A slender rod $AB$ of mass $m$ and length $l$ is hinged at its end $A$ to a square plate $CDEF$ with sides of length $S$ as shown in Figure 1. The hinge at $A$ is designed in such a way that the rod does not rest on the plate but swings in a plane just above the plate (see Figure 2). That is, all the contact forces between the rod and plate are carried by the hinge at $A$. A torsional spring of stiffness $k$ that is unstretched when $\phi = 0$ is attached to the rod at $A$. Corners $C$ and $D$ of the plate move along a circular horizontal track of radius $R$ ($R >> S$) at a known constant angular rate of $\Omega$. The inclination of the plate with respect to the horizontal is a constant angle $\theta$.

1. Using the Lagrangian approach, write a differential equation for the angle $\phi$ that describes the rod’s swinging motion on the plate.

2. Using the Newtonian approach, write a differential equation for the angle $\phi$ that describes the rod’s swinging motion on the plate.

3. Determine the reaction forces at $A$.

4. Find the equilibrium values of $\phi$ (Assume that at $t = 0$, the deflection of $\phi$ is equal to a very small angle $\epsilon$ in the direction shown in Figure 1).

5. Determine the frequency of small oscillations in $\phi$ in the case these are possible.
Figure 2: Side View of Rod and Plate Assembly.