1. Consider a two-dimensional piston pushed at constant speed $U$ within a long closed rectangular channel. One end of the channel is shown below. (Assume that the channel and piston have infinite depth perpendicular to the page). The channel is filled with a viscous, incompressible fluid with density $\rho$ and viscosity $\mu$. The channel width is $D$. The piston width and length are $d$ and $L$ respectively. The gap between the piston and the channel is the same on both sides and has magnitude $h$. Note that $h \ll d$ and $h \ll L$. Also, the pressure upstream ($P_2$) and downstream ($P_1$) of the piston can be assumed uniform.

a. Give a brief qualitative description of key features in the flow. What are the main forces on the piston?

Use the governing equations to answer parts b-d below. Make sure to state any additional assumptions you need.

b. Find an expression for the velocity profile across the gap, and show a plot of your result.

c. Find an expression for the shear stress across the gap, and show in a plot.

d. Find the force per unit depth required to drive the piston at the constant speed $U$. Express the force in terms of the fluid material properties, $U$, and the geometric quantities given.

e. Keeping the problem geometry fixed, under what conditions might the assumptions stated explicitly above or made by you break down? Discuss how the flow might differ from what you found above.
2. A wedge with a 90° corner projects out from a bounding wall into a channel as shown below. Initially, there is no flow. Then, a large piston is started which drives a flow in the channel from left to right as shown. Then, the piston is stopped. Given this scenario, describe the flow in the vicinity of the wedge. Use equations wherever helpful to justify your reasoning.