Problem 1

A uniform slender rod with length $2a$, mass $m$, and principal moment of inertia $I$ has a pin at one end. The pin slides without friction in a slot as shown in the diagram. The height of the centerline of the slot is described by a given function $Y(x)$. The motion is restricted to the plane of the diagram. Thus, this dynamical system has two degrees of freedom.

1. Choose generalized coordinates for the two degrees of freedom, and define the generalized coordinates by drawing a diagram and labeling the coordinates.
2. Write the Lagrangian for this system.
3. Identify any constants of the motion.
4. Write the Lagrange equation of motion for either one of the two generalized coordinates.
Problem 2

A uniform slender Euler-Bernoulli beam is simply supported at its ends. The beam has a length L, an elastic modulus E, an area moment of inertia I, a cross sectional area A, and a mass density $\rho$. The beam has a constant axial force $P$ applied at each end and a concentrated transverse force $F \sin(pt)$ applied at $L/2$. Neglect gravity.

a. If $y(x,t)$ is the transverse displacement of the beam, use any valid method to show that the equation of motion for the beam is

$$\rho A \frac{\partial^2 y}{\partial t^2} - P \frac{\partial^2 y}{\partial x^2} + EI \frac{\partial^4 y}{\partial x^4} = F \sin(pt) \delta(x-L/2)$$

where $\delta(\cdot)$ is the delta function.

b. What are the boundary conditions for this problem?

c. Determine the natural frequencies of vibration and the mode shapes.

d. Assume that there is sufficient unmodeled damping to damp out the natural response of the system, determine the steady state response due to the sinusoidal force $F \sin(pt)$ applied at $L/2$. 

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