Towards LES of High Reynolds Number External Flows on Unstructured Grids

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ABSTRACT

Large eddy simulation (LES) using the dynamic Smagorinsky model (DSM) (Germano et al., 1991) and discretely kinetic energy conserving numerical methods (Mahesh et al., 2004) has successfully predicted complex flows such as gas turbine combustors and marine propulsor crashback (e.g. Výšohlil and Mahesh (2005); Chang et al. (2008); Jang and Mahesh (2010, 2012); Verma et al. (2012)). This paper discusses two developments towards reliably using LES for inhomogeneous and attached flows: (1) a dynamic Lagrangian model where a dynamic procedure is proposed for the Lagrangian timescale and (2) a wall model where in addition to the Germano-identity error, external Reynolds stress is also imposed as a constraint on the ensemble-average subgrid-scale stress. Both developments are for unstructured grids.

INTRODUCTION

Many practical flows of engineering interest such as flow past a submarine, ship wakes, etc. are high Reynolds number (Re) flows. It has been estimated that the grid requirement for a Direct Numerical Simulation (DNS) scales with the Reynolds number as \( Re_{x}^{9/4} \). High Reynolds number flows exhibit such a large range of length and time scales that DNS are rendered impossible for the foreseeable future. Large eddy simulation (LES) is a viable analysis and design tool for complex flows due to advances in massive parallel computers and numerical techniques. LES is essentially an under-resolved turbulence simulation using a model for the subgrid-scale (SGS) stress to account for the inter-scale interaction between the resolved and the unresolved scales. The success of LES relies on the dominance of the large, geometry dependent, resolved scales in determining important flow dynamics and statistics.

In LES, the large scales are directly accounted for by the spatially filtered N-S equations and the small scales are modeled. The spatially filtered incompressible Navier-Stokes equations are

\[
\frac{\partial \hat{\tau}_{ij}}{\partial t} = \frac{\partial}{\partial x_i} (\hat{u}_i \hat{\tau}_{ij}) - \frac{\partial}{\partial x_j} (\hat{u}_j \hat{\tau}_{ij}) - \frac{\partial \hat{\tau}_{ij}}{\partial x_j} - \frac{\partial \hat{\tau}_{ij}}{\partial x_j} - \frac{\partial \hat{\tau}_{ij}}{\partial x_j} \tag{1}
\]

\[
\hat{\tau}_{ij} = \hat{\tau}_{ij} - \frac{1}{3} \hat{\tau}_{kk} \delta_{ij} = -2(C_s \Delta)^2 |\hat{S}| \hat{S}_{ij} - 2\nu_t |\hat{S}| \tag{2}
\]

where \( C_s \) is a global, adjustable, model coefficient, \( \Delta \) is the filter width, \( |\hat{S}| \) is the strain rate tensor, \( |\hat{S}| = (2\hat{S}_{ij}\hat{S}_{ij})^{1/2} \) and \( \nu_t = (C_s \Delta)^2 |\hat{S}| \) is the eddy viscosity.

The Dynamic Smagorinsky model (DSM) (Germano et al., 1991) is a widely used model. It is based on the Germano identity and dynamically computes the model coefficient from the resolved flow and allows it to vary in space and time. The dynamic procedure to obtain the SGS model coefficient \( C_s \) attempts to minimize the Germano-identity error (GIE),

\[
\epsilon_{ij} = T_{ij}^d - \hat{\tau}_{ij}^d - L_{ij}^d
\]

\[
= 2(C_s \Delta)^2 \left[ |\hat{S}| \hat{S}_{ij} - \left( \frac{\hat{\Delta}}{\Delta} \right)^2 |\hat{S}| \hat{S}_{ij} \right] - L_{ij}^d \tag{3}
\]

\[
= (C_s \Delta)^2 |\hat{S}| \hat{S}_{ij} - L_{ij}^d,
\]

where \( \hat{\Delta} \) denotes test filtering at scale \( \Delta \) and is usually taken to be \( \hat{\Delta} = 2\Delta \), deviatoric parts (denoted by \((^d)\)) of \( \tau_{ij} \) and \( T_{ij} \) are modeled by using...
the Smagorinsky model at scales $\Delta$ and $\hat{\Delta}$, $M_{ij} = 2 \left[ \langle S \rangle_{ij} - \left( \frac{\hat{\Delta}}{\Delta} \right)^2 \langle S \rangle_{ij} \right]$, and $L_{ij} = \bar{u}_i \bar{u}_j - \bar{u}_i \bar{u}_j$.

Since $\epsilon_{ij}(C_s) = 0$ is a tensor equation, $C_s$ is overdetermined. The original DSM due to Germano et al. (1991) satisfies $\epsilon_{ij}S_{ij} = 0$ to obtain $C_s$. Lilly (1992) found the equations to be better behaved when minimizing $\epsilon_{ij}$ in a least-square sense, yielding

$$(C_s \Delta)^2 = \frac{\langle L_{ij} M_{ij} \rangle}{\langle M_{ij} M_{ij} \rangle}$$

where $\langle \cdot \rangle$ denotes averaging over homogeneous direction(s) and was required to regularize $C_s$.

However the requirement of averaging over at least one homogeneous direction is impractical for complex inhomogeneous flows. To enable averaging in inhomogeneous flows, Meneveau et al. (1996) developed a Lagrangian version of DSM (LDSM) where $C_s$ is averaged along fluid trajectories. Lagrangian averaging is physically appealing considering the Lagrangian nature of the turbulence energy cascade (Meneveau and Lund, 1994; Choi et al., 2004).

In essence, the Lagrangian DSM attempts to minimize the pathline average of the local GIE squared. The objective function to be minimized is given by

$$E = \int \text{pathline} \epsilon_{ij}(z) \epsilon_{ij}(z) dz = \int_{-\infty}^{t} \epsilon_{ij}(z(t'), t') \epsilon_{ij}(z(t'), t') W(t - t') dt'$$

where $z$ is the trajectory of a fluid particle for earlier times $t' < t$ and $W$ is a weighting function to control the relative importance of events near time $t$, with those at earlier times.

Choosing the time weighting function of the form $W(t - t') = T^{-1} e^{-(t - t')/\tau}$ yields two transport equations for the Lagrangian average of the tensor products $L_{ij} M_{ij}$ and $M_{ij} M_{ij}$ as $I_{LM}$ and $I_{MM}$ respectively:

$$\frac{DI_{LM}}{Dt} = \frac{\partial I_{LM}}{\partial t} + \bar{u}_i \frac{\partial I_{LM}}{\partial x_i} = \frac{1}{T} (L_{ij} M_{ij} - I_{LM}),$$

$$\frac{DI_{MM}}{Dt} = \frac{\partial I_{MM}}{\partial t} + \bar{u}_i \frac{\partial I_{MM}}{\partial x_i} = \frac{1}{T} (M_{ij} M_{ij} - I_{MM}),$$

whose solutions yield

$$(C_s \Delta)^2 = \frac{I_{LM}}{I_{MM}}.$$  

Here $T$ is a time scale which represents the ‘memory’ of the Lagrangian averaging. Meneveau et al. (1996) proposed the following time scale:

$$T = \theta \Delta (I_{LM} I_{MM})^{-1/2}; \quad \theta = 1.5. \quad (8)$$

This procedure for Lagrangian averaging has also been extended to the scale-similar model by Anderson and Meneveau (1999); Sarghini et al. (1999) and the scale-dependent dynamic model by Stoll and Porté-Agel (2006).

**DYNAMIC LAGRANGIAN TIME SCALE**

The time scale for Lagrangian averaging proposed by Meneveau et al. (1996) (henceforth, $T_{LDSM}$) contains an adjustable parameter which is typically chosen to be $\theta = 1.5$. This value was chosen based on the autocorrelation of $L_{ij} M_{ij}$ and $M_{ij} M_{ij}$ from DNS of forced isotropic turbulence. This arbitrariness is acknowledged to be undesirable by the authors and infact they document results of turbulent channel flow at $Re_\tau = 650$ to be marginally sensitive to the value of $\theta$, with $\theta = 1.5$ appearing to yield the best results. You et al. (2007) tested three different values of the relaxation factor $\theta$ and concluded $T_{LDSM}$ was ‘reasonably robust’ to the choice of $\theta$ for a $Re_\tau = 180$ channel flow. Over the years, choosing a value for $\theta$ has demanded significant consideration by many practitioners who have found the results to be sensitive to $\theta$, especially in complex flows (Inagaki et al., 2002).

The extension of the Lagrangian averaging procedure to other models has also presented the same dilemma. In simulations of turbulent channel flow at $Re_\tau = 1050$ using a two-coefficient Lagrangian mixed model, Anderson and Meneveau (1999) and Sarghini et al. (1999) note that a different parameter in $T_{LDSM}$ might be required for averaging the scale similar terms. Vasilyev et al. (2008) proposed extensions to the Lagrangian dynamic model for a wavelet based approach and used $\theta = 0.75$ for incompressible isotropic turbulence.

Park and Mahesh (2009) note that $T_{LDSM}$ has a high dependence on the strain rate through the $L_{ij}$ and $M_{ij}$ terms. They however show that the time scale of the GIE near the wall and the channel centerline are similar. Thereby they argue that strain rate may not be the most appropriate quantity for defining a time scale for Lagrangian averaging of the GIE. It seems only natural that the averaging time scale should be the time scale of the quantity being averaged which in this case is the GIE. Park and Mahesh (2009) therefore, proposed a dynamic time scale $T_{\text{SC}}$, called “surrogate-correlation based time scale” $T_{\text{SC}}$. 

2
However the Park and Mahesh (2009) formulation was in the context of a spectral structured solver, and considered their dynamic Lagrangian time scale model along with their proposed control-based Corrected DSM. The present work considers the dynamic Lagrangian time scale model in the absence of control-based corrections. This procedure for computing a dynamic Lagrangian time scale is extended to an unstructured grid framework.

**Surrogate-correlation based time scale**

Let us assuming knowledge of the local and instantaneous values of the GIE squared ($E = \epsilon_{ij} \epsilon_{ij}$) at five consecutive events along a pathline:

$$E^0(x,t), \quad E^{\pm 1}(x \pm u \Delta t, t \pm \Delta t), \quad E^{\pm 2}(x \pm 2u \Delta t, t \pm 2\Delta t).$$

(9)

At each location, the following surrogate Lagrangian correlations for three separation times ($0, \Delta t, 2\Delta t$) can be defined by computing a running time average up to the current time $t_n$:

$$C(l \Delta t) = \sum_{t=0}^{t_n} \left( \frac{1}{5} \sum_{k=-2}^{2-l} (E^{k,t} - E^t)(E^{k+l,t} - E^t) \right)$$

where $E^t = \sum_{\tau=0}^{t_n} \left( \frac{1}{5} \sum_{k=-2}^{2} E^{k,\tau} \right)$

(10)

This leads to converged correlations after sufficiently long times and is a consistent and general method to compute the surrogate Lagrangian correlations. These correlations are then normalized by the zero-separation correlation $C(0)$ to obtain

$$\rho(0) = 1, \quad \rho(\Delta t) = \frac{C(\Delta t)}{C(0)}, \quad \rho(2\Delta t) = \frac{C(2\Delta t)}{C(0)}.$$

(11)

An osculating parabola can be constructed passing through these three points and it can be described by

$$\rho(\delta t) = a(\delta t)^2 + b(\delta t) + 1$$

(12)

where $a, b$ can be written in terms of $\rho(0) = 1, \rho(\Delta t), \rho(2\Delta t)$ and $\Delta t$. Note that $\rho(\delta t)$ is an approximate correlation function (of separation time $\delta t$) for the true Lagrangian correlation. Thus the time scale based on the surrogate correlation $T_{SC}$ is defined as the time when $\rho(\delta t) = 0$ i.e. the positive solution

$$T_{SC} = \frac{-b - \sqrt{b^2 - 4a}}{2a}$$

(13)

If the surrogate Lagrangian correlations $C$ have enough samples, $1 > \rho(\Delta t) > \rho(2\Delta t)$ is satisfied which leads to $a < 0$. As a result $T_{SC}$ is always positive. In the initial stages of a simulation, there are not enough time samples. $1 > \rho(\Delta t) > \rho(2\Delta t)$ may not be satisfied and $a$ could be positive. In such cases, $T_{SC}$ is obtained by constructing the osculating parabola to be of the form $1 + a(\delta t)^2$ and passing through either of the two points $\rho(\Delta t), \rho(2\Delta t)$:

$$T_{SC} = \min \left( \frac{dt}{\sqrt{1 - \rho(\Delta t)}}, \frac{2dt}{\sqrt{1 - \rho(2\Delta t)}} \right)$$

(14)

The minimum of the time scales is chosen so that the solution has lesser dependence on past values and can evolve faster from the initial transient stage.

**Lagrangian approximation**

The proposed dynamic time scale requires the values of the Germano-identity error (GIE) squared $E$ at five events along a pathline. Rovelstad et al. (1994) suggest the use of Hermite interpolation. Meneveau et al. (1996) use multilinear interpolation to obtain the values of $I_{LM}$ and $I_{MM}$ at a Lagrangian location. Both Hermite and multilinear interpolation get expensive in an unstructured grid setting. The use of an expensive interpolation method just to compute the time scale for Lagrangian averaging may be unnecessary. As a result, a simple material derivative relation as proposed by Park and Mahesh (2009) is used to approximate Lagrangian quantities in an Eulerian framework:

$$\frac{DE}{Dt} = \frac{\partial E}{\partial t} + \vec{w} \cdot \frac{\partial E}{\partial x_i}$$

(15)

A simple first order in time and central second order in space, finite-volume approximation for the convective term is used to approximate values of $E$ in eqn. 9 in terms of the local $E(x,t) = E^{0,n}$ and $E(x,t - \Delta t) = E^{0,n-1}$. The Green-Gauss theorem is used to express the convective term in conservative form and evaluate it as a sum over the faces of a computational volume.

**NUMERICAL METHOD**

Eq. 1 is solved by a numerical method developed by Mahesh et al. (2004) for incompressible flows on unstructured grids. The algorithm is derived to be robust without numerical dissipation. It is a finite volume method where the Cartesian velocities and pressure are stored at the centroids of the cells and the face normal velocities are stored independently at the centroids of the faces. A predictor-corrector approach is used. The predicted velocities
at the control volume centroids are first obtained and then interpolated to obtain the face normal velocities. The predicted face normal velocity is projected so that the continuity equation in eq. 1 is discretely satisfied. This yields a Poisson equation for pressure which is solved iteratively using a multi-grid approach. The pressure field is used to update the Cartesian control volume velocities using a least-squares formulation. Time advancement is performed using an implicit Crank-Nicolson scheme. The algorithm has been validated for a variety of problems over a range of Reynolds numbers (Mahesh et al., 2004). To improve results on skewed grids, the viscous terms and the pressure Poisson equation are treated differently. The Generalized Improved Deferred Correction method by Jang (2011) is used to calculate the viscous derivatives and the right-hand side of the pressure Poisson equation.

RESULTS: LAGRANGIAN DSM

The performance of the Lagrangian DSM with dynamic time scale $T_{SC}$ (eq. 13) is evaluated by applying it to problems of increasing complexity: turbulent channel flow, cylinder flow, and flow past a marine propulsor attached to an upstream hull, operating in an off-design condition.

Turbulent channel flow

LES of a turbulent channel flow is performed at three Reynolds numbers; $Re_{\tau} = 590, 1000, 2000$ and different grid resolutions. Here $Re_{\tau} = u_{\tau}\delta/\nu$ where $u_{\tau}$ denotes friction velocity, $\delta$ channel half-width and $\nu$ viscosity. Table 1 lists the $Re_{\tau}$ and grid distribution for the various runs. All LES cases have a domain of $2\pi\delta \times 25 \times 2\pi\delta$ and a uniform spacing in $x$. The cases with ‘tl’ indicate that a 4 : 2 transition layer has been used in $z$ along $y$. A transition layer allows transition between two fixed edge ratio computational elements. It allows a finer wall spacing to coarsen to a fixed ratio coarser outer region spacing. All other cases have a uniform spacing in $z$. Our LES results are compared to the DNS of Moser et al. (1999) for $Re_{\tau} = 590$, del Alamo et al. (2004) for $Re_{\tau} = 1000$, and Hoyas and Jimenez (2006) for $Re_{\tau} = 2000$ whose grid parameters are also included in the table for comparison. Note that the LES have employed noticeably coarse resolutions.

It will be shown that Lagrangian averaging using $T_{SC}$ is able to predict better results and achieve the regularization effect of plane averaging while retaining spatial localization. For a given problem, as the grid becomes finer, the results obtained using different averaging schemes for DSM tend to become indistinguishable from one another. On a fine grid, the effect of averaging and Lagrangian averaging time scale is small. Case 590c is a very coarse grid, and shows difference between the different averaging schemes. Fig. 1(a) shows that the mean velocity shows increasingly improving agreement with DNS as the averaging scheme changes from averaging along homogeneous directions (plane) to Lagrangian averaging using $T_{LDSM}$ and finally $T_{SC}$. Note that though $\theta = 1.5$ was chosen for $T_{LDSM}$, using $\theta = 3.0$ produced results only marginally different (not shown). This just re-affirms that $T_{LDSM}$ is marginally sensitive to the choice of $\theta$ for the given problem. The fact that Lagrangian averaging performs better than plane averaging has been demonstrated by Meneveau et al. (1996) and Stoll and Porté-Agel (2008). The present results show that using $T_{SC}$ as the time scale for Lagrangian averaging yields as good as or even better results.

Stoll and Porté-Agel (2008) report that the Lagrangian averaged model using $T_{LDSM}$ has approximately 8% negative values for $\nu_t$ compared to 40% for the locally smoothed (neighbor-averaged) model.

<table>
<thead>
<tr>
<th>Case</th>
<th>$Re_{\tau}$</th>
<th>$N_x \times N_y \times N_z$</th>
<th>$\Delta x^+$</th>
<th>$\Delta z^+$</th>
<th>$\Delta y_{\text{min}}^+$</th>
<th>$\Delta y_{\text{cen}}/\delta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>590c</td>
<td>590</td>
<td>$64 \times 64 \times 64$</td>
<td>58</td>
<td>29</td>
<td>1.6</td>
<td>0.08</td>
</tr>
<tr>
<td>590tl</td>
<td>590</td>
<td>$160 \times 84 \times (200, 100)$</td>
<td>23.2</td>
<td>9.3, 18.5</td>
<td>1.8</td>
<td>0.04</td>
</tr>
<tr>
<td>1ktl</td>
<td>1000</td>
<td>$160 \times 84 \times (200, 100)$</td>
<td>39.3</td>
<td>15.8, 31.4</td>
<td>3.1</td>
<td>0.04</td>
</tr>
<tr>
<td>2ktl</td>
<td>2000</td>
<td>$320 \times 120 \times (400, 200, 100)$</td>
<td>39.3</td>
<td>15.7, 31.4, 62.8</td>
<td>2.0</td>
<td>0.04</td>
</tr>
</tbody>
</table>

Table 1: Grid parameters for turbulent channel flow.
in their simulations of stable atmospheric boundary layer. The percentage of time that $\nu_t$ is negative in our computations is shown in fig. 3(a). Plane averaged $\nu_t$ was never negative and hence is not plotted. Clearly, $\nu_t$ averaged using $T_{SC}$ has the least number of negative values up until $y^+ \sim 100$ (which contains 50% of the points). Even after $y^+ \sim 100$, $\nu_t$ averaged using $T_{SC}$ has lesser negative values than $T_{LDSM}$ with $\theta = 1.5$. It is also observed that increasing $\theta$ reduced the number of negative values, as expected intuitively. Therefore, $T_{SC}$ is able to achieve the smoothing effect of plane averaging while retaining spatial localization.

For this kind of relatively coarse near-wall resolution, GIE is expected to be high near the wall (as shown in fig. 2) and in addition, remain correlated longer because of the near-wall streaks. This results in a high correlation of GIE near the wall which leads to to a higher Lagrangian time scale. Consistent with this, fig. 3(b) shows that $T_{SC}$ is much higher near the wall than $T_{LDSM}$. On closer inspection, $T_{SC}$ is actually found to overlap with $T_{LDSM}$ for $\theta = 3.0$ for almost half the channel width. For this particular computation, $\theta = 3.0$ is therefore a preferable alternative to $\theta = 1.5$. This makes it entirely reasonable to suppose that other flows might prefer some other $\theta$ than just 1.5. The dynamic procedure used in this paper alleviates this problem.

It must be noted that computing a dynamic $T_{SC}$ for Lagrangian averaging the DSM terms does not incur a significant computational overhead. For case 590c, the total computational time required for computing $T_{SC}$ and then using it for Lagrangian averaging of the DSM terms is just 2% more than that when no averaging is performed.
A wall-resolved LES is performed using an unstructured zonal grid, which has a transition layer in Z along Y (case 590tl). Figs. 4(a)-(b) show that the results are in good agreement. The statistics (fig. 4(b)) have a small kink around $y^+ \sim 140$ where the grid transitions. This kink in the statistics is an artifact of numerical discretization and grid skewness and is present even when no SGS model is used.

The proposed model (eq. 13) is applied to turbulent channel flow at higher Reynolds numbers of $Re = 1000$ and $Re = 2000$. The grid used for case 1ktl is the same as used for case 590tl and hence the resolution in wall units is almost twice as coarse, as shown in table 1. The grid used for case 2ktl is based on similar scaling principles as case 590tl, which is to enable a wall-resolved LES. Hence, it has 2 transition layers to coarsen from a fine near-wall $\Delta z$ to a coarser outer region $\Delta z$. Fig. 5(a) shows good agreement for the mean velocity which indicates that the wall stress is well predicted. Fig. 5(b) compares the computed Lagrangian time scales for the three cases - 590tl, 1ktl and 2ktl. With increasing Reynolds number, the correlation of GIE increases which results in increasing $T_{SC}$. Overall, the results indicate that the Lagrangian DSM with $T_{SC}$ works well for high Reynolds number wall-bounded flows on grids where non-orthogonal elements are present and plane averaging is not straightforward.

**Flow past a circular cylinder**

The Lagrangian DSM with dynamic time scale $T_{SC}$ (eq. 13) is applied to flow past a circular cylinder. LES is performed in the turbulent regime at $Re_D = 3900$ (based on freestream velocity $U_\infty$ and cylinder diameter $D$). The computational domain and
Table 2: Flow parameters at $Re_D = 3900$. Legend for symbols: mean drag coefficient $<C_D>$, rms of drag and lift coefficient ($\sigma(C_D), \sigma(C_L)$), Strouhal number $St$ and base pressure $C_{P_b}$, separation angle $\theta_{sep}$, recirculation length $L_{rec}/D$.

<table>
<thead>
<tr>
<th></th>
<th>$&lt;C_D&gt;$</th>
<th>$\sigma(C_L)$</th>
<th>$St$</th>
<th>$-C_{P_b}$</th>
<th>$\theta_{sep}$</th>
<th>$L_{rec}/D$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_{SC}$</td>
<td>1.01</td>
<td>0.139</td>
<td>0.210</td>
<td>1.00</td>
<td>88.0</td>
<td>1.40</td>
</tr>
<tr>
<td>$T_{LDSM}$</td>
<td>0.99</td>
<td>0.135</td>
<td>0.208</td>
<td>1.00</td>
<td>87.0</td>
<td>1.63</td>
</tr>
<tr>
<td>Kravchenko and Moin (2000)</td>
<td>1.04</td>
<td>-</td>
<td>0.210</td>
<td>0.94</td>
<td>88.0</td>
<td>1.35</td>
</tr>
<tr>
<td>Experiment (taken from Mahesh et al. (2004))</td>
<td>0.99</td>
<td>-</td>
<td>0.215</td>
<td>-</td>
<td>86.0</td>
<td>1.40</td>
</tr>
</tbody>
</table>

boundary conditions used the simulation are shown in fig. 6. The domain height is $40D$, spanwise width $\pi D$ and extends $50D$ downstream and $20D$ upstream of the center of the cylinder. An unstructured grid of quadrilaterals is first generated in a plane, such that computational volumes are clustered in the boundary layer and the wake. This two-dimensional grid is then extruded in the spanwise direction to generate the three-dimensional grid; 80 spanwise planes are used for both the simulations and periodic boundary conditions imposed in those directions. Uniform flow is specified at the inflow, and convective boundary conditions are enforced at the outflow. The smallest computational volume on any spanwise station of the cylinder is of the size $2.0e^{-3}D \times 5.2e^{-3}D$ but stretches to $3.9e^{-2}D \times 2.9e^{-2}D$ at a downstream location of $5D$.

Fig. 7 shows that the instantaneous GIE also follows the pattern of the Karman vortex street. The top shear layer can be seen to roll up (within one diameter) to form the primary vortex. The GIE is highest in the turbulent shear layers where scales are smaller. Downstream, as the turbulence becomes more developed and scales become bigger, GIE diminishes. As the grid becomes coarser downstream, DSM plays a more dominant role, providing a higher value of $\nu_t$ which reduces GIE. It appears that GIE follows the dominant structures in the flow and hence it is reasonable that Lagrangian averaging uses a time scale based on a correlation of the GIE.

To compare performance of different Lagrangian averaging based methods, results are computed using both the proposed surrogate correlation based time scale $T_{SC}$ and the standard time scale $T_{LDSM}$. Integral quantities using $T_{SC}$ show good agreement with the B-spline computation of Kravchenko and Moin (2000) and the experiments of Lourenco & Shih (taken from Mahesh et al. (2004)) as shown in table 2. Note that $T_{LDSM}$ also shows good agreement for the wall quantities; however, $L_{rec}/D$ which depends on the near-field flow, shows discrepancy. This points towards a difference in the values of the time scales away from the cylinder.

There have been numerous studies comparing the time averaged statistics for flow over a cylinder. However different authors have used varying
40 shedding periods are required to obtain converged mean flow statistics in the neighborhood of the cylinder. Tremblay et al. (2000) averaged over 60 shedding cycles in their DNS using an immersed boundary method. In the current work, statistics are obtained over a total time of $404/D/U_\infty$ ($\sim 85$ shedding periods) and then averaged over the spanwise direction for more samples. Converged mean flow and turbulence statistics using $T_{SC}$ show good agreement with the B-spline computations of Kravchenko and Moin (2000) up to $x/D = 2$ as shown in figs. 8. Results using $T_{LDSM}$ are also shown for comparison. Difference in the statistics between the two time scales seem to be significant in the near-wake.

These differences could be attributed to the contribution of the SGS model. Differences in the computed eddy viscosity arise due to different time scales for Lagrangian averaging of the DSM terms. Both $T_{SC}$ and $T_{LDSM}$ are found to increase almost linearly downstream after $x/D > 5$ as shown in fig. 9, though for different reasons. Based on the surrogate correlation of the GIE, increasing $T_{SC}$ is consistent with the flow structures becoming bigger as they advect downstream. Whereas, strong dependence of $T_{LDSM}$ on the strain rate gives it a linear profile both ahead of and behind the cylinder. It can be argued that perhaps a different value of the relaxation factor $\theta$ would be more appropriate for this flow. In fact, fig. 9 makes it apparent that if $T_{LDSM}$ were to be doubled, its value would be closer to that of $T_{SC}$. Again, it can be crudely estimated that scaling the value of $\theta$ by a factor of two or so ($\theta \geq 3.0$) will result in $T_{LDSM}$ being close to $T_{SC}$ after $x/D > 5$. However, it is clear that $T_{LDSM}$ would still not show the appropriate trend ahead of the cylinder and in the recirculation region. Note that, as expected, $T_{SC}$ is high just behind the cylinder ($x/D \sim 1$) in the recirculation region and low in the high acceleration region ahead of the cylinder.
Marine propeller in crashback

Propeller crashback is an off-design operating condition where the marine vessel is moving forward but the propeller rotates in the reverse direction to slow down or reverse the vessel. The crashback condition is dominated by the interaction of the free stream flow with the strong reverse flow from reverse propeller rotation; this interaction forms an unsteady vortex ring around the propeller. Crashback is characterised by highly unsteady forces and moments on the blades due to large flow separation and hence is a very challenging flow for simulation. Výšohlídek and Mahesh (2005, 2006) performed one of the first LES of a marine propeller in crashback. Chang et al. (2008) coupled the unsteady blade loads with a structural solver to predict shear stress and bending moment on the propeller blades during crashback. Jang and Mahesh (2010, 2012) studied crashback at three advance ratios and proposed a flow mechanism. Verma et al. (2011, 2012) explained the effect of an upstream hull on a marine propeller in crashback. These simulations were performed using locally-regularized DSM.

In the current work, LES of a marine propeller, attached to an upstream submarine hull is performed using the Lagrangian averaged DSM with the proposed dynamic time scale (eq. 13). Preliminary results are shown at a Reynolds number of \( Re = 480,000 \) and advance ratio of \( J = -0.7 \). Here

\[
Re = \frac{UD}{\nu} \quad \text{and} \quad J = \frac{U}{nD}
\]

where \( U \) is the free-stream velocity, \( n \) is the propeller rotational speed, and \( D \) is the diameter of the propeller disk.

Simulations are performed in a frame of reference that rotates with the propeller with the absolute velocity vector in the inertial frame. The computational domain is a cylinder with diameter 7.0\( D \) and length 14.0\( D \) as shown in fig. 10(a). Free-stream velocity boundary conditions are specified at the inlet and the lateral boundaries. Convective boundary conditions are prescribed at the exit. Boundary conditions on the rotor part, blades and hub are specified as \( u = \omega \times r \), where \( \omega = 2\pi n \) and \( r \) is the radial distance from the propeller center. No-slip boundary conditions are imposed on the hull body. An unstructured grid with 7.3 million cvs is used as shown in fig. 10(b). The propeller surface is meshed with quadrilateral elements. Four layers of prisms
Table 3: Propeller in crashback: Computed and experimental values of mean and rms of coefficient of thrust $K_T$, torque $K_Q$ and side-force $K_S$ on propeller blades.

<table>
<thead>
<tr>
<th></th>
<th>$\langle K_T \rangle$</th>
<th>$\sigma(K_T)$</th>
<th>$\langle K_Q \rangle$</th>
<th>$\langle K_S \rangle$</th>
<th>$\sigma(K_S)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>LES</td>
<td>-0.358</td>
<td>0.113</td>
<td>-0.067</td>
<td>0.046</td>
<td>0.024</td>
</tr>
<tr>
<td>Experiment (Bridges et al., 2008)</td>
<td>-0.340</td>
<td>0.085</td>
<td>-0.060</td>
<td>0.044 - 0.048</td>
<td>0.019 - 0.021</td>
</tr>
</tbody>
</table>

Figure 12: $J = -0.7$. (a) Contours of time scale $T_{SC}$ with streamlines, (b) Time averaged turbulent kinetic energy field with streamlines at a constant radial plane of $r/R = 0.4$.

The Lagrangian time scale $T_{SC}$ is seen to be physically consistent with the flow. It is high in the low-momentum wake behind the propeller where flow structures are expected to be more coherent. $T_{SC}$ is low in the unsteady vortex ring region around the propeller blades. The cylindrical region around the blades is where the grid transitions from tetrahedral to hexahedral volumes. Turbulent kinetic energy ($k$) is a measure of three-dimensional unsteadiness and turbulence in the flow. Fig. 12(b) shows the resolved turbulent kinetic energy within the blade passage at a radial plane of $r/R = 0.4$. $k$ is highest near the leading edge of the blades, related to the unsteadiness caused by the reverse flow separating at the sharp leading edge. The performance of $T_{SC}$ for such complex flows is encouraging.
REYNOLDS STRESS CONSTRAINED WALL MODEL

The Lagrangian averaged DSM with a dynamic time scale gives better results over existing averaged DSM methods. However, it does not solve the wall modeling problem. Fig. 13 shows that the GIE from the 3 cases 590t, 1kt, and 2ktl (which use relatively ‘wall-resolved’ grids) is still high in the near-wall region; the error increases as the grid coarsens. This is indicative of greater SGS modeling errors near the wall, especially when coarser near-wall grids are employed for LES. It is well known that LES with simple eddy viscosity model works poorly for wall-bounded flows (Piomelli et al., 1996; Templeton et al., 2006; Park and Mahesh, 2008a). This is primarily due to the fact that near the wall, flow structures scale in viscous units. If the near-wall grid is fashioned to resolve the large or integral length scales of the flow, these near-wall structures remain unresolved. Moreover, near-wall flow structures tend to be anisotropic and simple SGS models fail to accurately represent the turbulent stress near the wall. It has been estimated that the grid requirement for a wall-resolved LES scales as $Re^{2}_{\tau}$ (Baggett et al., 1997); comparable to that for a DNS which scales as $Re^{9/4}_{\tau}$. In order to overcome this severe resolution requirement, various wall modeling approaches have been suggested and summarized in various review articles (Piomelli and Balaras, 2002; Piomelli, 2008). One such approach is that of hybridizing Reynolds Averaged Navier-Stokes (RANS) and LES formulations. The present study is motivated by (1) the inherent limitations of the existing hybrid RANS-LES methodologies and (2) the challenges in implementing a robust hybrid RANS-LES framework for complex flows on unstructured grids.

Detached-Eddy Simulation (DES) by Spalart et al. (1997) is a widely used approach for high Reynolds number flows. The idea behind DES is to compute the boundary layer using RANS and use LES away from the wall (in the ‘separated’ region). Many hybrid RANS-LES type formulations also use essentially what is a RANS-type eddy viscosity near the wall and ‘blend’ it with LES eddy viscosity away from the wall. The basic idea behind the present work can be summarized as:

- Using a RANS model directly in the near-wall region produces excessive dissipation (Park and Mahesh, 2008b). A less dissipative “subgrid scale model” is needed which leads solution to a target quantity prescribed from external data only in the mean sense. This target quantity may be the wall stress, Reynolds stress or mean velocity.
- The intention is to perform LES in the whole computational domain using a simple yet robust wall model. In general, LES is superior to RANS even with coarse resolutions away from the wall. The external Reynolds stress constraint should be imposed in a limited region (near the wall) where LES is expected to be erroneous.

A hybrid dynamic SGS model constrained by externally prescribed Reynolds stress is formulated in the next section. The proposed model is then applied to turbulent channel flow at various Reynolds numbers and grid distributions.

A CONSTRAINED DYNAMIC SGS MODEL

An advantage of the dynamic procedure is that various terms can be easily incorporated to form dynamic mixed models. Ghosal et al. (1995) proposed a dynamic localization model by including a non-negative constraint on the model coefficient. Shi et al. (2008) imposed an energy dissipation constraint on the dynamic mixed similarity model. In the present work, Reynolds stress is considered to be provided as an externally prescribed constraint. More particularly, only a time average of the Reynolds stress needs to be provided and hence it could be sourced from RANS, DNS, experimental statistics or even empirical closures/fits. A simple and efficient hybrid SGS model was first pro-
posed by Park and Mahesh (2008b) that combines the dynamic Smagorinsky model (DSM) approach and Reynolds stress constraints. However, they used averaging along homogeneous directions in the context of a spectral, structured solver. The current work extends this formulation to unstructured grids without averaging along homogeneous directions. A Lagrangian averaged version of this formulation is used to show improved results for turbulent channel flow in the next section.

**Formulation**

For simplicity, let us assume that Reynolds stress $R_{ij} = \langle u_i u_j \rangle_E - \langle u_i \rangle_E \langle u_j \rangle_E$ is given from an external RANS solution. Here, $\langle \cdot \rangle_E$ denotes an ensemble average, which is equivalent to $\langle \cdot \rangle_t = $ time averaging. The ensemble average of SGS stress satisfies

$$\langle \tau_{ij} \rangle_E = \langle \pi_i \pi_j \rangle_E - \langle \tau_{ij} \rangle_E = R_{ij}$$  \hspace{1cm} (16)

However, imposition of this condition on unsteady simulations is not straightforward. Consider therefore, an instantaneous version of (16) with SGS model $\tau_{ij}^\text{SGS}$:

$$\epsilon^{\text{R}}_{ij} = \pi_i \pi_j - U_i U_j + \tau_{ij}^\text{M} - R_{ij},$$  \hspace{1cm} (17)

where $\epsilon^{\text{R}}_{ij}$ is the error (and $\mathcal{R}$ denotes RANS), and

$$U_i = \frac{1}{T} \int_0^T \pi_i dt \equiv \langle \pi_i \rangle_T$$  \hspace{1cm} (18)

is cumulative, ensemble-averaged velocity up to current time $T$. When $T$ is sufficiently large, $\epsilon^{\text{R}}_{ij}$ in (17) represents deviation from (16) due to SGS modeling error. Thus, the minimization of $\epsilon^{\text{R}}_{ij}$ seems to be a proper RANS constraint.

On the other hand, error from the Germano identity is

$$\epsilon^{\mathcal{L}}_{ij} = T_{ij} - \tau_{ij}^\text{M} - L_{ij},$$  \hspace{1cm} (19)

where $\mathcal{L}$ denotes ‘LES’. Then, cost function to be minimized can take the form

$$\mathcal{J}(C_s) = \int_{\Omega} \epsilon^{\mathcal{L}}_{ij} \epsilon^{\mathcal{L}}_{ij} d\mathbf{x} + \omega^\mathcal{R} \int_{\Omega} \langle \epsilon^{\mathcal{R}}_{ij} \rangle_T \langle \epsilon^{\mathcal{R}}_{ij} \rangle_T d\mathbf{x},$$  \hspace{1cm} (20)

where we consider a one–parameter SGS model $\tau_{ij}^\text{M} = \tau_{ij}^\text{M}(C_s)$, $\Omega$ is the domain, and $\omega^\mathcal{R}$ is the weight function for RANS constraints.

Then, the optimal $C_s$ is given by

$$\delta \mathcal{J} (C_s) = \int_{\Omega} \frac{\partial}{\partial C_s} \left[ \epsilon^{\mathcal{L}}_{ij} \epsilon^{\mathcal{L}}_{ij} + \omega^\mathcal{R} \langle \epsilon^{\mathcal{R}}_{ij} \rangle_T \langle \epsilon^{\mathcal{R}}_{ij} \rangle_T \right] \delta C_s d\mathbf{x} = 0,$$  \hspace{1cm} (21)

which implies that

$$\frac{\partial}{\partial C_s} \left[ \epsilon^{\mathcal{L}}_{ij} \epsilon^{\mathcal{L}}_{ij} + \omega^\mathcal{R} \langle \epsilon^{\mathcal{R}}_{ij} \rangle_T \langle \epsilon^{\mathcal{R}}_{ij} \rangle_T \right] = 0.$$  \hspace{1cm} (22)

Eq. (22) is a general relation that can be used for complex flows and one–parameter SGS models. Note that the above relation can be easily modified to be applicable with the homogeneous averaged and the Lagrangian averaged DSM (eq. 5). Let us consider $\epsilon^{\mathcal{L}}_{ij}$ and $\epsilon^{\mathcal{R}}_{ij}$ for the Smagorinsky model (SM) (Smagorinsky, 1963)

$$\tau_{ij} - \frac{1}{3} \tau_{kk} = -2C_s \Delta^2 |S| \delta_{ij},$$  \hspace{1cm} (23)

where $|S| = \sqrt{2S_{ij} S_{ij}}$.

First, Germano identity error (GIE) is

$$\epsilon^{\mathcal{L}}_{ij} \epsilon^{\mathcal{L}}_{ij} = C_s \left(-2\Delta^2 |S| \delta_{ij} + 2\Delta^2 |S| S_{ij} \right) - L_{ij},$$  \hspace{1cm} (24)

Here, all tensors are inherently or made traceless. Therefore, the first part in the bracket of Eq. (22) is

$$\epsilon^{\mathcal{L}}_{ij} \epsilon^{\mathcal{L}}_{ij} = C_s^2 M_{ij} M_{ij} - 2C_s L_{ij} M_{ij} + L_{ij} L_{ij},$$  \hspace{1cm} (25)

which leads to

$$\frac{\partial \epsilon^{\mathcal{L}}_{ij} \epsilon^{\mathcal{L}}_{ij}}{\partial C_s} = 2C_s M_{ij} M_{ij} - 2L_{ij} M_{ij}. $$  \hspace{1cm} (26)

Obviously, equating eq. (26) to zero results in the standard DSM (Germano et al., 1991; Lilly, 1992). Next, RANS Reynolds–stress reconstruction error (eq. 17) is considered:

$$\langle \tau_{ij} \rangle_T = \langle r_{ij} \rangle_T - 2C_s \Delta^2 |S| \delta_{ij} \langle R_{ij} \rangle_T$$

$$= \langle r_{ij} \rangle_T - 2\Delta^2 C_s \langle |S| \delta_{ij} \langle R_{ij} \rangle_T - R_{ij} \rangle_T \rangle$$

$$\approx \langle r_{ij} \rangle_T - R_{ij} - 2\Delta^2 \langle |S| \delta_{ij} \rangle_T C_s$$

$$= A_{ij} - B_{ij} C_s,$$

where $r_{ij} = \pi_i \pi_j - U_i U_j$ and $C_s$ is assumed constant in time. Similar to $\epsilon^{\mathcal{L}}_{ij}$, the second part of eq. (22) is

$$\frac{\partial \langle \epsilon^{\mathcal{R}}_{ij} \rangle_T \langle \epsilon^{\mathcal{R}}_{ij} \rangle_T}{\partial C_s} = 2C_s B_{ij} B_{ij} - 2A_{ij} B_{ij}.$$  \hspace{1cm} (28)

Inserting eqs. (26) and (28) in eq. (22) yields $C_s$ as

$$C_s = \frac{L_{ij} M_{ij} + \omega^\mathcal{R} A_{ij} B_{ij}}{M_{ij} M_{ij} + \omega^\mathcal{R} B_{ij} B_{ij}}.$$  \hspace{1cm} (29)
In principle, the above expression for $C_s$ (eq. 29) is applicable throughout the flow. However, as mentioned earlier, the intention is to apply the external Reynolds stress constraint only in a limited region where LES is expected to be erroneous. We propose the Germano-identity error (eq. 19) as a measure of accuracy of LES utilizing a dynamic Smagorinsky SGS model. Figs. 2 and 13 show respectively, that the instantaneous and time-averaged GIE is very high near the wall, so that the validity of the Smagorinsky SGS model (eq. 23) in this region can be questioned. The external Reynolds stress constraint should be active in such regions where the GIE is deemed too high; to be determined by the weight function $\omega^R$. Note that, to transition from RANS to LES, DES uses purely grid parameters such as the wall distance and local grid spacing; its variants incorporate some flow information. The current proposal to use GIE is explicitly dependent on the flow and the underlying SGS model.

Let us denote $\mathcal{E} = \epsilon_{ij}^c \epsilon_{ij}^c / \tau_{ij}^M \tau_{ij}^M$ as the Germano-identity error normalized by the modeled SGS stress (fig. 14). The weight function $\omega^R$ is then proposed to be of the form,

$$\omega^R = C_\omega \max(\mathcal{E} - \mathcal{E}_t, 0),$$

where $C_\omega$ is a scaling coefficient and $\mathcal{E}_t$ is the threshold value. Nominally, $\omega^R$ is determined using $C_\omega = 0.1$ and $\mathcal{E}_t = 100$ is chosen to impose the constraints in the near-wall region.

Hence $\omega^R \neq 0$ implies the Reynolds stress constraint is active only in the region where the normalized Germano-identity error $\mathcal{E}$ exceeds a certain threshold $\mathcal{E}_t$. Fig. 14 shows that the constraint would be active only in the near-wall region (for cases 590un, 1kun, and 2kun as described in table 4). Obviously, $\omega^R = 0$ retrieves the standard DSM.

**RESULTS: WALL MODEL**

The goal of wall modeling is to relax the grid requirement scaling with Reynolds number. DES hopes to achieve this by operating on a RANS near-wall grid where the wall-parallel spacing is large compared to the boundary-layer thickness ($\Delta_\parallel \gg \delta$) but the wall-normal grid spacing requirement is stricter ($\Delta_\perp \sim O(1)$). Nikitin et al. (2000) followed this guideline for their DES of channel flow and showed results with $\Delta_\parallel = 0.16$ and $\Delta y^*_w < 1$. Further savings could be obtained by relaxing the wall-normal grid spacing requirement. When the first off-wall grid point is in the log layer, the filter width is much larger than the local turbulent integral scales. Hence, wall stress models are required to compensate for the SGS modeling errors in this region. Nicoud et al. (2001) and Templeton et al. (2005) use walls stress models on coarse LES grids.
Our motivation is to perform LES at high Reynolds numbers using no-slip boundary conditions at the wall with a slightly relaxed near-wall grid requirement. Results are shown with grids where the first off-wall grid point is in the viscous layer ($\Delta y^+ \leq 5$). In what follows, DSM denotes Lagrangian averaged Dynamic Smagorinsky Model and CDSM denotes Constrained DSM (with Lagrangian averaging for the DSM terms) which is the proposed constrained model (eq. 29). CDSM is applied to turbulent channel flow at three Reynolds numbers; $Re_\tau = 590, 1000, 2000$ to show improvement over DSM. Table 4 lists the $Re_\tau$ and grid distribution for the various runs. All LES cases have uniform spacing in $x$; ‘un’ indicates that an unstructured grid has been used near the wall in the spanwise direction ($z$) to allow flexibility in the near-wall grid while maintaining a fixed coarse outer-region grid. The LES results are compared to DNS whose grid parameters are also included in the table for comparison. The numerical method used is the same as described earlier.

Fig. 15 shows results for case 590un with parameters $C_\omega = 0.1$, $\mathcal{E}_t = 100$ and Reynolds shear stress from the DNS of Moser et al. (1999) as the constraint. CDSM shows marginal improvement over DSM for mean and rms streamwise velocity at this grid resolution. With CDSM, resolved shear stress (fig. 15(c)) reduces slightly near the wall but is compensated by higher SGS stress such that the total shear stress is closer to the DNS constraint. This establishes that the constrained formulation CDSM (i) is successful in constraining the total shear stress to an externally provided constraint in the mean and (ii) shows improvement over DSM.

However the improvement is significant when the near-wall grid resolutions for the LES are coarse. To this end, simulations are performed at $Re_\tau = 1000$ using both DSM and CDSM with a coarser $\Delta z^+$ and $\Delta x^+$. Fig. 16 shows results at $Re_\tau = 1000$ from case 1kun with parameters $C_\omega = 0.1$, $\mathcal{E}_t = 100$. The CDSM results for mean and rms $u$-velocity are in good agreement with unfiltered DNS. As with case 590un, fig. 16(c) shows that with CDSM, the resolved shear stress reduces slightly near the wall but is compensated by higher SGS stress; the total Reynolds shear stress computed is closer to the imposed constraint especially near the wall which is where the constraint is being activated due to high GIE. CDSM predicts a higher near-wall SGS stress due to higher eddy viscosity near the wall as shown in fig. 17(a). The constrained minimization of the GIE with an external constraint also reduces the GIE near the wall (fig. 17(b), right side, red) for

Figure 15: Mean statistics from turbulent channel flow at $Re_\tau = 590$ - Case 590un: (a) mean velocity, (b) rms velocity fluctuations, (c) Reynolds stress.
Figure 16: Mean statistics from turbulent channel flow at $Re_τ = 1000$ - Case 1km: (a) mean velocity, (b) rms velocity fluctuations, (c) Reynolds stress.

Figure 17: Mean statistics from turbulent channel flow at $Re_τ = 1000$ - Case 1km: (a) eddy-viscosity, (b) Germano-identity error.

CDSM. It also shows that only a few points near the wall have threshold $E_t > 100$ (left side, black). Thus the constraint is active only at a few points near the wall ($y^+ < 100$).

At higher Reynolds numbers, the target Reynolds stress may not be easily available a priori from DNS, RANS or even experiments. A more convenient alternative is to use a model for Reynolds stress. Such models need only be reasonably accurate in the near-wall region as the constraint is only intended to be applied there. Results are shown at $Re_τ = 1000, 2000$ with the Reynolds stress constraint obtained using the method described by Perry et al. (2002). Fig. 18 shows results at $Re_τ = 2000$ from case 2km with parameters $C_ω = 0.1$, $E_t = 100$. Note that the grid is almost the same as cases 590km and 1km. This implies an even coarser
\[ \Delta z^+ \text{ and } \Delta x^+ \] for this higher Reynolds number (as seen in table 4). The CDSM results for mean and rms u-velocity are in good agreement with unfiltered DNS. Though not shown here, DSM at such coarse resolution is expected to show significant differences with DNS.

Fig. 19 shows that skin-friction coefficient and wall pressure fluctuations obtained from CDSM are in reasonable agreement with the available DNS data and empirical fits. The skin-friction coefficient is based on the centerline velocity \( C_f = 2 / U_{\|}^{+2} \). The fit shown is extrapolated from the DNS of Moser et al. (1999) by assuming \( U_{\|}^{+} = 21.26 + \log(Re_{\|}/587)/0.41 \). This is done following Nikitin et al. (2000). Importantly, CDSM shows improvement over DSM. This improvement is expected to get very significant for LES at high Reynolds number on very coarse near-wall grids (such as for case 2kun). CDSM also predicts reasonable wall pressure fluctuations \( \sigma(p) / \tau_w \) when compared to unfiltered DNS. The fit shown is taken from Bull (1996). It is encouraging that CDSM predicts unsteady behavior down to the wall along with quantities of engineering interest such as skin friction and wall pressure fluctuations.

**CONCLUSION**

This paper discusses two developments towards reliably using LES for attached high Reynolds number flows: (1) a dynamic Lagrangian model where a dynamic procedure is proposed for the Lagrangian timescale and (2) a wall model where in addition to the Germano-identity error, external Reynolds stress...
is also imposed as a constraint on the ensemble-average subgrid-scale stress. Both developments are towards performing LES of complex flows on unstructured grids. They are in the context of the Dynamic Smagorinsky model (DSM) (Germano et al., 1991; Lilly, 1992) and exploit the Germano-identity error.

A dynamic Lagrangian averaging approach is developed for the dynamic subgrid scale model. The standard Lagrangian dynamic model (Meneveau et al., 1996) uses a Lagrangian time scale \( T_{LDSM} \) which contains an adjustable parameter \( \theta \). We propose to use a dynamic time scale \( T_{SC} \) based on a “surrogate-correlation” of the Germano-identity error (GIE) (Park and Mahesh, 2009). The proposed model is applied to LES of turbulent channel flow at moderately high Reynolds numbers and relatively coarse grid resolutions. Good agreement is obtained with unfiltered DNS data. Improvement is observed when compared to other averaging procedures for the dynamic Smagorinsky model, especially at coarse resolutions. The model is also applied to flow over a cylinder at a high Reynolds number. \( T_{SC} \) shows good agreement of turbulence statistics with previous computations and experiments, and is shown to outperform \( T_{LDSM} \). It is established that \( T_{SC} \) is physically consistent with flow structures and hence a more apt choice for Lagrangian averaging. \( T_{SC} \) reduces the number of times ad-hoc clipping operations need to be performed on the computed eddy viscosity. Finally, the extra computational overhead incurred by the proposed Lagrangian averaging is negligible compared to when no averaging is used.

The strong scaling of the computing cost of LES with Reynolds number is an impediment to LES being applied to attached wall-bounded flows. However LES for wall-bounded flows offers the advantage of computing fluctuating quantities on the wall such as wall pressure fluctuations. A wall model (CDSM) is proposed to enable LES at coarse near-wall grid resolutions. The proposed model approaches the mean modeled behavior of RANS through a constraint on what is essentially an SGS model. Primarily, it allows hybridization of the LES methodology with a desired or expected mean target quantity; external Reynolds stress constraints are incorporated into the Dynamic Smagorinsky model. Secondly, this target quantity may be imposed in a small region near the wall for wall-bounded flows where SGS modeling errors are expected to be large; normalized Germano-identity error is used as a measure of SGS modeling errors and hence as a weight for the constraint. CDSM is applied to turbulent channel flows at various Reynolds numbers and grid resolutions. CDSM outperforms DSM and this improvement becomes more significant as the near-wall grid coarsens. CDSM achieves better predictions than DSM by constraining the total Reynolds stress to an \( a \ priori \) imposed target. It has been shown that this target Reynolds stress can be obtained from DNS and approximate near-wall models. Imposition of the Reynolds stress constraint in a small region near the wall increases the eddy viscosity and reduces the Germano-identity error near the wall. Importantly, this procedure does not force the instantaneous flow to a mean quantity but only constrains the mean behavior. Hence CDSM predicts unsteady behavior down to the wall and is a reliable tool to predict quantities of engineering interest such as skin friction and wall pressure fluctuations. For future work, CDSM will be applied to complex geometries and separated flows.

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