A slender ring of mass $2m$ and radius $R$ is supported at point $O$ by a frictionless pin. The small bead of mass $m$ slides without friction along an inextensible massless cord of length $2a$ which is attached to the ring at points $A$ and $B$, as shown. If one assumes that the cord remains taut at all times, the sum of the distances to the bead from points $A$ and $B$, respectively, is constant. This means that the bead moves along an elliptic path, in the reference frame of the ring, with foci at $A$ and $B$. Therefore the following constraint is satisfied:

$$r = \frac{a(1-e^2)}{1-e \cos \theta}$$

where $e=R/a$ (equation for the ellipse). 

a) How many degrees of freedom does the system have? 
b) Write the equations of motion. 
c) Verify that the position where the bead is directly under $O$, and $\phi=0$, is an equilibrium position. 
d) Denoting the equilibrium value for $\theta$ by $\theta_e$, linearize the equations of motion about the equilibrium position.