Three problems in the large–eddy simulation of complex flows

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Abstract

This paper discusses: (i) an algorithm that addresses the problems posed by low Mach numbers and high Reynolds numbers in large–eddy simulation of compressible turbulent flows, (ii) the near–wall modeling problem for attached turbulent boundary layers, and (iii) a simple kinematic model that possibly explains why large–eddy simulation predicts turbulent mixing accurately, even though the viscous processes are not being represented.

1 An algorithm for large–eddy simulation of compressible turbulent flows

A key issue in turbulence simulation is ensuring robustness without the use of numerical dissipation. Considerable attention has been devoted to this problem for incompressible flows, where algorithms that discretely conserve kinetic energy have been found to be very successful in reliably performing large–eddy simulation (LES). However, the compressible equations do not conserve kinetic energy; energy is exchanged between kinetic and potential energy. Also, small Mach numbers result in the compressible equations becoming very stiff. A solution that addresses both problems is proposed. The compressible equations are cast in a form that reduces to the incompressible equations in the limit of zero Mach number. This is achieved by non–dimensionalizing pressure using an incompressible scaling. Furthermore, the energy equation is expressed as an equation for the divergence of velocity. The resulting non–dimensional equations therefore naturally yield the incompressible equations in the limit of very small Mach number. Also, all spatial derivatives are in divergence form, and hence conservative. The resulting set of governing equations are therefore very attractive in that at high Mach numbers, they would yield the proper jump in variables across shock waves, and at very small Mach numbers, variations on the fast, acoustic time–scale would be projected out at time–steps larger than the acoustic time–scale.

A projection approach has been developed to solve the equations, that is discretely energy conserving in the incompressible limit. The resulting compressible algorithm is very robust at high Reynolds numbers. Validation examples will be shown. The algorithm is currently being applied to simulate a supersonic boundary layer. Results will be presented.
2 The near–wall modeling problem

Large–eddy simulations of attached boundary layers at high Reynolds numbers require very fine near–wall resolution when the LES equations are integrated down to the wall. We consider the question of whether common subgrid models are modeling the dominant physical/numerical effect of the subgrid scales in the inner–layer region. Most subgrid models are required to model the net nonlinear transfer of energy from the resolved scales to the subgrid scales. However, Kline et al. (1967, J. Fluid Mech., 30: 741–773), Uzkan & Reynolds (1967, J. Fluid Mech., 28: 803–821), and Lee et al. (1990, J. Fluid Mech., 216: 561–583) suggest that streaks, which dominate the near–wall region, are produced by the linear mechanism of rapid straining of turbulent fluctuations. Lee et al. in particular, establish a close connection between turbulence in the viscous sub–layer, and homogeneous turbulence that is sheared at very high shear–rates. They also show that the evolution of rapidly sheared homogeneous turbulence is well described by linear rapid distortion theory (RDT) and that the RDT can reasonably predict the Reynolds stress anisotropy and structural features of near–wall turbulence.

We therefore consider the possibility that the errors involved when numerically solving the RDT equations on a coarse mesh might correspond to the errors in the near–wall region on coarse meshes. The discretized RTD equations, thus obtained, can be solved analytically using the notion of ‘modified wave–number’. The numerical RTD results for the case where even the large–scales are not resolved show similar trends to channel flow simulations on coarse grids; i.e. at coarse resolutions, the streamwise component of kinetic energy is higher, while the vertical and spanwise components are lower than their exact values. The RDT equations are used to suggest that the dominant source of error arises from the discrete Poisson equation for pressure since pressure is the only variable being differentiated in the linear homogeneous limit. Both truncation and discretization error yield inaccurate representations for the Laplacian operator in the pressure equation. Inversion of the Laplacian operator yields inaccurate pressure fields which in turn yields inaccurate velocity fields. An equivalent interpretation is that the pressure field is obtained by constraining the velocity field to be divergence–free. However, each of the individual derivatives, $\partial u_\alpha / \partial x_\alpha$ is incorrect due to truncation and discretization. The sum of the three gradients is still constrained to be zero. In terms of the Reynolds stresses, this error shows up in the pressure–strain correlation. Truncation error appears to suppress the transfer of energy from the streamwise component of velocity to the vertical and spanwise components, yielding higher values of the streamwise component, and lower values of the vertical and spanwise components.

This suggests that, near the wall, it is probably more important to account for the effect of the subgrid scales on the non–local effects of pressure than it is to model their nonlinear effects due to advection. One way to achieve this might be to use the equivalent of Reynolds stress modeling for LES, where the pressure–strain correlation would be explicitly modeled. Another possibility is to retain presently used models for the subgrid stress, but allow the velocity field to have a finite–divergence. This divergence could be modeled in a variety of ways, e.g. $\sim C \Delta \frac{\partial^2 u_\alpha}{\partial x_\alpha \partial x_j}$ where $C$ is a constant that could be obtained from direct numerical simulation (DNS) data, or obtained dynamically, and $\Delta_j$ denotes the filter width in each coordinate direction, which ensures that the velocity field is divergence–free in the DNS limit.
3 Passive scalar mixing

We discuss why large-eddy simulation yields very good predictions for scalar fluctuations even though the details of the viscous processes that are thought to be essential to mixing are not captured.

Direct numerical simulation of passive scalar transport in a spatially evolving turbulent jet is performed at Reynolds number of 2400 and Schmidt number of unity. The computational domain extends upstream of the jet exit plane to allow for entrainment near the exit plane. Good comparison with experimental data is obtained for the mean velocity, mean scalar concentration, and fluctuations of velocity and scalar. The instantaneous radial profiles of velocity and passive scalar are examined, and related to entrainment from the free-stream. The role of diffusion in scalar transport is studied. Diffusion-dominated regions are very thin near the jet center, but are fairly thick and ‘brush-like’, near the jet edge. Longer residence times near the jet edge are proposed as a reason for this behavior. A simple kinematic model is proposed, that predicts the experimentally observed variation of scalar fluctuations with Reynolds number. The model assumes that scalar fluctuations at a fixed location in the jet result from the oscillation of scalar fronts, whose thickness depends on Reynolds and Schmidt numbers, and whose oscillation amplitude depends on the level of turbulent fluctuations. The value of \( \frac{c_{\text{rms}}}{\tau} \) is predicted to decrease, and asymptote to a constant value as the ratio of the oscillation amplitude of the scalar fronts to their thickness increases. This prediction is consistent with the experimental data discussed by Dimotakis (2000, J. Fluid Mech., 409: 69–98) in the context of mixing transition. The model results also suggest that Reynolds number and Schmidt number dependencies are likely to be stronger, away from the jet centerline, where the scalar fronts are thicker and the levels of turbulence smaller. The fact that the model only relies on convective motions to predict the proper trends for scalar intensities offers an explanation for the success of LES in predicting turbulent mixing even though the viscous processes are not being represented.

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