Consider a Cartesian $N \times N \times N$ finite element mesh consisting of trilinear elements, and with $N >> 1$. We are using this mesh to discretize a linear, static problem with one unknown per node. In the resulting linear equation system

$$Ax = b,$$  
(1)

$x$ is the vector of nodal values of the unknown, and

$$A = \sum_{e=1}^{N^3} A^e,$$  
(2)

$$b = \sum_{e=1}^{N^3} b^e,$$  
(3)

where $A^e$ and $b^e$ are the contributions from Element $e$ to Matrix $A$ and Vector $b$, respectively. Do not assume that $A$ is symmetric.

1. Assuming that Eq. (1) is being solved by using a direct (non-iterative) method, give an estimate of the amount of memory needed to store Matrix $A$.

2. Now assume that Eq. (1) is being solved by using an iterative method, based on massively parallel implementation, with each element assigned to a different processor (or virtual processor). Assume that the evaluation of the residual

$$r = b - Ax$$  
(4)

is the only computational stage where matrix $A$ appears.

(a) Give an estimate of the amount of memory needed for the storage requirement attributable to Matrix $A$.

(b) Describe at least one scenario of carrying out the computations given by Eq. (4) in a massively parallel way.

3. Can the computations given by Eq. (4) be carried out without the memory need for storage attributable to Matrix $A$? If your answer is “Yes”, explain how.