Consider a boundary-value problem in its strong form:

Given $f : \Omega \rightarrow \mathbb{R}$ and constants $g$ and $h$, find $u : \tilde{\Omega} \rightarrow \mathbb{R}$, such that

\[
\begin{align*}
    u_{xx} + f &= 0 \quad \text{on } \Omega \\
    u(1) &= g \\
    -u_x(0) &= h
\end{align*}
\]

and its weak form:

Given $f$, $g$ and $h$ as before, find $u \in \mathcal{S}$, such that for all $w \in \mathcal{V}$

\[
\int_0^1 w_{xx}u_{xx} \, dx = \int_0^1 wf \, dx + w(0)h
\]

where $\mathcal{S} = \{u | u \in H^1, u(1) = g\}$, $\mathcal{V} = \{w | w \in H^1, w(1) = 0\}$ and $\Omega = ]0, 1[$. In a standard finite element course we prove the equivalence of $(S_1)$ and $(W_1)$ in two steps: $(S_1) \Rightarrow (W_1)$ and $(S_1) \Leftarrow (W_1)$. In the second step, starting with $(W_1)$, we show that $u$ must satisfy both $u_{xx} + f = 0$ and $u_x(0) + h = 0$. In this step we make an assumption that the functions involved are sufficiently smooth, with continuous first derivatives.

Repeat the proof for the case when the functions involved are smooth on element interiors, i.e., $]x_A, x_{A+1}[$, $A = 1, \ldots, n$, but the continuity of the derivatives at element boundaries cannot be assumed.