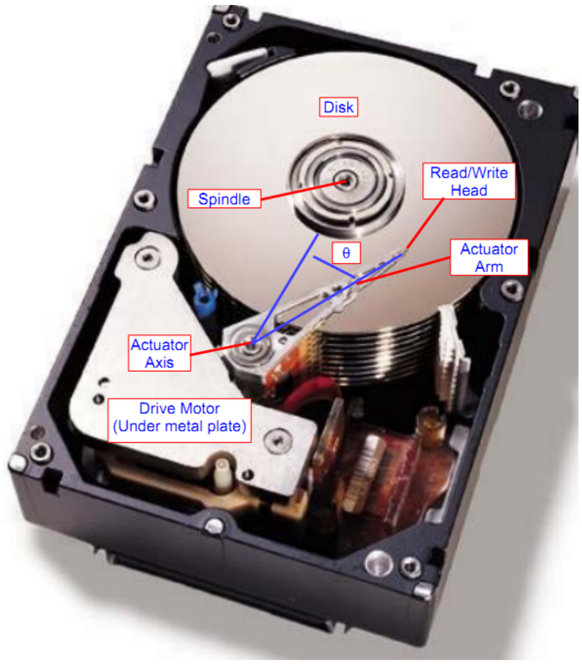

Figure 2: Disk Drive Components

(a) What is the closed-loop transfer function from r to θ? Compute the poles and zeros of the closed-loop system. Is the system stable? Feel free to use the following Matlab functions to perform the calculations: `tf`, `feedback`, `pole`, and `zero`.

(b) The closed-loop system has a "fast" pole and a slower complex pair of poles. Use the natural frequency and damping ratio of the "slow" complex pair to sketch the approximate response $\theta(t)$ due to a unit step reference command. Compare your sketch with the actual response computed with the `step` command. Comment on the effect of the closed-loop zero.

(c) In actuality the actuator arm and drive shaft motor are not rigid. There is some flexibility in these mechanical parts. The details of this flexibilty are complex and typically result in several resonant modes. A model that includes the first resonant mode of the flexible dynamics is:

$$G_{flex}(s) = G(s)\frac{2\zeta_1\omega_1 s + \omega_1^2}{s^2 + 2\zeta_1\omega_1 s + \omega_1^2} \tag{4}$$

where $\zeta_1 = 0.05$ and $\omega_1 = 440$rad/sec. Note that G_{flex} is the serial interconnection of the rigid body model $G(s)$ and a lightly damped second order system that models the flexibility. What are the poles of the closed-loop system if we replace the rigid body model $G(s)$ with the more accurate model $G_{flex}(s)$? Is the closed-loop system still stable?

3. **Bode Plots**

Sketch the Bode plots (magnitude and phase) for the following systems:

(a) $2\dot{x} + 0.6x = \dot{u} + 30u$

(b) $2\dot{x} + 0.6x = -\dot{u} + 30u$

(c) $\ddot{x} + 0.2\dot{x} + 4.01x = -u$

4. **System Identification**

Earlier in the course we discussed ODE models developed from basic physical laws, e.g. Newton's laws, Kirchoff's voltage and current laws, etc. We briefly mentioned that an alternative is to construct an ODE model from experimental data. This is known as system identification. In this problem you'll use your knowledge of frequency reponses to construct an ODE model for a system from input-output data. This technique is used to construct models in many different applications.

The `Simulink` model `IdentModel.mdl` and file `IdentSystem.p` were posted with this homework. The `p`-file must be in the directory to simulate the model.

(a) Simulate the system with sine wave inputs of many different frequencies. Record the output for each simulated input frequency. Use these results to sketch the Bode plot (magnitude and phase) for the system.

(b) Derive the system transfer function based on the Bode plot generated in the previous part. What is the ODE that models the system dynamics?