Please explain all your design decisions. You may use any or all of the classical control design tools such as root-locus method, Bode plot, etc. Good luck.

1. Consider the following feedback system

\[
\begin{align*}
\frac{-10}{s+1} & \quad e^{-sT} \\
\end{align*}
\]

How large may \( T \) get before causing instability?
2. An inverted pendulum of mass $m$ is hinged at $A$. A gyro with spin angular momentum, $h$, is attached to the pendulum but is free to rotate about the pendulum axis (angle $\phi$) as shown in the figure. A control torque $Q$ can be applied to the gyro from the pendulum. The equations of motion are:

\[
\begin{align*}
I\ddot{\Theta} &= mgl\Theta - h\dot{\Theta} \\
J\ddot{\Theta} &= h\dot{\Theta} + Q
\end{align*}
\]

where

$I = $ the moment of inertia of the pendulum plus gyro about $A$

$J = $ the moment of inertia of the gyro about axis $AC$

$C = $ the mass center of the pendulum plus gyro

(a) Compute the transfer function from $Q(\cdot)$ to $\phi(\cdot)$ and $Q(\cdot)$ to $\Theta(\cdot)$.

(b) Show that the system is controllable by $Q$, observable with $\phi$ and unobservable with $\Theta$.

(c) Show that the system is always unstable.
3. Consider the standard feedback system shown below,

\[ \hat{c} \quad \hat{p} \quad y \quad \hat{p}(s) = \frac{1}{s(s+4)} \]

(a) Find the constant-gain controller, \( \hat{c}(s) = \bar{c} \) (a real number) such that the closed-loop poles are at \(-2 \pm 2j\).

(b) Plot a Nyquist plot of \( \bar{c}\hat{p}(s) \) for the standard contour with indentation at the origin, shown below.

(c) Compute the value of \( \omega \) such that \( |\bar{c}\hat{p}(j\omega)| = 1 \). Mark this on your Nyquist plot, and determine the phase margin of the closed loop system.

(d) Using partial fractions, determine the output \( y(t) \) when the input \( r(t) \) is a unit step function. Plot the response \( y \).

(e) Repeat parts 3b, 3c, and 3d for \( \hat{c} = \frac{1}{4}\bar{c} \), and then \( \hat{c} = 4\bar{c} \). Comment on the relationship between phase margin, overshoot in step response, and rise time that occurs in this particular problem.