A massless ring of radius $R$ rotates at a known constant angular speed $\Omega$. A collar of mass $m$ slides on the ring as shown in Figure 1. The coefficients of static and kinetic friction between the ring and collar are $\mu_s$ and $\mu_k$, respectively. The friction forces depend on the normal force in the $\hat{b}_3$ direction only. Initially $\Omega = 0$ and the collar is at rest on the collar stop at an angle of $-\theta_0$. Suddenly, the ring accelerates and achieves an angular velocity of $\Omega$ instantaneously. This problem deals with the motion of the collar after the ring achieves an angular speed of $\Omega$. 

Figure 1: A collar on a rotating frame.
The $\hat{a}_1$-$\hat{a}_2$-$\hat{a}_3$ frame is attached to the ring with origin at point $O$. The $\hat{b}_1$-$\hat{b}_2$-$\hat{b}_3$ system also has its origin at point $O$ but is attached to the collar and rotates with it. **Note that $\theta$ is measured starting from the $\hat{a}_1$ axis.**

1. Determine the equations of motion using the Newtonian approach and derive an expression for the reaction forces between the ring and collar in the $\hat{b}_1$, $\hat{b}_2$ and $\hat{b}_3$ directions.

2. Verify the equations of motion for the collar using the Lagrangian (analytical) approach. Determine any required forces using constraint-relaxation.

3. What is the value of $\Omega$ for which the collar will just start moving away from the collar stop?

4. For given values of $\theta$ and $\dot{\theta}$, determine the external moment $M$ that would need to be applied to the ring’s vertical shaft in order to keep $\Omega$ constant.

5. Find the values of $\theta$ for which the collar can stay at equilibrium with respect to the ring in the range $[-\theta_0, \pi/2]$. Assume $\Omega$ is sufficiently large.

6. Derive an equation for small motions $\delta \theta(t)$ around one of the points you identified in part(5).