Final report on research on “Aerodynamic Breakup of Liquids”

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Aerodynamic Breakup of Liquids

I have been doing some work to support the interpretation of the results from shock tube studies for Douglas R. Sommerville as specified under Schedule of Supplies/Services on the second page of the purchase order.

I have analyzed Rayleigh-Taylor stability of breaking drops in a high speed air stream behind a shock wave. The analysis is given in the paper “Breakup of a liquid drop suddenly exposed to a high speed airstream” by Joseph, Belanger & Beavers which is here submitted for documentation in this second invoice. The analysis in this paper is mathematically rigorous and in excellent agreement with experiments. The analysis leads to a statement of the smallest size of drops which can arise from RT instability (see equations [39] and [35] of the enclosed paper).

A revised and improved paper on breakup mentioned in the last paragraph has been accepted for publication in the International Journal of Multiphase Flow and will be published in November. This paper is included here as documentation for this final report.

The breakup paper did not explain the experimental measurements in the case of viscoelastic (thickened) liquid.

The experimental results we found for the thickened liquids were unexpected. The time to breakup was much smaller than the time to breakup Newtonian liquid and the drop fragments after breakup were larger and stringier than the Newtonian case. I carried out an analysis of Rayleigh-Taylor instability using the popular Oldroyd B model of viscoelastic fluid. First, I did an analysis of viscoelastic potential flow and found that the analysis given in our paper on “Breakup of a liquid drop suddenly exposed to a high speed airstream” for Newtonian fluids could be corrected by replacing the viscosity with the viscosity times the factor:

\[
\frac{1 + \lambda_2n}{1 + \lambda_1n}
\]

Where \( n \) is the growth rate for \( e^{\nu} \), \( \lambda_1 \) is the relaxation time and \( \lambda_2 \) the retardation time of an Oldroyd B fluid. This replacement is made in equation [28] of the enclosed manuscript.
I also calculated the dispersion relation for the fully viscoelastic case. The relation was calculated for 2% Polyox; the relevant material and dynamic parameters are given in Table 1 and 2 (the relaxation time was measured on our wave speed meter) of the enclosed manuscript. The dispersion relations are given in the two figures A and B in this report. There is no essential difference in the dispersion relations, but the potential flow analysis is much easier. The growth rates are much larger implying shorter breakup times in viscoelastic fluids in agreement with observations. The analysis also predicts much shorter wave lengths than in a Newtonian fluid with the same viscosity.

To check the foregoing prediction we image processed the polyox drop (Figure C) and found an instability wave length of approximately 0.25 mm. We are able to match this wave length using measured values of acceleration, viscosity and the relaxation time with a retardation time $1/1000$ times the relaxation time $\lambda_2 = \lambda_1 /1000$. All the qualitative features of the drop breakup are obtained by analysis.