**Problem 1.**

(i) Consider incompressible flow, i.e. $\nabla \cdot \vec{u} = 0$, in a streamtube. Invoke the divergence theorem to show that the volume flow rate through any cross-section is constant.

(ii) Next, consider a vortex tube. Use similar arguments as above to show that the circulation around any cross-section is constant.
Problem 2.

An axisymmetric Venturi nozzle (the cross section is round) is shown below. A steady uniform, flow enters the nozzle as shown. The flow is incompressible.

![Diagram of a Venturi nozzle with labeled sections A, B, C, D, E and velocity profile u.]

(a). First assume that the flow is inviscid. Make careful qualitative plots of the pressure vs. x for the center streamline and for a streamline close to the upper surface. Justify the curves in your plots using appropriate equations. Carefully state any assumptions that you might need to make your justification.

(b). Now assume that viscosity is present in the fluid. You do not need to solve any equations, but please justify your responses to the following questions using appropriate equations.

i. Sketch velocity profiles at locations A, B, C, D, E.

ii. What is the vorticity distribution across the flow at locations C,D, E?

iii. How will the wall shear stress vary at locations A, B, C, D, E?

iv. Are there any locations where the flow might separate? If not, why not? If so, where and why?

(c). Assume that your answers in (b) corresponded with a water flow. Assume that the inlet velocity U remains constant but the fluid is now air. The flow is still incompressible. How do your plots and answers in (b) change and why?