Two identical uniform links of mass, $m$, and length, $l$, are attached to one another by a frictionless pin at point B. The upper link is attached to a non-accelerating point A by a frictionless pin. The links remain in the XY plane as shown; however, this plane rotates about the Y axis with a constant angular velocity $\Omega$. Gravity acts downward in the negative Y direction. Each link has a moment of inertia about its center of mass $I_{y'y'} = I_{z'z'} = (1/12) ml^2$, and $I_{x'x'} = 0$.

Determine

(1) The differential equations of motion in terms of the angles $\Theta_1$ and $\Theta_2$

(2) The frequencies of vibration for small perturbations about $\Theta_1 = \Theta_2 = 0$ if $\Omega^2 = g/l$

(3) The range of values of $\Omega$ for which $\Theta_1 = \Theta_2 = 0$ is stable (has eigenvalues which do not have positive real parts)
Before starting the specific problem you are to solve, we give background
equations and notations for the restricted three-body problem. In this
problem, we assume that there are two "primary" masses $m_1, m_2$ and a third mass
$m_3$, with $m_3 << m_1$ and $m_3 << m_2$. The two primaries are assumed to
move only under their mutual gravitational attraction:

$$m_1 \ddot{r}_1 = -\frac{G m_1 m_2}{|r_1 - r_2|^2} \hat{r}_{2r1}$$  \hspace{1cm} (1)

$$m_2 \ddot{r}_2 = -\frac{G m_1 m_2}{|r_1 - r_2|^2} \hat{r}_{1r2}$$  \hspace{1cm} (2)

The third mass moves subject to the gravitational attraction of the other
two:

$$m_3 \ddot{r}_3 = -\frac{G m_1 m_3}{|r_1 - r_3|^2} \hat{r}_{1r3} - \frac{G m_2 m_3}{|r_2 - r_3|^2} \hat{r}_{2r3}$$  \hspace{1cm} (3)

If in equations (1), (2) we change variables to the relative position vector
$\vec{r} = r_1 - r_2$ and the mass center positions for $m_1, m_2$, then we can (a)
choose an inertial frame with origin at the mass center and (b) reduce the two-body
problem to the solution of the equivalent one-body equation

$$\ddot{\vec{r}} + \frac{\mu \hat{r}}{r^2} = 0, \hspace{1cm} (4)$$

where $\mu = G(m_1 + m_2)$.

Restrict the motion of the primaries to a circular motion at the appro-
priate constant angular velocity for a given distance $r_0 = |\vec{r}|$ between them
and introduce the rotating coordinate system $\vec{i} \equiv \hat{r}, \vec{j}, \vec{k}$ with origin at the
mass center, $\vec{i}$ along the line of the primaries, $\vec{j}$ in their orbital plane, and
$\vec{k}$ perpendicular to it. In this coordinate system, call $r_1 = -x_1 \vec{i}$, $r_2 = x_2 \vec{i}$,
$r_3 = x_3 \vec{i} + y_3 \vec{j} + z_3 \vec{k}$, where $x_1 > 0$, $x_2 > 0$.

The problem you are to solve is:

(a) Show that the point $r_3^* = (x_2 - x_1) \vec{i}$ is a position of equilibrium
for $m_3$ in the rotating coordinate system; and

(b) Study the stability of small motions near this point.