A. Consider a flow field with an axisymmetric vorticity distribution, as sketched below. The effect of viscosity is to cause the vorticity to diffuse over time. For some vorticity fields, the diffusion may be counteracted by placing a sink at the origin (approximated by placing a porous pipe along the \( \hat{z} \) axis). For what distributions of vorticity is this possible? You may assume that the flow is uniform in the axial (\( \hat{z} \)) direction, that the flow is incompressible, that fluid is Newtonian, that the viscosity is constant, and that the vorticity field is finite – in particular that, as \( r \to \infty \), \( \partial \omega_z / \partial r \to 0 \) at least as fast as \( 1/r \), and \( |\bar{\omega}| \to 0 \).

B. Another method to cancel viscous diffusion is to (strain) stretch the flow field along the \( \hat{z} \) axis. Replacing the two-dimensional sink in part (A) by a uniform strain rate \( \partial u_z / \partial z \equiv \Delta = \text{constant} \), determine the appropriate problem for the vorticity distribution. Do not attempt to solve this problem; rather, give a physical interpretation of the different terms in your equations. Describe the similarities and the differences in the physical mechanisms present in parts (A) and (B).

C. Use your insight and the results from parts (A) and (B) to describe the behavior of a vortex ring as it travels directly towards an impermeable wall. (Assume the vorticity in the vortex ring is distributed in space.) How might this flow be similar to the approach of a pair of two-dimensional line vortices? We are not looking for equations here; rather, we expect a description of the impact of the physical processes present in parts (A) and (B) on the motion at different stages in the trajectories.
In cylindrical coordinates, where

\[ \vec{A} = A_r \hat{r} + A_\theta \hat{\theta} + A_z \hat{z}, \]
\[ \vec{B} = B_r \hat{r} + B_\theta \hat{\theta} + B_z \hat{z}, \]

and

\[ \hat{z} = \hat{r} \times \hat{\theta} \]

the following vector operations are defined

\[ \nabla \cdot \vec{A} = \frac{1}{r} \frac{\partial}{\partial r} (r A_r) + \frac{1}{r} \frac{\partial A_\theta}{\partial \theta} + \frac{\partial A_z}{\partial z} \]

\[ \nabla \times \vec{A} = \left[ \frac{1}{r} \frac{\partial A_z}{\partial \theta} - \frac{\partial A_\theta}{\partial z} \right] \hat{r} + \left[ \frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r} \right] \hat{\theta} + \frac{1}{r} \left[ \frac{\partial}{\partial r} (r A_\theta) - \frac{\partial A_r}{\partial \theta} \right] \hat{z} \]

\[ \vec{A} \cdot \nabla \vec{B} = \left[ A_r \left( \frac{\partial B_r}{\partial r} \right) + A_\theta \left( \frac{1}{r} \frac{\partial B_r}{\partial \theta} - \frac{B_\theta}{r} \right) + A_z \left( \frac{\partial B_r}{\partial z} \right) \right] \hat{r} \]
\[ + \left[ A_r \left( \frac{\partial B_\theta}{\partial r} \right) + A_\theta \left( \frac{1}{r} \frac{\partial B_\theta}{\partial \theta} + \frac{B_r}{r} \right) + A_z \left( \frac{\partial B_\theta}{\partial z} \right) \right] \hat{\theta} \]
\[ + \left[ A_r \left( \frac{\partial B_z}{\partial r} \right) + A_\theta \left( \frac{1}{r} \frac{\partial B_z}{\partial \theta} \right) + A_z \left( \frac{\partial B_z}{\partial z} \right) \right] \hat{z} \]

\[ \nabla^2 \vec{A} = \left[ \frac{\partial}{\partial r} \left( \frac{1}{r} \frac{\partial}{\partial r} (r A_r) \right) + \frac{1}{r^2} \frac{\partial^2 A_r}{\partial \theta^2} + \frac{2}{r^2} \frac{\partial A_\theta}{\partial \theta} - \frac{2}{r^2} \frac{\partial A_\theta}{\partial \theta} \right] \hat{r} \]
\[ + \left[ \frac{\partial}{\partial r} \left( \frac{1}{r} \frac{\partial}{\partial r} (r A_\theta) \right) + \frac{1}{r^2} \frac{\partial^2 A_\theta}{\partial \theta^2} + \frac{2}{r^2} \frac{\partial A_r}{\partial \theta} \right] \hat{\theta} \]
\[ + \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial A_z}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 A_z}{\partial \theta^2} + \frac{\partial^2 A_z}{\partial z^2} \right] \hat{z} \]