A disk is fixed to a shaft that is free to rotate around its axis. The radius of the disk is \( a \), the mass of the disk is \( m_1 \) and the moment of inertia around its axis is \( I_1 \). A rod is attached to the disk by a pin joint at the edge of the disk. The rod is free to rotate in a horizontal plane around the pin. The distance from the pin to the center of mass of the rod is \( b \), the mass is \( m_2 \), and the moment of inertia around the center of mass is \( I_2 \). Use the angles \( \Theta \) and \( \Phi \), shown in the figure, to describe the configuration of the system.

1. Write the Lagrangian for the system, and derive the equations of motion.

2. Identify the constants of the motion and write expressions for these constants. (The identity, \( \sin \Theta \sin \Phi + \cos \Theta \cos \Phi = \cos (\Theta - \Phi) \), may be useful. Please do not spend more than 15 minutes on this question unless you finish the remaining questions and can return to it.)

3. Describe the equilibrium solutions for the system. That is, describe the conditions for which

\[
\dot{\Theta} = 0, \quad \dot{\Phi} = \text{const} \\
\ddot{\Theta} = 0, \quad \ddot{\Phi} = \text{const}
\]

4. Find the frequencies of oscillation for small displacements from an equilibrium solution. (Exclude the case \( \dot{\Theta} = \dot{\Phi} = 0 \).)

5. A small motor is installed in the bearing of the vertical shaft. This exerts a moment \( M_1(t) \) on the shaft. A second motor is installed in the pin. This exerts a moment \( M_2(t) \) on the rod at the pin joint. Write the equations of motion that include these moments.