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D. D. Joseph, Luigi Preziosi

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D. D. Joseph

Department of Aerospace Engineering and Mechanics, University of Minnesota, Minneapolis, Minnesota 55455

Luigi Preziosi

Dipartimento di Matematica e Applicazioni, Universitá di Napoli, Via Mezzocannone 8, 80134 Napoli, Italy

The concept of transmission of heat by waves is reviewed and interpreted. The notion of an effective thermal conductivity, an effective heat capacity, and relaxation functions for heat and energy is introduced along lines used recently to describe the elastic response of viscous liquids. An annotated bibliography of the literature on heat waves, from the beginning until now, gives a complete or nearly complete survey of the history of this subject.

CONTENTS

I. INTRODUCTION

We are considering the problem of heat transported by conduction in which the heat pulses are transmitted by waves at finite but perhaps high speeds. We are interested in applying some ideas that seem appropriate for describing shear waves in liquids (Joseph *et al.*, 1986) to the problem of propagation of heat. In this theory, one may have fast waves carrying small amounts of heat and slower speeds carrying larger amounts of heat. In the linearized theory, the heat flux is determined by an integral over the history of the temperature gradient weighted against a relaxation function called the heat-flux kernel. The area under the curve giving the monotonic relaxation of the heat-flux kernel is the thermal conductivity. The energetic effect of modes that decay rapidly at times long for the fast modes and short for the slow modes is absorbed by an effective thermal conductivity associated with the portion of the relaxation that has already relaxed. The effective thermal conductivity can be acknowledged explicitly in mathematical formulation analogous to the well-known model of Jeffreys for the stress and strain rate in liquids. In an idealized solid, for example, thermal energy is transported by two different mechanisms: by quantized electronic excitations, which are called free electrons, and by the quanta of lattice vibrations, which are called phonons. These quanta undergo collisions of a dissipative nature, giving rise to thermal resistance in the medium. A relaxation time τ is associated with the average communication time between these collisions for the commencement of resistive flow. There are different times of relaxation, so that the mean relaxation time τ is not generally known. Indeed, there may be a spectrum of relaxation times in most solids giving rise to different speeds of propagation of heat. For such solids, it would be more important to know what modes carry the most heat, so we want the dominant rather than the mean mode of relaxation.

The notion of an effective thermal conductivity and an effective heat-flux kernel with related concepts for the internal energy will be treated later. For now, it will suffice to establish some common notations.

θ	temperature
$\mathbf{q}$	heat flux
τ	relaxation time
$k=k_1+k_2$	thermal conductivity
k_1	effective thermal conductivity
k_2	elastic conductivity
e	internal energy
γ	heat capacity
c	wave speed
c_1	sound speed
c_2	speed of temperature waves, second sound

Another group of notations, to be used later, is introduced in Sec. VI.

In Secs. I–VII of this paper, we deal with linear theories.[1] A heat-flux equation of the Jeffreys type can be expressed as

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[1]We have tried to review all works on heat waves, linear and nonlinear, in our chronology (Sec. VIII) of thoughts about heat waves. It is certain that we have not found every reference and equally certain that we have not missed many. Our reviews of these papers are slightly personal; we took the liberty of expressing some opinions and some ideas of our own.

$$\tau\frac{\partial\mathbf{q}}{\partial t}+\mathbf{q}=-k\nabla\theta-\tau k_1\frac{\partial}{\partial t}\nabla\theta\ . \tag{1.1}$$

The physical ideas leading to Eq. (1.1) will be discussed in Sec. VI. If $k_1=0$, then Eq. (1.1) reduces to

$$\tau\frac{\partial\mathbf{q}}{\partial t}+\mathbf{q}=-k\nabla\theta\ . \tag{1.2}$$

We shall call Eq. (1.2) Cattaneo's equation. When $\tau=0$, Eq. (1.2) reduces to Fourier's law and, if $de=\gamma\, d\theta$, as for a solid, then the energy equation[2]

$$\frac{de}{dt}=-\mathrm{div}\mathbf{q} \tag{1.3}$$

leads to diffusion,

$$\frac{\partial\theta}{\partial t}=\kappa\nabla^2\theta\ , \tag{1.4}$$

where $\kappa=k/\gamma$ is the thermal diffusivity. The diffusion equation has the unphysical property that if a sudden change of temperature is made at some point on the body, it will be felt instantly everywhere, though with exponentially small amplitudes at distant points. In a loose manner of speaking, we may say that diffusion gives rise to infinite speeds of propagation.

The temperature of a body is the macroscopic consequence of certain kinds of vibratory motions, the motions of molecules of a gas or the vibrations of a lattice in a solid on microscopic scales. Heat is transported by near-neighbor excitation in which changes of momentum and energy on a microscopic scale are propagated as waves.

An "inertial" theory of heat conduction can be obtained from Eq. (1.2) in an appropriate limit $\tau\to\infty$, $k\to\infty$, $k/\tau=\beta$ finite

$$\frac{\partial\mathbf{q}}{\partial t}=-\beta\nabla\theta\ . \tag{1.5}$$

This shows already that a finite thermal conductivity arises from damping. A wave, rather than diffusion, equation is implied by Eqs. (1.3) and (1.5),

$$\frac{\partial^2\theta}{\partial t^2}=c^2\nabla^2\theta\ , \tag{1.6}$$

where, assuming constant properties, $c=\sqrt{k/\tau\gamma}$ is the speed of the thermal wave. The wave equation has the unacceptable property that wave pulses are propagated without attenuation; even worse, we could never establish steady heat flow with temperature varying from point to point.

If no terms are omitted from Eq. (1.2), we may combine Eqs. (1.3) and (1.2) to form a telegraph equation,

$$\frac{\partial^2\theta}{\partial t^2}+\frac{1}{\tau}\frac{\partial\theta}{\partial t}=c^2\nabla^2\theta\ . \tag{1.7}$$

Equation (1.7) is hyperbolic and it transmits waves of temperature with a speed c. The waves are attenuated as a result of relaxation, and steady heat flow may be induced by temperature gradients. It is obvious that Cattaneo's law has many desirable properties.

The development of ideas about heat waves is a stream with many tributaries (see Sec. VIII for a chronology). Two problems are the source of this stream: the problem of infinite wave speeds and the problem of second sound. The problem of second sound arose first in studies of Tisza (1938) and Landau (1941) of heat waves in liquid helium II.

The problem that infinite speeds of propagation are generated by diffusion evidently first appeared in the work of Cattaneo (1948) and was apparently addressed independently by Morse and Feshbach (1953) and Vernotte (1958a). Their objections to diffusion seem not to have generated any resistance, and, as nearly as we can tell, everyone agrees that heat pulses ought to be transported by waves. This does not mean that there are big movements afoot to discard Fourier's law. The relaxation time in Eq. (1.2) is thought to be very small in nearly all practical and even exotic applications, so that as a practical matter it is believed that we get Fourier's law even on the shortest time scales of our daily lives. In fact, in our view, an understanding of times scales is the central object of scientific investigations of heat waves, and is only imperfectly understood.

Cattaneo's equation (1.2) has been derived in different ways by different authors. Derivations based on kinetic theory can be found in the works of Maxwell (1867), Cattaneo (1948), and Grad (1958). Maxwell cast out the time derivative because it "... may be neglected, as the rate of conduction will rapidly establish itself." In fact, the works of Maxwell and Grad are rather more general than Cattaneo's, and they did not come to grips with the problem of heat propagation. "More general" is not necessarily "better." Cattaneo's equation is written down by Vernotte (1958a) as the most obvious and simple generalization of Fourier's law that will give rise to finite speeds of propagation. Rate laws like Cattaneo's are very well known and extensively used in the theory of viscoelastic fluids and solids and in relaxing gas dynamics.

The time derivative in Eq. (1.2) can be described as a thermal "inertia" [see Eq. (1.5)]. Nernst (1917) suggested that in good thermal conductors at low temperatures heat may have sufficient "inertia" to give rise to "oscillatory discharge." Onsager (1931) noted that the Fourier law contradicted the principle of microscopic reversibility used in his thermodynamics, a contradiction that "... is removed when we recognize that [the Fourier law] is only an approximate description of the process of conduction, neglecting the time needed for acceleration of the heat flow" (Onsager, 1931, p. 419). A theory in which "thermal" inertia is postulated in the context of a

[2]Equation (1.3) is the energy equation for a rigid conductor. Various other terms appear in the balance of energy for deformable bodies. Constitutive equations like Eq. (1.2) can be postulated for both rigid and deformable conductors, but Eq. (1.3) applies only to rigid bodies.

dynamical generalization of Onsager's theory is given by Kaliski (1965). After some approximations, he arrives at a telegraph equation for the temperature. Earlier, Nettleton (1960) postulated Cattaneo's equation for liquids and showed it to be compatible with irreversible thermodynamics. Different modern authors have postulated a constitutive equation like Cattaneo's, or generalized forms of it, and derived consistency relations for one or another form of thermodynamics; for example, see Gurtin and Pipkin (1968), Müller (1969), Meixner (1970), Morro (1980a, 1980b), Coleman and co-workers (1982, 1987). One aim of the first and last mentioned authors was to determine nonlinear effects, if any, on the propagation of heat, with one result given by Gurtin and Pipkin and the opposite one by Coleman, Fabrizio, and Owen (1982).

Boltzmann's equation for the distribution function for particles—molecules in a gas, electrons and phonons in solids—plays an important role in the chronology of thought about heat waves. It was first used by Ward and Wilks (1952) to derive a wave equation for second sound in helium II. Tavernier (1962) derived an equation like Eq. (1.2) from Boltzmann's equation using a finite difference approximation to the collision term. Callway's approximation of the collision term with two relaxation times forms the basis of the study of Guyer and Krumhansl (1964) of the frequency window for the passage of temperature waves through dielectric crystals at low temperatures (second sound). A more complete solution using eigenfunctions of the normal-process collision operator was given by Guyer and Krumhansl (1966a). This last paper and one by Kwok (1967) lead to different macroscopic equations for heat flow and are based on Boltzmann's equation. Hardy (1970) solved the complete linearized Boltzmann equation in terms of eigenvectors of the collision matrix, not only for normal processes but with umklapp processes included. Maurer (1969) derived a relaxation model leading to a telegraph equation for the heat flux in metals from the quantum-mechanical form of the Boltzmann transport equation. Beck (1975) wrote an extended and critical review of the physics literature, including applications of Boltzmann's equation to second sound and related thermal conduction phenomena.

It is perhaps important, and is certainly interesting, that the telegraph equation, which is the simplest mathematical model combining waves and diffusion, can be derived as limiting cases in the problem of random walks. The 1951 paper of Goldstein is evidently the first effort in this direction. He treats several versions of the problem of random walks in one dimension and derives the telegraph equation with leakage from another. In both cases, the directions in any two consecutive intervals are correlated. If this correlation is relaxed, a diffusion equation rather than a telegraph equation appears. Goldstein draws analogies between solutions of the telegraph equation for heat conduction, but does not suggest that the diffusion equation for heat ought to be replaced. A similar analysis of a one-dimensional random walk with correlation, leading to yet another form of the telegraph equation, in which first as well as second spatial derivatives appear, was given by Weyman (1965). His work was motivated by the desire to correct the infinite propagation speeds associated with diffusion.

Turning now to second sound, we note that whereas Cattaneo proposed to correct diffusion for effects associated with thermal inertia, which in gases could be expected to be important only for surpassingly small times, Band and Meyer (1948), the same year as Cattaneo, and Osborne (1950), only two years later, proposed exactly the same telegraph equation (1.7), but with the first time derivative added to the wave equation to account for dissipative effects in liquid helium II. We have just said that in 1948 the diffusion equation was corrected for infinite wave speeds by adding the second derivative, and the wave equation was corrected for the lack of dissipation by adding a first derivative. The Tisza-Landau prediction of temperature waves in liquid helium II and the subsequent verification of this prediction by Peshkov (1944) and others stimulated interest among physicists in propagation of waves of heat. This interest might have waned if the phenomenon of heat waves were confined to helium II. However, already in 1946, Peshkov noted that "a gas of thermal quanta capable of performing vibrations similar to those of sound should exist." Not long after, Ward and Wilks (1951) derived the Landau expression, $c_2=c_1/\sqrt{3}$, for the speed of second sound without recourse to a two-fluid model. In 1952, they derived a wave equation for the propagation of second sound in a phonon gas by neglecting dissipation and using Boltzmann's equation for the distribution function for phonons. All of the work on heat waves since the early studies of liquid helium II have been motivated in one way or another by the problems of either infinite speeds or second sound in solids, with one exception.

The exception is the molecular dynamic calculations of wave propagation on a lattice of atoms under forces for an iron alloy done by MacDonald and Tsai in the 1970s. They were interested in extremely high temperatures and pressure. Their approach is fundamentally different from all the others. The degree of agreement between the usual theories of second sound at low temperatures and their computation of propagation on a lattice at high temperature, but short times, is astonishing.

II. AN EFFECTIVE THERMAL CONDUCTIVITY AND RELAXATION KERNEL FOR CONDUCTORS OF THE JEFFREYS TYPE

Cattaneo's equation (1.2) can be expressed as an integral over the history of the temperature gradient,

$$\mathbf{q}=-\frac{k}{\tau}\int_{-\infty}^{t}\exp\left[-\frac{t-t'}{\tau}\right]\nabla\theta(\mathbf{x},t')dt' . \tag{2.1}$$

A more general form for the heat flux is

$$\mathbf{q}=-\int_{-\infty}^{t}Q(t-t')\nabla\theta(\mathbf{x},t')dt' , \tag{2.2}$$

where $Q(s)$ is a positive, decreasing relaxation function that tends to zero as $s \to \infty$. Integral expressions like Eq. (2.2) are used in Boltzmann's theory of linear viscoelasticity to express the present value of the stress in terms of past values of the strain or strain rate (Joseph, 1986). Many different constitutive models arise from different choices of $Q(s)$. If $Q(s)=k\delta(s)$ where $\delta(s)$ is a one-sided Dirac delta function,

$$\int_0^{0_+} \delta(s)ds = 1 ,$$

then $\mathbf{q}=-k\nabla\theta$ is Fourier's law. Gurtin and Pipkin (1968) were the first to write Eq. (2.2), but under assumptions that disallow a delta function in the kernel. In this case, $Q(0)$, the instantaneous modulus, which we call the heat rigidity, is bounded. Nunziato (1971) added a Fourier term to Eq. (2.2), producing an equation that is the same as Eq. (2.2) with a delta function in the kernel.

The thermal conductivity k is defined for steady temperature in which $\theta(\mathbf{x},t)$ is independent of t. In this case, Eq. (2.2) implies that

$$\mathbf{q}=-k\nabla\theta , \tag{2.3}$$

where

$$k=\int_{-\infty}^{t} Q(t-t')dt' = \int_0^{\infty} Q(s)ds . \tag{2.4}$$

Since k is given by the area under the curve, the slower the relaxation for a given ridigity, the larger the value of k.

It would be a miracle if for some real conductor the relaxation kernel could be rigorously represented by an exponential kernel with a single time of relaxation, as is required by Cattaneo's model. The nature of the thermal response, as we shall see, depends critically on what is assumed about Q.

We may define a kernel of the Jeffreys type

$$Q(s)=k_1\delta(s)+\frac{k_2}{\tau}e^{-s/\tau} , \tag{2.5}$$

where $\delta(s)$ is a Dirac delta function and k_1 and k_2 are constants. The kernel Eq. (2.5) gives rise to a heat-flux law of the Jeffreys type,

$$\mathbf{q}=-k_1\nabla\theta(\mathbf{x},t)-\frac{k_2}{\tau}\int_{-\infty}^{t} \exp\left[-\frac{t-t'}{\tau}\right]\nabla\theta(\mathbf{x},t')dt' , \tag{2.6}$$

in which an effective Fourier conductivity k_1 is explicitly acknowledged. Evaluation of Eq. (2.6) on steady flow gives rise to

$$\mathbf{q}=-(k_1+k_2)\nabla\theta(\mathbf{x}) . \tag{2.7}$$

It follows that thermal conductivity

$$k=k_1+k_2 \tag{2.8}$$

corresponding to (2.6) is the sum of an effective conductivity k_1 and an elastic conductivity

$$k_2=\int_0^{\infty} Q_2(s)ds , \tag{2.9}$$

where

$$Q_2(s)=\frac{k_2}{\tau}\exp\left[\frac{-s}{\tau}\right] . \tag{2.10}$$

We call Eq. (2.5) a kernel of the Jeffreys type because it is the integrated form of Eq. (1.2), and Eq. (1.3) is the differentiated form of Eq. (2.5). It is perhaps necessary to remark that the name Jeffreys is attached to Eq. (2.5) by analogy; Jeffreys wrote these things about stress and deformation, but said not one word about propagation of heat.

All the foregoing will be generalized in Sec. VI. It is necessary first to discuss some dynamic consequences of our constitutive models (Sec. III) and to interpret our parameters in terms of the parameters used by physicists to describe the propagation of heat at low temperatures in dielectric crystals (Sec. IV).

III. THERMAL RESPONSE OF CATTANEO-TYPE AND JEFFREYS-TYPE CONDUCTORS TO A SUDDEN CHANGE OF TEMPERATURE

The thermal response of motionless conductors following from constitutive equations that have an effective conductivity [Eqs. (1.2) or (2.5)] is determined by a second-order partial differential equation of the Jeffreys type,

$$\frac{\partial^2\theta}{\partial t^2}+\frac{1}{\tau}\frac{\partial\theta}{\partial t}=c^2\nabla^2\theta+\kappa_1\nabla^2\frac{\partial\theta}{\partial t} , \tag{3.1}$$

where $c^2=k/\gamma\tau$, $\kappa_1=k_1/\gamma$. When $\kappa_1=0$, Eq. (3.1) reduces to a telegraph equation (1.7), which is a hyperbolic equation that allows for propagation of discontinuities with constant speed c.

When $\kappa_1\neq 0$, Eq. (3.1) is parabolic and discontinuities are smoothed by diffusion associated with the effective thermal conductivity k_1. If $\kappa_1/\tau c^2=1$, then Eq. (3.1) reduces to a diffusion equation

$$\frac{\partial\phi}{\partial t}=\kappa_1\nabla^2\phi, \quad \phi=\frac{\partial\theta}{\partial t}+\frac{c^2}{\kappa_1}\theta . \tag{3.2}$$

To understand well the difference between Eq. (3.1) and the telegraph equation (1.7) to which Eq. (3.1) reduces when $\kappa_1=0$, we consider the problem of the transient distribution in a semi-infinite heat conductor, $x>0$, after a sudden change of temperature at $x=0$. We would need to solve

$$\frac{\partial^2\theta}{\partial t^2}+\frac{1}{\tau}\frac{\partial\theta}{\partial t}=c^2\frac{\partial^2\theta}{\partial x^2}+\kappa_1\frac{\partial^2}{\partial x^2}\left[\frac{\partial\theta}{\partial t}\right] ,$$

$$\theta=\begin{cases}0 & \text{when } t=0 \text{ for all } x\geq 0,\\ 1 & \text{at } x=0 \text{ when } t=0,\\ 0 & \text{at } x=\infty .\end{cases} \tag{3.3}$$

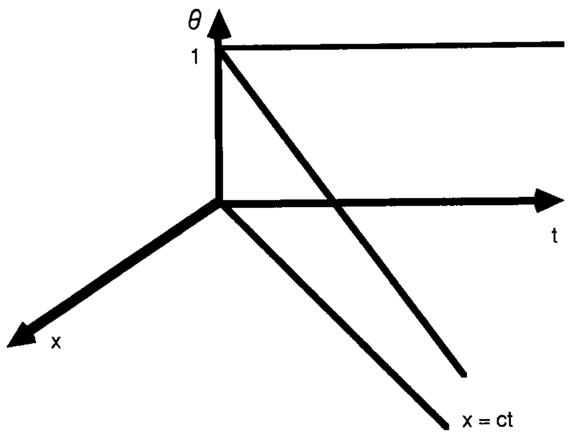

FIG. 1 Solution surface for Eq. (3.3) when $\kappa_1=0$.

This problem is the analog of Stokes's first problem in fluid mechanics. An extensive discussion of solutions in this and more complicated cases can be found in recent papers (Narain and Joseph, 1982; Joseph, Narain, and Riccius, 1986); Preziosi and Joseph, 1987). It is enough to note that, when $\kappa_1=0$, the discontinuity imposed by the sudden jump of temperature at $x=0$ and $t=0_+$ propagates at constant speed as a discontinuity without smoothing. The amplitude $\theta(ct,t)=\exp(-t/2\tau)$ at the shock front $x=ct$ of the wave decays exponentially, as shown in Fig. 1.

The wave propagation shown in Fig. 1 will not be possible when $\kappa_1\neq0$. In this case, we get diffusion with immediate smoothing of the wave and the information of a change in the temperature at $x=0$ is felt everywhere immediately. There is, of course, a singular perturbation type of continuity with κ_1 as it is increased through small values from zero. For very small values of κ_1, the solution surface is almost as shown in Fig. 2, except that the sharp front is rounded and θ is not zero when $x>ct$, $t>0$. The effect of a small κ_1 at the shock front is to smooth the discontinuity, creating a shock structure whose thickness is proportional to $\sqrt{k_1x/k}$ (see Fig. 2). This type of smoothing is analogous to the smoothing by viscosity of shock waves in gas dynamics. The speed of the smoothing wave at small values of x is c. A small effective conductivity will not extinguish effective wave

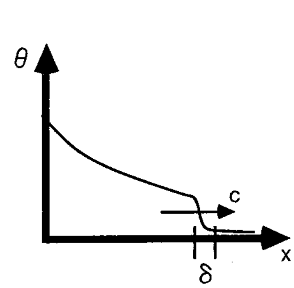

FIG. 2. Smoothing of a shock wave due to an effective conductivity, $c=(k/\gamma\tau_1)^{1/2}$, $\delta=(k_1x/k)^{1/2}$.

propagation at small times and distances. Numerical computations of solutions of Eq. (3.3) showing the effects of changing κ_1 have been carried out by Preziosi and Joseph (1987).

IV. MACROSCOPIC EQUATIONS OF GUYER AND KRUMHANSL FOR SECOND SOUND IN DIELECTRIC CRYSTALS

Guyer and Krumhansl (1966a) have solved the linearized Boltzmann equation for the pure phonon field in terms of the normal-process collision operator. They neglected electronic conduction, which would be important in metals but not in dielectrics, and they neglected other interactions in which momentum is lost from the phonon system. Their goal was to identify the parameters favorable to the passage of heat waves in dielectric crystals at low temperatures. They found the following macroscopic equations relating θ and $\mathbf{q}$:

$$\gamma\frac{\partial\theta}{\partial t}+\operatorname{div}\mathbf{q}=0\ , \tag{4.1}$$

$$\frac{\partial\mathbf{q}}{\partial t}+\frac{c^2}{3}\nabla\theta+\frac{1}{\tau_R}\mathbf{q}=\frac{\tau_Nc^2}{5}(\nabla^2\mathbf{q}+2\nabla\operatorname{div}\mathbf{q})\ . \tag{4.2a}$$

c^2 is the average (sound) speed of the phonons, τ_R is a relaxation time for momentum-nonconserving processes (the umklapp processes in which momentum is lost from the phonon system), and τ_N is a relaxation time for normal processes that preserve phonon momentum. In the regime of low temperatures where Eqs. (4.1) and (4.2a) are to apply, the heat flux is proportional to the momentum flux $\mathbf{p}$, $\mathbf{q}=c^2\mathbf{p}$, of the phonon gas.

Of course, Eq. (4.1) is the usual energy equation for a rigid conductor and Eq. (4.2a) is a constitutive expression that is supposed to apply under particular conditions specified in its derivation.

Equation (4.2) is close to, but not the same as, a heat flux equation (1.1) of the Jeffreys type, and it does not reduce to Fourier's law for steady flow. There is a new law of heat conduction,

$$\frac{c^2}{3}\nabla\theta+\frac{1}{\tau_R}\mathbf{q}=\frac{\tau_Nc^2}{5}\nabla^2\mathbf{q},\quad \operatorname{div}\mathbf{q}=0\ . \tag{4.2b}$$

Here $\mathbf{q}$ is the momentum flux of a viscous fluid, and θ is like the pressure in an incompressible fluid. The heat flow of Eq. (4.2b) is not necessarily down the temperature gradient. Sussman and Thellung (1963) and Gurzi (1964) have shown that, under conditions such that normal processes dominate umklapp process, heat transport in a stationary temperature gradient is mainly convective and due to a phonon drift of the Poiseuille flow type. The conditions require that $\tau_N\Omega\ll1\ll\Omega\tau_R$, where Ω is a frequency so that the second term in the above equation is small. This new type of heat transport has subsequently been found experimentally in helium IV crystals by Mezhov-Deglin (1964). Guyer and Krumhansl (1966b) solved the steady-state problem for one-dimensional flow

$\mathbf{q}=\mathbf{e}_1 q(r)$ in a cylinder with $q(R)$ where R is the radius of the cylinder. They showed how τ_N can be computed from measuring the thermal conductivity when the τ_R term is negligible.

We may learn something about the thermal response of Eqs. (4.1) and (4.2) by eliminating $\mathbf{q}$. We find a diffusion equation of the Jeffreys type (3.1),

$$\frac{\partial^2\theta}{\partial t^2}+\frac{1}{\tau_R}\frac{\partial\theta}{\partial t}=\frac{c^2}{3\gamma}\nabla^2\theta+\tfrac{3}{5}\tau_N c^2\nabla^2\frac{\partial\theta}{\partial t}\,, \tag{4.3}$$

with an effective thermal diffusivity

$$\kappa_1=\tfrac{3}{5}\tau_N c^2\,. \tag{4.4}$$

Equation (4.3) will not permit the propagation of waves unless $\kappa_1=0$. When $\kappa_1\neq 0$, the last term of Eq. (4.3) smooths discontinuities. Equation (4.3) with $\kappa_1\neq 0$ is not hyperbolic and has the conceptual problems of diffusion equations. A sudden change of temperature at some point is felt instantly everywhere (see Fig. 2).

The thermal response of Eq. (4.3) to a sudden change of temperature was described under Eq. (3.3). Small values of τ_N lead to small values of the effective diffusivity. Evidently the diffusivity of normal processes is a dominating feature of the thermal response, smoothing discontinuities. When $\kappa_1=0$, we get a propagating shock front with constant speed $c/\sqrt{3}$ whose amplitude decreases like $\exp(-t/2\tau_R)$ where τ_R is the relaxation time for processes that do not conserve phonon momentum. When τ_N is small, the speed is unchanged at lowest order, but a smooth layer (Fig. 2) of thickness $[(9\tau_N/5\tau_R)x]^{1/2}$ replaces the shock. It appears that the diffusivity of normal processes is even more effective in destroying wave propagation than the so-called momentum-nonconserving umklapp processes.

The effective conductivity can be associated with the viscosity of the phonon gas which leads to the broadening of the thermal pulse in the experiments of Ackerman and Guyer (1968) and to diffusive effects in the experiments of Rogers (1971).

Sussman and Thellung (1963) and Kwok (1967) have derived equations of the form (4.1) and (4.2) except that they both have a term proportional to $\nabla^2\theta$ in the energy Eq. (4.1). Such a term appears to be inconsistent with the balance of energy in rigid conductors. Guyer and Krumhansl seem to indicate that the term arises from an inconsistent approximation.

V. THE EQUATIONS OF GURTIN AND PIPKIN

Gurtin and Pipkin (1968) gave a general constitutive theory for rigid heat conductors that propagate waves. Their theory was an application of mathematical methods then in use in continuum mechanics and thermodynamics. They said that their theory differed from others "... in that the heat-flux, like the entropy, is determined by the functional for the free energy." Their method required that they characterize domain space of functionals, and they chose the weighted L_h^2 space (Coleman and Noll, 1960). This is a space of functions of s whose squares are integrable on $s\in[0,\infty]$ against a decaying positive weight $h(s)$ such that $s^2h(s)$ is integrable. The Riesz representation theorem then implies that linearized flux laws may be represented as in Eq. (2.2), and application of Schwarz's inequality then shows that $Q(\ )/h(\ )$ must also be quadratically integrable against $h(s)$. This means that there can be no Dirac measures in the kernel, so that an effective viscosity is ruled out. Many other choices for the allowed domain of functionals are possible, and each one leads to different laws for heat conduction (Saut and Joseph, 1983). Another method is to let the heat flux depend on the instantaneous value of heat flux and the history of the heat flux in the same L_h^2 setting. This is the method followed by Nunziato (1971). In this case, it is not true that the heat flux is determined by a functional of the free energy.

At the end of the analysis of Gurtin and Pipkin (1968), after linearization, the expressions for the internal energy $e(\mathbf{x},t)$ and the heat flux $\mathbf{q}(\mathbf{x},t)$ are

$$e(\mathbf{x},t)=b+\gamma\theta(\mathbf{x},t)+\int_0^\infty F(s)\theta(\mathbf{x},t-s)ds \tag{5.1}$$

and

$$\mathbf{q}(\mathbf{x},t)=-\int_0^\infty Q(s)\nabla\theta(\mathbf{x},t-s)ds\,, \tag{5.2}$$

where $F(0)$ and $Q(0)$ are bounded, and θ and $\nabla\theta$ are quadratically integrable functions of s on a weighted L_h^2 space. The energy equation $\partial e/\partial t=-\mathrm{div}\mathbf{q}$ and Eqs. (5.1) and (5.2) imply that

$$\gamma\frac{\partial\theta}{\partial t}+\int_0^\infty F(s)\frac{\partial\theta(\mathbf{x},t-s)}{\partial t}ds=\int_0^\infty Q(s)\nabla^2\theta(\mathbf{x},t-s)ds\,. \tag{5.3}$$

After some manipulation Eq. (5.3) may be written as

$$\gamma\frac{\partial^2\theta(x,t)}{\partial t^2}+F(0)\frac{\partial\theta(\mathbf{x},t)}{\partial t}+\int_0^\infty F'(s)\frac{\partial\theta(\mathbf{x},t-s)}{\partial t}ds=Q(0)\nabla^2\theta(\mathbf{x},t)+\int_0^\infty Q'(s)\nabla^2\theta(\mathbf{x},t-s)ds\,. \tag{5.4}$$

Equation (5.4) is hyperbolic; discontinuities will propagate with constant speed,

$$c=\sqrt{Q(0)/\gamma}\,. \tag{5.5}$$

In the special circumstance under which the heat flux and internal energy kernels are both exponential with

$$Q(s)=Q(0)e^{-s/\tau},\quad F(s)=F(0)e^{-s/\lambda}\,, \tag{5.6}$$

we may simplify Eq. (5.4). First, we find with $f=e-b$ that

$$\lambda\frac{\partial f}{\partial t}+f=\lambda\gamma\frac{\partial\theta}{\partial t}+[\lambda F(0)+\gamma]\theta\,,$$

$$\frac{\partial f}{\partial t}=-\mathrm{div}\mathbf{q},\quad \tau\frac{\partial\mathbf{q}}{\partial t}+\mathbf{q}=-\tau Q(0)\nabla\theta\,.$$

After eliminating f and $\mathbf{q}$ from these equations, we find that

$$\gamma\frac{\partial^3\theta}{\partial t^3}+\left[F(0)+\frac{\gamma}{\tau}+\frac{\gamma}{\lambda}\right]\frac{\partial^2\theta}{\partial t^2}+\left[\frac{F(0)}{\tau}+\frac{\gamma}{\lambda\tau}\right]\frac{\partial\theta}{\partial t}=Q(0)\nabla^2\frac{\partial\theta}{\partial t}+\frac{Q(0)\nabla^2\theta}{\lambda}\ . \quad (5.7)$$

Equation (5.7), like Eq. (5.4), is a hyperbolic equation with a constant wave speed given by Eq. (5.5).

Gurtin and Pipkin (1968) assumed conditions that imply that $Q(s)$ is a bounded kernel with no delta function, as in Eq. (2.5), and that $\gamma\neq 0$. If either of these assumptions is relaxed, hyperbolicity is lost. For example, if $\gamma=0$, then Eq. (5.7) reduces to a diffusion equation of the Jeffreys type, and if, in addition, $\tau=\lambda$ then

$$\frac{\partial\theta}{\partial t}=\frac{Q(0)}{F(0)}\nabla^2\theta\ .$$

It could be argued that consistency requires that the heat flux and the internal energy both depend on present values or that they both do not. In either case, we lose hyperbolicity and finite wave speeds. In Sec. VI below, we first assume that they both are independent of present values and show how the equations arising from this assumption lead to an effective dependence on present values.

VI. ORIGIN OF EFFECTIVE CONDUCTIVITY, EFFECTIVE CAPACITY, ELASTIC CONDUCTIVITY, ELASTIC CAPACITY

Now we rewrite Eq. (5.1) as

$$e(\mathbf{x},t)=b+\int_0^\infty E(s)\theta(\mathbf{x},t-s)ds \quad (6.1)$$

and allow that

$$E(s)=\gamma_1\delta(s)+E_2(s)\ , \quad (6.2)$$

where γ_1 is the effective capacity. [Clearly Eq. (6.1) is exactly the same as Eq. (5.1) with $\gamma_1=\gamma$, $E_2=F$.] At the same time, we write

$$Q(s)=k_1\delta(s)+Q_2(s)\ ,$$

where k_1, as we already know, is the effective conductivity. We have

$$e-b=\gamma_1\theta(\mathbf{x},t)+\int_0^\infty E_2(s)\theta(\mathbf{x},t-s)ds \quad (6.3)$$

and

$$\mathbf{q}=-k_1\nabla\theta(\mathbf{x},t)-\int_0^\infty Q_2(s)\nabla\theta(\mathbf{x},t-s)ds\ . \quad (6.4)$$

In steady flow,

$$e-b=(\gamma_1+\gamma_2)\theta(\mathbf{x})\ ,$$

$$\mathbf{q}=-(k_1+k_2)\nabla\theta(\mathbf{x})\ ,$$

where $\gamma_2=\int_0^\infty E_2(s)ds$ is the elastic capacity, $k_2=\int_0^\infty Q_2(s)ds$ is the elastic conductivity, $\gamma=\gamma_1+\gamma_2$ is the heat capacity, and γ_1 is the effective heat capacity. We may now express the energy equation $\partial e/\partial t=-\mathrm{div}\mathbf{q}$ as

$$\gamma\frac{\partial\theta(\mathbf{x},t)}{\partial t}+\int_0^\infty E_2(s)\frac{\partial\theta(\mathbf{x},t-s)}{\partial t}ds=k_1\nabla^2\theta(\mathbf{x},t)+\int_0^\infty Q_2(s)\nabla^2\theta(\mathbf{x},t-s)ds\ . \quad (6.5)$$

Nunziato's equation (6.5) is a generalized equation of the Jeffreys type.

The presence of an effective conductivity in the theory of heat transmission has exactly the same conceptual problem as pure diffusion; there is an immediate response to a disturbance at distant points. This conceptual problem is not relieved by diffusion equations of the Jeffreys type even when the effective conductivity is small and the diffusive response is wavelike (see Fig. 2).

We shall now adopt and pursue the view that ultimately there is no diffusion. This means that $Q(0)$, the rigidity, is finite, but possibly huge, and that heat waves in ordinary materials at room temperature propagate with finite but possibly huge speeds $c=\sqrt{Q(0)/\gamma}$. In this view, we are obliged to set the effective conductivity $k_1=0$.

An effective viscosity $k_1\neq 0$ can be a useful concept even if, strictly speaking, $k_1=0$. To understand this, we must first understand fast modes and slow modes. It is all a question of time scales. In problems characterized by one relaxation time, we mean to judge the time of response of the material in terms of time units of an experiment or another external process. There is an external clock. In problems in which different substructures in a material relax at different rates, we may judge fast or slow for one relaxation process in terms of the clock defined by another. In this case, there is an internal clock.

The point made in the foregoing paragraph is illustrated by common ideas about heat conduction in solids. Thermal energy is transported in a solid by two different mechanisms: by quantized electronic excitations, which are called free electrons, and by the quanta of lattice vibrations, which are called phonons. These quanta undergo collisions of a dissipative nature, giving rise to thermal resistance in the medium. The relaxation time τ_0 is associated with the average communication time between these collisions for the commencement of resistive flow.

The magnitude of the relaxation time has been estimated for particular types of collision processes. Peierls (1955) states that at room temperature the longest collision time occurs for a phonon-electron interaction and is of the order of 10^{-11} sec, while the collision times of phonon-phonon and free-electron interactions are both of the order of 10^{-13} sec. However, these times are reduced by imperfections and impurities (e.g., alloying substances) existing in the medium, so that the mean relaxation time τ_0 is not generally known. It is obvious that 10^{-11} sec is a long time relative to 10^{-13} sec. We can ask, "What is the effect of the modes that have decayed at $t_0>10^{-13}$

sec on the subsequent transfer of heat?" The answer is that these decayed modes continue to play a role, producing diffusion with an effective viscosity k_1 associated with the (possibly small) area under $Q(s)$, $0 \le s \le s^*$ where $s^* = O(10^{-13}\ \text{sec})$.

The next point that needs to be made in our argument motivating the introduction of effective moduli is about the rigidity $Q(0)$, which by hypothesis is finite. The heat rigidity $Q(0)$ can be determined from the speed $c = \sqrt{Q(0)/\gamma}$ of heat waves. Another way to determine $Q(0)$ is through high-frequency small oscillations. In looking at the problem of harmonic waves of frequency ω, one finds that the complex conductivity

$$k^*(\omega) = \int_0^\infty Q(s) e^{-i\omega s} ds$$

plays an important role. For small ω, we get

$$k^*(\omega) = k + O(\omega)$$

where k is the area under Q. For large frequencies,

$$k^*(\omega) = -i\frac{Q(0)}{\omega} + O\left[\frac{1}{\omega^2}\right] . \tag{6.6}$$

It would be very hard indeed to measure $Q(0)$ with the method of small oscillations. We would need to invent devices to detect temperature oscillations in the range of 10^{13} rad/sec. Even in dielectric crystals at very low temperatures with electronic conduction suppressed, we would need to be able to deal with frequencies of 10^5 rad/sec or greater. In fact, the method of small oscillations does not appear to have an important place in the measurement of second sound, and heat pulses, giving rise to c, are used.

If we know the rigidity $Q(0)$ and the conductivity $k = \int_0^\infty Q(s) ds$, then we may compute a mean time

$$\tau = k / Q(0)$$

of relaxation. For metals at room temperature, with τ small and k modest, $Q(0)$ is huge. In this situation, the theory of wave propagation is of no apparent practical utility.

It is of interest to bring into play the idea of the internal clock leading to an effective thermal conductivity, an effective relaxation kernel, and an effective time of relaxation τ_2. We may imagine a relaxation function of the type shown in Fig. 3. It has a fast relaxation followed by a slow relaxation.

We next decompose the relaxation function into a fast and a slow part,

$$Q(s) = Q_1(s) + Q_2(s) . \tag{6.7}$$

The decomposition is certainly not unique, but it is not entirely arbitrary if fast and slow modes can be identified. For example, in metals, we might put $\tau_1 = 10^{-13}$ sec and $\tau_2 = 10^{-11}$ sec.

The thermal conductivity of the conductor in Fig. 3 is given by

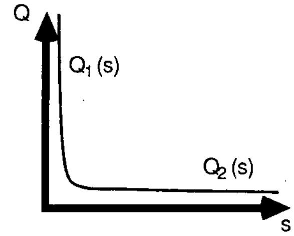

FIG. 3. Example of relaxation function with a fast and slow relaxation. The time τ locates the end of the fast relaxation.

$$k = k_1 + k_2 , \tag{6.8}$$

where k_1 is the area under the fast mode $Q_1(s)$ and k_2 the area under the slow mode Q_2, $s \ge \tau_1$. The fast relaxation will enter into the thermal response for times $\tau_1 \le s < \tau_2$ as an effective thermal conductivity. We can model the kernel in Fig. 3 by a double step as in Fig. 4 with $k = k_1 + k_2$ and $k_2 = Q_2(0)(\tau_2 - \tau_1)$. For times larger than τ_2, everything has relaxed into pure diffusion with conductivity k. For the three-step kernel, we would see an elastic response corresponding to the rigidity $Q_1(0)$ in the time interval $0 \le s < \tau_1$. When $\tau_1 \le s < \tau_2$, the elastic response is to a rigidity $Q_2(0)$ smoothed in an effective conductivity $k_1 = Q_1(0)\tau_1$. When $\tau_2 < s < \tau_3$, the elastic response is to a rigidity $Q_3(0)$ smoothed in an effective conductivity $k_1 + k_2$, $k_2 = Q_2(0)(\tau_2 - \tau_1)$. For $s > \tau_3$, the evolution of the temperature change is governed by pure diffusion with conductivity $k = k_1 + k_2 + k_3$, $k_3 = Q_3(0)(\tau_3 - \tau_2)$.

The models using multiple steps of relaxation are meant to be qualitative. In fact, deeper investigations of these models raise difficulties on the mathematical side and are difficult to interpret on the physical side when the actual relaxation does not exhibit the plateaulike regions shown in Fig. 3 and, in an exaggerated form, in Fig. 4. The effective viscosity k_1 is meant to represent all the modes that have decayed so rapidly as to be useful only as a delta-function contribution at the origin.

We are thinking of kernels for which $Q(s) = Q_1(s) + Q_2(s)$ such that for some small time s_0, $Q_1(s_0)/Q_2(s_0) \ll 1$ such that $Q_2(s_0) \approx Q_2(0)$. At times $t > s_0$ the integrand of the integral

$$\int_0^\infty Q_1(s) \nabla\theta(\mathbf{x}, t-s) ds \tag{6.9}$$

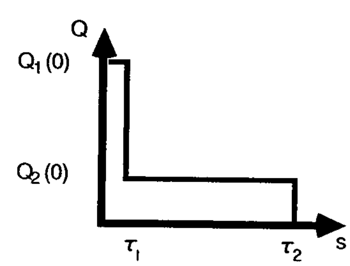

FIG. 4. Double step of relaxation.

is nearly zero and may be replaced by

$$\sim \int_0^{s_0} Q_1(s)\nabla\theta(\mathbf{x},t-s)ds \ . \tag{6.10}$$

If s_0 is small, $t-s\sim t$, and this reduces to

$$\sim \nabla\theta(x,t)\int_0^{\infty} Q_1(s)ds = k_1\nabla\theta(\mathbf{x},t) \ . \tag{6.11}$$

Hence

$$\begin{aligned} -\mathbf{q} &= \int_0^{\infty}[Q_1(s)+Q_2(s)]\nabla\theta(\mathbf{x},t-s)ds \\ &\sim k_1\nabla\theta(\mathbf{x},t)+\int_0^{\infty} Q_2(s)\nabla\theta(\mathbf{x},t-s)ds \ . \end{aligned} \tag{6.12}$$

Unlike the step relaxation, $Q_2(s)$ decays slowly for $0<s<s_0$, but it decays. In fact, the decay of $Q_2(s)$ could be relatively rapid on a time scale in which the fast modes look as if they decayed in the distant past. It may not be good to put the relatively fast decay of $Q_2(s)$ into the delta function on time scales in which $Q_2(0)$ can be viewed as an effective rigidity. We could judge whether the relaxation of $Q_2(s)$ is fast or slow by an effective time of relaxation given by

$$\tau = k_2/Q_2(0) \ .$$

If $k_2 \gg k_1$, then

$$\tau = k/Q_2 \ ,$$

where k is the conductivity of the body in steady heat flow.

A similar decomposition of the heat-capacity kernel

$$E(s)=E_1(s)+E_2(s)$$

leads directly to Eq. (6.2), using identical arguments. Almost nothing is known about the memory dependence of the internal energy. Without knowing more, it seems sensible to assume that $E(0)$ is bounded and to look for a delta function in the relaxation of fast modes.

The idea behind the decomposition Eq. (6.7) and the multiple steps is that an effective thermal conductivity arises at times that are long relative to the relaxation times of fast modes but short on the time scales characterizing the relaxation of slow modes. It is natural to represent these fast modes by a Dirac measure at the origin. This leads directly to $Q_1(s)=k_1\delta(s)$ and to the generalized Jeffreys equation with an effective relaxation function $Q_2(s)$, effective rigidity $Q_2(0)$, and effective relaxation time $\tau=k/Q_2(0)$.

VII. SOLUTION OF CANONICAL PROBLEMS FOR WAVE PROPAGATION

There are three canonical problems for wave propagation: (i) plane harmonic waves, (ii) propagation of weak singularities, and (iii) propagation of strong singularities. These canonical problems may each be framed as a problem of propagation into a semi-infinite region $x>0$ of data prescribed at the boundary $x=0$ of a semi-infinite solid. The first problem is to find the plane-wave response compatible with prescribed oscillation of the temperature at $x=0$. The solution of this problem by the method of plane harmonic waves is well known in electrical engineering, polymer mechanics, and ultrasound and is elementary. The solution of this problem when Eq. (6.5) governs was given by Nunziato (1971).

A weak singularity is a jump in the first derivatives of $\mathbf{q}$ or θ while the functions themselves are continuous. The method of weak singularities is the most readily understood method for generating characteristic curves in quasilinear hyperbolic systems. Unfortunately, this method is not useful when $k_1\neq 0$ in the sense that, after doing it, we learn merely that the discontinuity we assumed was, after all, not possible. For example, if $\mathbf{q}$ is continuous and the Fourier law holds, then $\nabla\theta$ must be continuous. When $k_1=0$, this method can be used to solve the problem of following the jump, say, of the temperature rate $[\partial\theta/\partial t]$ which is prescribed at $x=0$ when $t=0$. When $k_1=0$ and the governing system is hyperbolic, this jump will propagate. This type of analysis has been performed by Achenbach (1968), Chen (1969), Chen and Gurtin (1970), Nunziato (1971), and Morro (1980a, 1980b).

A discontinuity in θ is stronger than a discontinuity in $\partial\theta/\partial t$. The problem here is to describe the response of the semi-infinite solid to a sudden change of temperature at $x=0$, as in Eq. (3.3). This problem can be solved with Laplace transforms, and it has been solved with $k_1=0$ by many authors. The solution for propagation of higher-order singularities can be obtained by differentiating the solution of the problem corresponding to a discontinuity one order lower (see property 4 below). For example, we could prescribe $\theta(x,t)$ at $x=0$ as

$$\theta(0,t)=\begin{cases}0, & t<0,\\ \theta'(0)t, & t>0,\end{cases} \tag{7.1}$$

and find $\theta(x,t)$ by Laplace transforms [see, for example, Amos and Chen (1970)]. A temperature rate discontinuity could then be obtained as the partial time derivative at fixed x of the solution corresponding to Eq. (7.1). In the same way, we can generate the solution for heat pulses from the solution for a sudden change of temperature. This method of bootstrapping solutions of problems with discontinuous initial data by differentiation works for linear problems even if $k_1\neq 0$, but, of course, waves propagate only if $k_1=0$. One of the main conclusions that can be drawn from the bootstrapping argument is that, when $k_1=0$, the wave speed Eq. (7.9) and attenuation Eq. (7.11) are the same for discontinuities of all orders. The following argument shows that these formulas also hold in the high-frequency limit of plane harmonic waves.

Nunziato (1971) looked for conditions on the real-valued constants η, ω, and ξ such that

$$\theta(x,t)=\theta_0\exp(\eta x)\exp i(\omega-\xi x)$$

can satisfy Eq. (6.3). For this it is enough that

$$i\omega\gamma^* = -(\xi + i\eta)^2 k^* , \tag{7.2}$$

where

$$k^* = k_1 + \int_0^\infty Q_2(s) e^{-i\omega s} ds \overset{\text{def}}{=} k'(\omega) - ik''(\omega) ,$$

$$\gamma^* = \gamma_1 + \int_0^\infty E(s) e^{-i\omega s} ds \overset{\text{def}}{=} \gamma'(\omega) - i\gamma''(\omega) .$$

Equation (7.2) may be solved for the wave number ξ,

$$\xi = [\omega(\gamma' k'' - \gamma'' k' + AB)/2A^2]^{1/2} ,$$

and the attenuation

$$\eta = -\omega(\gamma'' k'' + \gamma' k')/2\xi A^2 , \tag{7.3}$$

where $A^2 = k'^2 + k''^2$, $B^2 = \gamma'^2 + \gamma''^2$.

The phase speed $v(\omega)$ of the wave is given by

$$v(\omega) = \frac{\omega}{\xi(\omega)} = \left[\frac{2\omega A^2}{\gamma' k'' - \gamma'' k' + AB} \right]^{1/2} . \tag{7.4}$$

For large ω, we find that

$$k' = k_1 - \frac{Q_2'(0)}{\omega^2} + O(\omega^{-4}), \quad k'' = \frac{Q_2(0)}{\omega} + O(\omega^{-3}) ,$$

and

$$\gamma' = \gamma_1 - \frac{E'(0)}{\omega^2} + O(\omega^{-4}), \quad \gamma'' = \frac{E(0)}{\omega} + O(\omega^{-3}) .$$

When $k_1 \neq 0$ and $\omega \to \infty$, we find, to leading order, that the phase velocity is given by

$$v = \left[\frac{2k_1\omega}{\gamma_1} \right]^{1/2} + O(\omega^{-1/2}) \tag{7.5}$$

with attenuation

$$\eta = \left[\frac{\gamma_1\omega}{2k_1} \right]^{1/2} + O(\omega^{-1/2}) . \tag{7.6}$$

The speed and attenuation of plane harmonic waves of frequency ω is unbounded, proportional to $\sqrt{\omega}$. Equations (7.5) and (7.6) are independent of $Q_2(s)$ and $E_2(s)$; hence, as $\omega \to \infty$, Nunziato's model reduces to Fourier diffusion.

When $k_1 = 0$ and $\omega \to \infty$, we find that

$$\begin{aligned} &v = [Q_2(0)/\gamma]^{1/2} , \\ &\eta = -\tfrac{1}{2}\gamma_1^{-1/2} \frac{E_2(0)Q_2(0) - \gamma Q_2'(0)}{Q_2^{3/2}(0)} , \end{aligned} \tag{7.7}$$

with an error of order $1/\omega$. The speed and attenuation of plane harmonic waves of frequency ω have finite limits as $\omega \to \infty$. These limits are identical to the speed and attenuation of shock waves and heat pulses. The heat-capacity relaxation function has no effect on the speed but does enter into the attenuation.

The difference between $k_1 = 0$ and $k_1 > 0$ is the difference between diffusion and waves, parabolic equations and hyperbolic ones. We again draw attention to the fact that, despite appearances, there is a sense in which the limit $k_1 \to 0$ is continuous; but it is a singular limit, which gives rise to propagation of layers with smooth but rapid variation, aping shocks.

We now consider the problem of the temperature distribution in a semi-infinite rigid conductor after a step change of temperature. The mathematical formulation of this initial-boundary-value problem is given by Eq. (3.3) with Nunziato's equation (6.5) replacing the Jeffreys equation. This problem can be solved by Laplace transforms. The solution is similar to that given in recent papers (Narain and Joseph, 1982; Joseph, Narain, and Riccius, 1986; Preziosi and Joseph, 1987) for propagation of shear waves into a liquid after a sudden increase in velocity. After making some obvious changes of notation to convert their problem to ours, we find that

$$\begin{aligned} \theta(x,t) &= \frac{1}{2\pi i} \int_{\gamma - i\infty}^{\gamma + i\infty} \exp[\omega t - x\eta(\omega)] \frac{d\omega}{\omega} \\ &= \frac{1}{2} + \frac{1}{\pi} \int_0^\infty \frac{d\zeta}{\zeta} \exp[-xW^-(\zeta)] \\ &\quad \times \{\sin[\zeta t - xW^+(\zeta)]\} , \end{aligned} \tag{7.8}$$

where

$$\eta(\omega) = \left[\frac{\gamma_1 + \gamma_2^*(\omega)}{k_1 + k_2^*(\omega)} \omega \right]^{1/2} ,$$

$$k_2^*(\omega) = \int_0^\infty Q_2(s) e^{-i\omega s} ds = p(\omega) - iq(\omega) ,$$

$$\gamma_2^*(\omega) = \int_0^\infty E_2(s) e^{-i\omega s} ds = r(\omega) - is(\omega) ,$$

$$W^{\pm}(\zeta) = \left[\frac{\zeta}{2} \right]^{1/2} \left[\left[\frac{(\gamma + r)^2 + s^2}{(k_1 + p)^2 + q^2} \right]^{1/2} \pm \frac{(\gamma + r)q - (k_1 + p)s}{(k_1 + p)^2 + q^2} \right]^{1/2} .$$

The following properties can be deduced from Eq. (7.8).

(1) When $Q_2(\ ) = 0$, $k_1 = k$ and the rigid conductor follows the Fourier law. If, in addition, $E(\ ) = 0$ then $de = \gamma\, d\theta$ and Eq. (7.8) reduces to the expression for the error function describing the diffusive response to a step change in temperature.

(2) When $k_1 = 0$, $Q_2(\ ) = Q(\ )$ and we get waves of heat which propagate with a velocity

$$c = \sqrt{Q(0)/\gamma} . \tag{7.9}$$

Near the trajectory $x = ct$, we have

$$\theta(x,t) \sim H(x - ct)\theta\left[x, \frac{x}{c}\right] , \tag{7.10}$$

where $H(x)$ is a Heaviside function $H(x) = 0$ for $x < 0$, $H(x) = 1$ for $x > 0$, and

$$\theta\left[x, \frac{x}{c}\right] = \exp\left[-\frac{x}{2c} \left[\frac{E_2(0)}{\gamma} - \frac{Q'(0)}{Q(0)} \right] \right] . \tag{7.11}$$

The wave speed Eq. (7.9) is independent of the elastic capacity $E_2(s)$, but the attenuation Eq. (7.11) depends on $E_2(0)$. The speed and attenuation of impulsive waves is the same as the phase and attenuation of harmonic waves in the high-frequency limit $\omega \rightarrow \infty$.

(3) When $k_1/k \ll 1$, $k=k_1+k_2$, the effective conductivity gives rise to a shock structure in which a layer of thickness $\sqrt{k_1 x/k}$ smooths the shock. The shock is entirely smoothed at large x.

(4) The solution of the problem of the transmission of heat in a semi-infinite solid following imposition of a pulse of heat can be obtained by differentiating Eq. (7.8) with respect to t^3. The temperature field

$$\theta(x,t)=\frac{1}{\pi}\int_0^\infty \exp[-xW^-(\zeta)]\cos[\zeta-xW^+(\zeta)]d\zeta \tag{7.12}$$

satisfies Eq. (6.3), $q(x,t)=0$ when $t<0$ for all $x\geq 0$, $q(x,t)=0$ for $x\rightarrow\infty$ and all $t>0$, and

$$\theta(0,t)=\delta(t)\ ,$$

where $\delta(t)$ is Dirac delta-function pulse. The solution of the pulse problem with delta-function initial data is the time derivative of the solution for a unit jump of temperature because the delta function is the derivative of a Heaviside function.

(5) Renardy (1982) did an important study of the propagation of shear waves in liquids. His work can be applied to the present problem when $k_1=0$ and $E_2(s)=0$. If the rigidity $Q(0)$ is finite and the slope $Q'(0)=-\infty$, then the solution lies on a compact support, as in Fig. 1. The support of the solution propagates with the usual speed $c=\sqrt{Q(0)/\gamma}$; but discontinuous data are smoothed, for clearly the amplitude $\theta(x,x/c)$ of the wave behind the front vanishes. Renardy exhibited a particular kernel of the type just described, which leads to a C^∞ smooth solution with a propagating support. Renardy kernels are interesting for heat, because they allow for finite propagation speeds but do not allow discontinuous temperatures.

Renardy kernels, like regular kernels, would be expected to give the large speeds of heat that are believed to characterize heat propagation of solids. There should be no difference between the two kernels at times long enough for the fast modes to have relaxed. Both kernels would give rise to equal effective conductivities if the area under the fast relaxing part of $Q(s)$ were the same; equal area means equal conductivity. The fast wave in the effective theory is a precursor wave; the second wave, which carries most of the heat, is smoothed under the action of the effective conductivity of modes already relaxed. The smoothing action of the Renardy kernel would work only at the front of the precursor wave.

Narain and Joseph (1983) showed that the effects of a Renardy kernel at small times could be modeled by a regular kernel plus a small effective conductivity. If two kernels $Q(s)$ are globally the same, differing only in a small neighborhood of the origin of s, then the dynamics to which they give rise is the same, except in a small neighborhood of the shock front.

VIII. CHRONOLOGY OF THOUGHT ABOUT HEAT WAVES

A list of papers on heat waves, arranged on a strictly chronological basis year by year and alphabetically within a given year, appears below. The contents of most of the papers in the list are briefly abstracted. In some of these abstracts, we participate more actively than in others, making some interpretation, expressing opinions—some critical—and occasionally suggesting a new idea or direction.

Many different types of efforts are represented in these papers: theoretical, mathematical, computational, and experimental. There are theoretical approaches based on kinetic theory, Boltzmann's equation for a phonon gas, molecular dynamics, thermodynamics, and educated guesses based on postulating equations with properties that are believed to be desirable. There are many theoretical papers dedicated to predicting conditions of temperature and frequency for which temperature waves may be observed; others explain what is observed; still others have more abstract goals not closely connected to experiments. These are papers on the mathematical properties of different models that have been postulated: existence, uniqueness, stability, and properties. There are computational papers showing how to solve particular problems that could arise in one or another application.

Experiments showing heat waves have been successfully carried out at low temperatures in liquid helium and in certain dielectric crystals. It appears that the response of dielectric crystals to oscillations in temperature is not clear enough to use ultrasound and acoustic methods. The experiments that appear to be successful use pulse inputs whose harmonic content is not perfectly known. In liquid helium II, there are very slow speeds, ranging from zero at the λ point 2.2 K to $O(10^4$ cm/sec) near absolute zero. In dielectric crystals at low temperatures, all measured speeds are $O(10^5$ cm/sec). In metals, where most of the heat is carried by electrons rather than phonons, it is believed that heat waves travel at speeds of $O(10^8$ cm/sec). Slow speeds in ordinary materials at room temperature have never been measured, and, even if they exist, they may be masked by diffusion arising from an effective thermal conductivity associated with modes of heat that have already relaxed.

An entirely different approach to the problem of propagation of heat waves has been taken by Tsai and MacDonald in a sequence of papers in the 1970s describing the results of numerical simulations of lattice dynamics using the equations of motion of individual atoms. They do not linearize their equations and they take full account of the anharmonicity of the interatomic potential for forces. Their results are extremely interesting because of the rich contact they make with second sound, theory and experiment, and with continuum theories reviewed here. Their calculation gives rise to new and different features.

Their motivating applications lie in the realm of the high temperature (10^4 K) and high pressures (100 GPa) that would arise in laser pellet implosion experiments in nuclear fusion research, in determining thermophysical properties of materials used in nuclear reactors, in the effects of cratering due to impact, and in other extremes. They get wave propagation for these conditions. It looks like second sound. Their work suggests that second sound could be observed in many materials, at any temperature, at time scales short enough to minimize damping. This conclusion is reached in different ways by nearly all workers in the field, starting with Maxwell. It suggests a kind of time and temperature equivalence that is well known in polymer physics. They also present some interesting results about thermal wave propagation behind strong shock waves and some other results, like the impossibility of propagation of solitons in two or more space dimensions. We think that their approach should be developed further.

1867, J. Clerk Maxwell, Philos. Trans. R. Soc. London **157**, 49.

Equation (1.2) is a truncated form of an equation (143) derived by Maxwell, who cast out the time derivative term with the casual remark that it "... may be neglected, as the rate of conduction will rapidly establish itself." Maxwell never pursued analysis of short-time relaxation effects. His book, *Theory of Heat,* is based on diffusion and Fourier's law. He did not note that diffusion is associated with infinite speeds of propagation.

1917, W. Nernst, *Die Theoretischen Grundlagen des n Wärmestatzes* (Knapp, Halle).

Nernst suggested that at low temperatures in good thermal conductors heat may have sufficient "inertia" to give rise to oscillatory discharge.

1931, Lars Onsager, Phys. Rev. **37**, 405.

Onsager argues that the Fourier law implies a contradiction to the principle of microscopic reversibility used in Onsager's thermodynamics, which

> ... is removed when we recognize that the (Fourier law) is only an approximate description of the process of conduction, neglecting the time needed for the acceleration of heat flow. This time t is probably rather small, e.g., in gases, it ought to be of the same order of magnitude as the average time spent by a molecule between collisions.

The same smallness argument, resulting in the removal of the second time derivative for the telegraph equation, was also invoked by Maxwell (1967).

1938, L. Tisza, C. R. Acad. Sci. **207** (22), 1035.

Tisza introduced superfluid helium II and two phases of liquid helium; he also derived a wave equation for heat and a formula for the speed of heat waves which predicted extremely small wave speeds for the propagation of heat in liquid helium II. This heat wave was called a "second sound" by Landau.

1941, L. Landau, J. Phys. **5**, 71.

Landau developed the two-fluid theory for liquid helium II. He found two speeds, one for ordinary sound, one for a second sound, which describe propagation waves of temperature. He showed that the second sound speed depends strongly on temperature, varying monotonically between $c_1/\sqrt{3}$ where c_1 is the speed of ordinary sound and zero at the λ point 2.2 K. In Landau's theory, there is no damping or dissipation, and both sound speeds are associated with wave equations rather than telegraph equations. Propagation of Landau waves in helium II is specifically a quantum phenomenon and does not imply that transport of heat in ordinary materials should proceed by wave propagation. Landau represented liquid helium near absolute zero by a phonon gas of elementary excitations. A compressional sound wave is propagated through the phonon gas; then there will be periodic variations of the phonon density corresponding to temperature variations in liquid helium, that is, second sound.

Landau and the Russian workers who immediately followed him seemed to think that propagation of heat was a special phenomenon connected to phonon excitation. No mention is made of the paradox of heat diffusion in Landau's early or later works.

1944, E. Lifshitz, J. Phys. **8** (2), 110.

The calculations of Lifshitz show that, in the usual mechanical method of generating sound, the second sound is masked by the ordinary one. On the contrary, the plate with a periodically varying temperature radiates an almost pure second sound. He also concluded that conditions for observing second sound from the variation in pressure are unfavorable and are extremely favorable from the variation in temperature.

1944, V. Peshkov, J. Phys. **8**, 381

Peshkov measured the velocity of the waves of temperature in helium II and found waves of 19 m/sec at 1.4 K close to the values 26 m/sec given by Lifshitz. The phenomenon of second-sound propagation was not observed above 2.2 K.

1946, V. Peshkov, in *International Conference on Fundamental Particles and Low Temperatures, Cavendish Laboratory, Cambridge, July 22–27, 1946:* Report (Taylor and Francis, London), p. 19.

Here Peshkov suggested that second sound might be observed in crystals. He reasoned that Landau's arguments about second sound were based only on the postulated existence of a phonon gas. Thermal waves are associated with compression waves in the phonon gas. Since phonon gas excitation exists in any solid, second sounds should be detectable in solids as well as liquid helium. He points out that it will be necessary to experiment with a crystal in which the scattering of phonons by inhomogenieties and irregularities is at a minimum.′ (This type of process may possibly lead to a type of wave propagation associated with a telegraph equation.)

1948, W. Band and L. Meyer, Phys. Rev. **73**, 226.

Band and Meyer follow an idea used by Einstein (1920) to treat sound transmission in a disassociating gas and introduce a time of relaxation in the equations for second sound. This leads to a telegraph equation in which $c/\sqrt{3}$ is the sound of propagation of heat waves associated with second sound. Analogies between second sound and ordinary sound are derived.

1948, Carlo Cattaneo, Atti Semin. Mat. Fis. Univ. Modena **3**, 3.

Cattaneo was the first to build an explicit mathematical theory to correct unacceptable properties of the Fourier theory of diffusion of heat. The diffusion equation has the property that a heat pulse given at the surface of a body is felt immediately at all parts of the body no matter how distant. One says that the velocity of propagation is infinite. Cattaneo uses arguments from the kinetic theory of gases and a second-order correction of this, of his own, to derive a rate equation for the flow q of heat in one space dimension,

$$\tau\frac{\partial q}{\partial t}=-kq-k^2\frac{\partial\theta}{\partial x}\,, \tag{8.1}$$

where θ is the temperature, k is the conductivity, and τ is a relaxation time. Kinetic theory expressions for k and τ are derived.

He writes the energy equation in one dimension

$$\gamma\frac{\partial\theta}{\partial t}=\frac{\partial q}{\partial x}\,, \tag{8.2}$$

where γ is the heat capacity of the gas. Equations (8.1) and (8.2) imply that the temperature θ and the heat flux satisfy one and the same telegraph equation, e.g.,

$$\tau\frac{\partial^2\theta}{\partial t^2}-\frac{k^2}{\gamma}\frac{\partial^2\theta}{\partial x^2}+k\frac{\partial\theta}{\partial t}=0\,. \tag{8.3}$$

The propagation speed associated with this equation is

$$c=\sqrt{k^2/\gamma\tau}\,. \tag{8.4}$$

Cattaneo does not refer to Maxwell's (1867) work or to other prior work like that of Landau (1941), in which second sound is identified as wave propagation of heat. In 1948, the work on second sound would have appeared to be a special topic, lacking generality. Cattaneo restricted his attention to gases; no mention is made of propagation of heat in liquids and solids. The mathematical consistency of his second-order approximation of the kinetic theory has been challenged by Vernotte (1961) and Kaliski (1965).

1950, K. R. Atkins and D. V. Osborne, Philos. Mag. **41**, 1078.

Atkins and Osborne measured the velocity of second sound in liquid helium and obtained the value 150 m/sec at absolute zero, which is in agreement with Landau's prediction of $c_1/\sqrt{3}$ where c_1 is the velocity of ordinary sound.

1950, D. V. Osborne, "Propagation of second sound below 1 °K," Low Temp. Phys. NBS (US) Circular No. 519, pp. 139–144.

This paper looks to a mathematical frame for the observation that the velocity of second sound in liquid helium II increases as the temperature is decreased below 2.2 K, but not to the exact value, speed of sound/$\sqrt{3}$, predicted by two-fluid theory. Using the mathematics for transmission line circuits, Osborne postulates, by analogy, that the telegraph equation

$$\frac{\partial^2\theta}{\partial t^2}+\frac{1}{\tau}\frac{\partial\theta}{\partial t}=c_2^2\frac{\partial^2\theta}{\partial x^2} \tag{8.5}$$

governs where c_2 is the speed of temperature waves known as second sound. He speculates that the first derivative becomes important at very low temperatures near zero, where viscosity or other dissipative effects neglected in the two-fluid theory become important.

1951, Sydney Goldstein, Q. J. Mech. Appl. Math. **IV** (2), 129.

Goldstein reports that most of the work in this paper was done in 1938 and 1939 at Caltech. He treats several versions of the random walk problem (drunkard's walk) in one dimension and derives the telegraph equation from one limiting process and the telegraph equation with leakage from another. In both cases, the directions in any two consecutive intervals are correlated (the particle is not completely drunk). If the correlation is relaxed, a diffusion equation rather than a telegraph equation appears. He draws analogies between the solution of telegraph equations and heat conduction, but does not suggest that the diffusion equation for heat ought to be replaced by a telegraph equation.

1951, J. C. Ward and J. Wilks, Philos. Mag. **42**, 314.

Ward and Wilks derived the Landau expression $c_1/\sqrt{3}$ for the speed of second sound directly from a phonon gas model without recourse to a two-fluid theory. Their derivation is important because it implies that, since phonon gas excitations exist in any solid and some liquids, second sound should be detectable in liquids and solids as well as in liquid helium. Their derivation gives no indication of the region in which the expression $c_2 = c_1/\sqrt{3}$ holds. They say it will fail if (a) the collisions between phonons do not satisfy the conservation laws of energy and momentum or (b) the excitations of the liquid cannot be adequately represented by phonons.

1952, R. B. Dingle, Proc. Phys. Soc. London Sect. A **60**, 374.

Dingle's derivation has a different starting point from that of Ward and Wilks (1952), who started with Boltzmann's equation. Dingle gets less precise results, which agree with previous results within distinguished limits.

1952, J. C. Ward and J. Wilks, Philos. Mag. **43**, 48.

In this paper, Ward and Wilks derive a wave equation for second sound,

$$\frac{\partial^2 E}{\partial t^2} = \frac{c_1^2}{3} \frac{\partial^2 E}{\partial x_i \partial x_i} , \tag{8.6}$$

where E is the momentum of localized phonons, f is the distribution function for phonons, and c_1 is the speed of sound. All phonons travel at a constant velocity c_1. The authors start with a Boltzmann equation for f and assume that there are many phonon collisions in a distance equal to the wavelength of any disturbance that may be propagated.

1953, P. M. Morse and H. Feshbach, *Methods of Theoretical Physics I* (McGraw-Hill, New York).

On p. 865 of this celebrated textbook, the authors state.

> As we have mentioned ... the diffusion equation governing the transmission of heat in a gas is an approximation to the rather complicated motion of the gas molecules. One of the immediately obvious shortcomings of the diffusion approximation is its prediction that the temperature of a body will rise instantaneously everywhere (though not equally) if heat is introduced at some point in the body .... As such instantaneous propagation of heat is impossible, we must assume that the diffusion equation is correct only after a sufficiently long time has elapsed. This time depends naturally upon the velocity of propagation of the heat, which in turn, depends upon the mean free path λ of the gas molecules. The velocity of propagation of a disturbance in a gas is, of course, the velocity of sound, c. Once the time required for the temperature to get to a point in question is exceeded, we may presume that then the diffusion equations apply. The partial differential equation which includes this effect is
>
> $$\nabla^2\theta = a^2 \frac{\partial \theta}{\partial t} + \frac{1}{c^2}\frac{\partial^2 \theta}{\partial t^2} .$$
>
> We may also arrive at this equation from another point of view, by considering the change in the sound wave equation due to absorption. We shall encounter this equation when we consider the effects of resistive losses in the vibration of a string and also the propagation of electromagnetic waves in conducting media.

1958, H. Grad, in *Handbuch der Physik 12: Thermodynamics of Gases,* edited by S. Flügge (Springer, Berlin), pp. 205–294.

The Cattaneo equation (1.2) is contained as a special case of Grad's Eq. (28.30), giving the heat flux from a sophisticated and elaborate study of the kinetic theory of gases.

1958, P. Vernotte, C. R. Acad. Sci. **246**, 3154.

Vernotte postulates a rate equation for the heat flux of the type (2.2) and derives a telegraph equation for the temperature.

1958 (July), C. Cattaneo, C. R. Acad. Sci. **247**, 431.

Here Cattaneo gives a summary of his 1948 work, noting his priority with respect to the 1958 work of Vernotte.

1958 (December), P. Vernotte, C. R. Acad. Sci. **247**, 2103.

Vernotte cites some earlier statistical works on other subjects which lead to a telegraph equation and acknowledges the priority of Cattaneo.

1960, A. C. Eringen, Phys. Rev. **117**, 1174.

Eringen finds a generalized law for heat conduction for a solid with internal constraints σ, which reduces to

$$-\frac{i}{T_0}\nabla\theta = \frac{1}{\kappa}\mathbf{q} + \sum_{\sigma} \int_0^t \frac{1}{\kappa^\sigma}\mathbf{q}(\mathbf{x},\tau)e^{-\lambda_0(t-\tau)}d\tau$$

in the linear isotropic medium. This is the first time a history integral appears in the heat-flow equation, but it does not lead to heat waves.

1960, R. E. Nettleton, Phys. Fluids **3**, 216.

Nettleton assumes Cattaneo's equation and shows that it is consistent with irreversible thermodynamics. He applies Debye's 1914 theory that heat in solids is carried by longitudinal elastic waves to thermal conduction in liquids. The speed of elastic waves is calculated for the bulk modulus, which is assumed to be much larger than the shear modulus. The modulus is associated with the high-frequency limit of longitudinal waves induced by small-amplitude oscillations. The dispersion relation for these waves implies a wave so short that the continuum is lost. Nettleton says that the elastic wave theory is consistent with the theory of Vernotte, i.e., Eq. (8.1), which he interprets as a force-flux process linking two irreversible processes.

1961, P. Vernotte, C. R. Acad. Sci. **252**, 2190.

Vernotte calls attention to an inconsistency in Cattaneo's derivation of the heat law in which the second-order correction of the Maxwellian distribution of molecules is ignored.

1962, Jean Tavernier, C. R. Acad. Sci. **254**, 69.

Tavernier derives a rate equation

$$\mathbf{q}=-\mathbf{k}\,\mathrm{grad}\theta-\tau\frac{\partial\mathbf{q}}{\partial t}\ , \tag{8.7}$$

where $\mathbf{k}$ is a conductivity tensor and τ is a mean time of relaxation, from a mathematical analysis of Boltzmann's equation,

$$\frac{\partial f}{\partial t}+\mathbf{u}\,\mathrm{grad}f=\left.\frac{\partial f}{\partial t}\right|_{\mathrm{coll}}\ , \tag{8.8}$$

for the partition function f in the absence of external forces. He approximates the collision term on the right-hand side of Eq. (8.8) by

$$-\frac{f-f_0}{\tau[\mathbf{u}]}\ , \tag{8.9}$$

where f_0 is the equilibrium distribution corresponding to the temperature at a point and τ is a relaxation term for particles with velocity $\mathbf{u}$. Some details of the integrations leading from Eqs. (8.8) and (8.9) to Eq. (8.7) are given. Tavernier indicates how a derivation, again leading to Eq. (8.7), can be carried out for solids in which thermal energy is transported by two types of particles, electrons and phonons. Naturally, Eq. (8.7) leads to a telegraph equation and a single speed for waves of heat.

1963, J. A. Sussman and A. Thellung, Proc. Phys. Soc. London **81**, 1122.

From the authors' summary,

> The thermal conductivity of a perfect but finite crystal is investigated at low temperatures when Umklapp processes may be neglected. The sample is taken to be large compared with the mean free path due to momentum conserving phonon-phonon processes. A mean free time approximation is used for the phonon distribution function and hydrodynamic equations for the phonon gas are derived. Heat flow is shown to consist of two contributions—one is the usual (diffusion-like) heat flow and the other is due to a drift motion of the phonon gas. A temperature difference between the ends of a long cylinder with a rough surface will lead to a Poiseuille flow of the phonon gas. An abrupt change of temperature in the immediate vicinity of a surface through which heat is flowing is obtained.

They derive the following set of linearized equations for the temperature perturbation $\theta(\mathbf{x},t)$ and local phonon velocity $\mathbf{u}(\mathbf{x},t)$ in the limit of slow time variations:

$$\frac{\partial\theta}{\partial t}+a\,\mathrm{div}\mathbf{u}-b\nabla^2\theta=0\ , \tag{8.10}$$

$$\frac{\partial\mathbf{u}}{\partial t}+d\nabla\theta-e(\nabla^2+f\nabla\,\mathrm{div})\mathbf{u}=0 \tag{8.11}$$

where

$$a=\frac{T_0}{3},\quad b=\frac{c_1^2c_2^2}{T_0}\frac{\tau_1c_2+\tau_2c_1}{c_1^2+c_2^3},\quad f=2\ ,$$

$$d=\frac{c_1^2c_2^2}{T_0}\frac{c_2^3+2c_1^3}{c_2^5+2c_1^5},\quad e=c_1^2c_2^2\frac{\tau_1c_2^3+2\tau_2c_1^3}{c_1^5+c_2^5}\ .$$

Here T_0 is the reference temperature, (c_1,τ_1) and (c_2,τ_2) are longitudinal and transverse sound velocities c and relaxation time τ. When dissipative effects are negligible, $\tau=b=e=0$ and

$$\frac{\partial^2\theta}{\partial t^2}=ad\nabla^2\theta\ , \tag{8.12}$$

where ad is second sound. The authors say that Eqs. (8.10) and (8.11) should not be used to calculate attenuation because of slowly varying terms that have been discarded. The same system of equations [(8.10) and (8.11)], but with different coefficients—a,b,d,e—was derived by Kwok (1967) to discuss dispersion and damping of second sound in solids.

Guyer and Krumhansl (1966) note that the $\nabla^2\theta$ term of Sussman and Thellung (and Kwok) is absent from their equation and say that "This difference is an essential one physically and arises from the linear energy-momentum relation for a phonon gas." The $\nabla^2\theta$ term in (8.10) does not arise in any other theory known to us and appears strange.

1963, Marvin Chester, Phys. Rev. **131**, 2013.

In this paper, for the first time, the work on the hyperbolic equation following from Cattaneo, Vernotte, etc., is brought together with work on the second sound. Chester follows the idea of Peshkov and Ward and Wilks that second sound will appear in any material that can be modeled as a phonon gas, including some solids as well as liquid helium. He postulates the Cattaneo rate equation (1.2). He says that wave propagation will be important when

$$\left|\frac{\partial\theta}{\partial t}\right|\gg\left|\frac{\theta}{\tau}\right|\ ,$$

and diffusion will dominate when the inequality is reversed. He reexpresses the criterion just mentioned with a critical frequency

$$\omega_c\doteq\frac{1}{2\pi\tau}\ ,$$

diffusion for lower frequencies, waves for higher frequencies.

He obtains two expressions for the wave speed c, and after equating these, he calculates τ; one value $c_1=\sqrt{k/\tau\gamma}$ comes from the telegraph equation, and another $c_2=c_1/\sqrt{3}$ follows from the same kind of physics when the transport of heat occurs via the phonon gas. The sound speed c_1 of phonons in liquid helium arises when phonons are viewed as gas particles, each of which moves with the same speed c_1, but in a random direction. The average root-mean-square velocity of a group of phonons in one particular direction is $c_2=c_1/\sqrt{3}$. Hence

$$c_2^2=k/\tau\gamma=c_1^2/3 , \tag{8.13}$$

giving τ in terms of c_1.

1964, B. Boley, in *High Temperature Structures and Materials,* edited by A. Freudenthal, B. A. Boley, and H. Liebowitz (MacMillan, New York), pp. 260–315.

Boley points out that, in most practical conduction problems, the effects of relaxation and hyperbolicity implicit in Cattaneo's model are negligible.

1964, R. A. Guyer and J. A. Krumhansl, Phys. Rev. **133**, 1411A.

From the authors' summary,

> In this paper the dispersion relation for second sound in solids is derived. The starting point of the analysis is a Boltzmann equation for a phonon gas undergoing a temperature perturbation $\delta\theta_0\exp(\mathbf{k}\cdot\mathbf{x}+\omega t)$; the Callaway approximation to the collision term is employed. We obtain a dispersion relation which explicitly exhibits the need for a "window" in the relaxation time spectrum. Further, the dispersion relation shows that measurement of the attenuation of second sound as function of frequency is a direct measurement of the normal process and Umklapp process relaxation times. We derive macroscopic equations for energy density and energy flux and show their relation to the macroscopic equation with which Chester has treated second sound.

The Callaway approximation is a more sophisticated difference approximation, using two times of relaxation for the collision term in the Boltzmann equation, than the one-term approximation used in the work of Tavernier (1962),

$$-\left.\frac{\partial N}{\partial t}\right|_{\text{coll}}=-\frac{N-N_\lambda}{\tau_N}-\frac{N-N_0}{\tau_0} , \tag{8.14}$$

where N, N_λ, and N_0 will be functions of position, wave number, and time. N, N_λ, and N_0 are, respectively, the distribution function of the phonon system, the distribution function of a uniformly drifting phonon gas, and the local equilibrium distribution function; τ_N and τ_0 are relaxation times for normal and momentum-nonconserving processes. Guyer and Krumhansl derive a macroscopic equation for the temperature,

$$\frac{\partial^2\theta}{\partial t^2}+\frac{1}{\tau_0}\frac{\partial\theta}{\partial t}-[1-\tfrac{4}{5}i\omega\tau_N+O(\varepsilon^2)+\cdots]\frac{k\nabla^2\theta}{\gamma\tau_0}=0 , \tag{8.15}$$

where γ is the specific heat, k is related to the thermal conductivity, and ε is the product of a frequency and a mean relaxation time. The dispersion relation $f(k,\omega)=\text{const}$ for this equation exhibits a window for the passage of second sound. Equation (8.15) is in the complex domain and is equivalent to two real second-order equations. This is the first theory with more than one time of relaxation. A more complete solution of the linearized Boltzmann equation is given by Guyer and Krumhansl (1966a).

1964, E. W. Prohofsky and J. A. Krumhansl, Phys. Rev. **133**, 1402A.

The authors present a theory that models solids with periodic structure by phonon dynamics. The theory is based on two relaxation times, one associated with momentum-preserving phonon interactions, called normal collisions, and the other with losses of momentum, called umklapp collisions. The theory is motivated by gas dynamics, with a body force proportional to velocity, which represents frictional damping and leads to a telegraph equation. The mathematical basis of the study is a linearized Boltzmann equation. The conditions necessary for the occurrence of second sound in solids are examined at some length. The results indicate that second sound can propagate at frequencies greater than the reciprocal umklapp relaxation time and smaller than the reciprocal normal relaxation time; the solutions are the same as those for normal thermal conductivity.

1965, S. Kaliski, Bull. Acad. Pol. Sci. **XIII** (4), 211.

Kaliski derives a telegraph equation by assuming (1) finite propagation velocity as an axiom and (2) that the equation governing heat conduction is a second-order partial differential equation of local character (not history dependent). To derive this equation, he modifies Onsager's symmetry relations to implement Onsager's idea of thermal inertia.

1965, A. V. Luikov, Inghenerno-fizicheskifi Zh. **9**, 287.

From the author's summary,

> On the basis of the phenomenological theory—the thermodynamics of irreversible process—and using particular data of kinetic and statistic theories, a consistent description is presented of transfer phenomena: heat conduction which accounts for a finite heat-propagation velocity, relaxation of stresses in visco-elastic bodies, moisture transfer in capillary-porous bodies, as well as turbulent transfer processes. Particular solutions of a

hyperbolic mass-transfer equation in porous bodies are given.

The irreversible thermodynamics used generalizes Onsager's thermodynamics [cf. Nettleton (1960), Kaliski (1965)].

1965, H. D. Weyman, Am. J. Phys. **35**, 488.

Weyman derives a telegraph equation as a continuous limit from the random walk problem when the number of steps increases without bound. Further assumptions are required to obtain diffusion. He does not mention the earlier work of Goldstein (1951), which treats a similar problem. In Goldstein's problem, the center of mass of particles does not move, while in Weyman's it does. The limiting equations are different; Weyman also gets first-derivative terms in time $\partial\theta/\partial t$ and in distance $\partial\theta/\partial x$.

1966, C. C. Ackerman, B. Bertman, H. A. Fairbank, and R. A. Guyer, Phys. Rev. Lett. **16**, 789.

These authors used a pulse technique to measure the speed of temperature waves in solid helium. This was the first apparently successful measurement of second sound in a solid.

1966, J. P. Brazel and E. J. Nolan, in *Proceedings of the 6th Conference on Thermal Conductivity, Dayton, Ohio, . . . 1966,* edited by M. L. Minges and G. L. Denman (U.S. Air Force Materials Laboratory, Materials Applications Division, Dayton), p. 238.

Brazel and Nolan applied Cattaneo's equation (1.2) to determine whether temperatures or heat flux overloads due to non-Fourier effects might produce structural defects. They considered the problem of a step jump in the heat flux and showed that these effects can be important when the heat flux is large in the hyperbolic theory, high transient temperatures would develop (see Mauer and Thompson, 1973).

1966, J. B. Brown, D. Y. Chung, and P. W. Matthews, Phys. Lett. **21**, 241.

Here propagation of heat pulses in dielectric materials is discussed for the low-temperature limit, where the mean free path is limited by the dimensions of the sample. Experimental results for liquid He II and for an Al_2O_3 crystal are compared with a theoretical prediction from a solution of the telegraph equation in which the relaxation time and conductivity are theoretically determined with no adjustable parameters. The authors find that the telegraph equation ". . . accounts very well for the experimental results of liquid He II at 0.25°K and may also apply to heat pulse propagation in Al_2O_3 crystals if additional assumptions are made."

1966a, R. A. Guyer and J. A. Krumhansl, Phys. Rev. **148**, 766.

The linearized Boltzmann equation for the pure phonon field is solved formally in terms of the eigenvectors of the normal-process collision operator. The solution is summarized by the two macroscopic equations (4.1) and (4.2) relating θ and the heat flux $\mathbf{q}$.

The paper by Hardy (1970) is an extension in some sense of the approach here. Hardy formally solves the complete linearized Boltzmann equation in terms of the eigenvectors of the collision matrix, not only for normal processes, but with umklapp processes and imperfections included.

1966b, R. A. Guyer and J. A. Krumhansl, Phys. Rev. **148**, 778.

Steady one-dimensional solutions $\mathbf{q}(r)=\mathbf{e}_x q(r)$ of Eqs. (4.1) and (4.2) are derived in a cylinder of radius $r=R$ with $q(R)=0$. When the τ_R term is negligible (umklapp processes are negligible relative to normal ones), their solution reduces to Poiseuille flow of the phonon gas. The authors derive a formula for thermal conductivity for this solution and use it to discuss and classify four different regions of heat transport: ballistic, Poiseuille flow, Ziman, and kinetic. If the conditions for Poiseuille flow are realized, a simple measurement of τ_N becomes possible, and second sound will propagate.

1967, M. Kranys, Nuovo Cimento B **50**, 48.

This paper may be the first to look at heat propagation in a relativistic setting. The idea is that signals should not propagate with speeds larger than the speed of light. This requirement is not satisfied in theories that use Fourier's law.

Kranys proposed a relativistic generalization of Cattaneo's law, and his system does not give rise to propagation at infinite velocity. Kranys wrote a number of works on this subject, which are mathematically satisfactory from most points of view. Maugin (1974) remarks that Kranys's earlier heat-flow equations are not properly invariant in the sense of Oldroyd (the equation is not form invariant to observers in different frames that move as rigid bodies). For more recent works see Kranys (1977).

1967, P. Kwok, Physics **3**, 221.

Kwok uses the Boltzmann equation for phonon distribution functions to study the acoustic-phonon collective mode or second-sound mode in nonisotropic solids. Expressions for the velocity and damping of the second sound are given in terms of acoustic-phonon spectra and their relaxation spectra. Kwok derives the macroscopic equations (8.10) and (8.11).

1967, H. W. Lord and Y. Shulman, J. Mech. Phys. Solids **15**, 299.

The Cattaneo rate equation for heat (1.2) ". . . which includes the time needed for acceleration of the heat flow" replaces the Fourier law in a theory of thermoelasticity. Onsager's 1931 work is mentioned first in this paper. Lord and Shulman are the first to introduce a tensor of relaxation times and to view the problem of second sound as a problem of anisotropic thermoelasticity. This point of view, which is only implicit in their paper, is developed fully in the paper by Pao and Banerjee (1973).

1967, E. B. Popov, J. Appl. Math. Mech. (USSR) **31**, 349.

This paper and that of Lord and Shulman (1967) are the first applications of the rate equation (1.2) for heat transport to problems of thermoelasticity in the linear approximation. The equations of thermoelasticity are coupled; temperature gradients force deformation gradients and vice versa. Temperature waves force stress waves and vice versa.

1968, J. D. Achenbach, J. Mech. Phys. Solids **16**, 273.

The propagation of discontinuities of the stresses and the temperature is studied in a one-dimensional medium in which displacement and temperature fields are coupled and the heat flux is governed by a rate equation (1.2) of the Cattaneo-Vernotte type. Achenbach finds that there are two waves, a mechanical wave and a thermal wave, and he derives expressions for the two wave speeds in terms of the elastic constants. Similar results were obtained by Popov (1967) and Lord and Shulman (1967).

1968, C. C. Ackerman and R. A. Guyer, Ann. Phys. (N.Y.) **50**, 128.

Ackerman and Guyer apply the criteria for second-sound propagation to solid helium and LiF crystals. They report experimental data for heat pulses in helium crystals that give rise to (1) values of thermal conductivity in agreement with steady-state measurements at temperatures above the conductivity peak, (2) speeds of propagation, wave forms, and multiple reflections expected of second sound at temperatures below this peak; they also derive an equation for a temperature pulse whose center of mass travels with the speed of second sound, which is broadened by N-process scattering when the temperature is below the peak. The magnitude of the broadening is temperature dependent. This effect is used to calculate τ_N. The values of τ_N found from these unsteady pulse experiments agree with values obtained from steady Poiseuille flow measurements.

1968, C. P. Enz, Ann. Phys. (N.Y.) **46**, 114.

From the author's summary,

> A description of propagation of heat and other thermal quantities in terms of lattice dynamics and of ensemble techniques is developed. First a general method for the construction of the microscopic energy, energy flux, and momentum densities is presented. Then the density matrices describing local thermal equilibrium, local phonon drift, and their perturbation by dissipation are discussed. For this purpose an effective Liouville equation is derived. In this formalism the Boltzmann equation does not occur explicitly so that the hydrodynamical balance laws are additional assumptions. The ensuring dispersion laws already given earlier are derived. In particular two types of second sound called "driftless" and "drifting" are found, and it is concluded that the recent discovery of this phenomenon is of the "drifting" type, while the realizability of the "driftless" type is uncertain.

Enz derives a theory that expresses the propagation of macroscopic thermal quantities such as local temperature, phonon drift, heat, and heat capacity in terms of the microscopic quantities of lattice dynamics. This paper is the first to identify two different temperature waves, with different speeds. He says that the two second sounds are encountered under different conditions, distinguished by frequency and temperature. "Driftless" second sound is associated with convection which, in the case of heat transport in a stationary temperature gradient, is due to a phonon drift of the Poiseuille flow type. Drifting second sound seems to be what is observed in experiments, whereas observations of "driftless second sound" are different, because it occurs only in high frequencies and is associated with a situation in which heat conduction dominates.

1968, M. E. Gurtin and A. C. Pipkin, Arch. Ration. Mech. Anal. **31**, 113.

Gurtin and Pipkin set up a theory for heat conduction using constitutive assumptions that lead to finite wave speeds and disallow effective thermal conductivity. The idea is to regard the heat flux as determined by the history of the temperature gradient, as in the modeling of viscoelastic materials with instantaneous elasticity. The instantaneous elastic part means that the heat-flux functional is always finite, even at the initial instant where it gives rise to a finite wave speed. This assumption also allows them to regard the heat flux as determined by a functional of the free energy. They make assumptions that lead to the conclusion that waves traveling in the direction of the heat-flux vector propagate faster than waves traveling in the opposite direction. They show that their assumptions about the heat-flux functional imply that the linearized constitutive equation for the heat flux $\mathbf{q}$ has the form

$$\mathbf{q}(\mathbf{x},t)=-\int_{-\infty}^{t} Q(t-t')\nabla\theta(\mathbf{x},t')dt' \ ,$$

where the kernel $Q(0)$ has a finite instantaneous value. They also introduce an internal energy functional that depends on the history of temperature and the heat flux in a differentiable way, so that the only scalar invariant $\mathbf{q}\cdot\mathbf{q}$ (and not $\sqrt{\mathbf{q}\cdot\mathbf{q}}$) for an isotropic material must appear linearly; it vanishes under linearization.

1968, F. R. Norwood and W. E. Warren, Q. J. Mech. Appl. Math. **22**, 283.

Norwood and Warren solve some problems of thermoelasticity for step inputs using Laplace transforms on Cattaneo's equation and the equations of Lord and Shulman (1967). No numerical results for applications are discussed.

1968, R. J. von Gutfeld, in *Physical Acoustics: Principles and Methods,* edited by W. P. Mason (Academic, New York/London), Vol. V, pp. 233–292.

The measurement of heat pulses in solids was made possible by a technology associated with transmission and detection of heat pulses at low temperatures. The technology is described and values for the speed of heat waves in different dielectric crystals—quartz, sapphire, NaCl, KCl, GeSi, NaF, and solid helium—are given. They are all of order 10^5 cm/sec. Some results for metals are reported. Electrons are important carriers of heat in metals, and they transmit heat more rapidly than the phonons that dominate heat transmission in dielectrics. Von Gutfeld reports speeds of $O(10^8$ cm/sec) in gallium.

1969, C. C. Ackerman and W. C. Overton, Jr., Phys. Rev. Lett. **22**, 764.

These authors report an experiment using heat pulses in which the speed of temperature waves was measured in solid helium-3.

1969, K. J. Baumeister and T. D. Hamill, J. Heat. Transfer **91**, 543.

Baumeister and Hamill solve the problem of the temperature in a semi-infinite $x>0$ solid following a sudden change of temperature at $x=0$ when the heat flux is governed by Cattaneo's equation (1.2). A mistake in this paper is corrected in Vol. 93, 126 (1971).

1969, P. Chen, Z. Angeu. Math. Phys. **20**, 448.

Chen applies the method of propagating discontinuities to the equation of Gurtin and Pipkin [our Eq. (5.4)]. He finds the wave speed Eq. (5.5) and the decay constant Eq. (7.11), even though he works in the frame of weak discontinuities in which $\partial\theta/\partial t$, rather than θ, is discontinuous. He also gets geometric factors for the decay in the jump of $\partial\theta/\partial t$. He obtains Eqs. (7.10) and (7.11), giving the form of the wave at the front [see also Amos and Chen (1970)].

1969a, N. Fox, Int. J. Eng. Sci. **7**, 437.

This paper develops a nonlinear constitutive theory for thermoelasticity, which extends the linearized theory of Lord and Shulman (1967). The material time derivative $d\mathbf{q}/dt$ of the heat flux $\mathbf{q}$ is assumed to depend on deformation gradients, $\mathbf{q}$, the temperature θ, and temperature gradients. Fox uses techniques of continuum mechanics to reduce this general dependence to a more explicit form in which

$$\frac{d\mathbf{q}}{dt}-\mathbf{W}\mathbf{q}=\mathbf{b}\ , \tag{8.16}$$

where $\mathbf{W}$ is the skew-symmetric part of the velocity gradient $\nabla\mathbf{u}$ and $\mathbf{b}$ is an isotropic vector-valued function of the variables in the above list, which reduces to

$$\frac{d\mathbf{q}}{dt}-\mathbf{W}\mathbf{q}=\alpha\mathbf{q}+\beta\nabla\theta\ .$$

Here α and β are functions of T and joint invariants of $\mathbf{q}$ and $\nabla\theta$. The term $\mathbf{W}\mathbf{q}$ is necessary to make the constitutive equation invariant under superposition of rigid rotation of the body as a whole and, of course, $\mathbf{W}=0$ for stationary rigid bodies. Some restrictions stemming from thermodynamics are also discussed. The invariant form

$$\frac{D\mathbf{q}}{Dt}=\frac{\partial\mathbf{q}}{\partial t}+(\mathbf{u}\cdot\nabla)\mathbf{q}-\mathbf{W}\mathbf{q} \tag{8.17}$$

appears first in this paper. Fox gives some simple exact solutions of the equations in this theory.

Fox did not completely solve the invariance problem. Using Oldroyd's method, we express $\mathbf{q}(\mathbf{x},t)$ in terms of time-independent coordinates ξ. Thus

$$\mathbf{q}=q_i(\xi,t)\mathbf{a}^i(\xi,t)=q^i(\xi,t)\mathbf{a}_i(\xi,t)\ ,$$

where $\mathbf{a}^i=\partial\mathbf{x}/\partial\xi_i$ are the contravariant base vectors and $\mathbf{a}_i=\partial\xi_i/\partial\mathbf{x}$ are the reciprocal vectors. The motion of these vectors describes the motion of material point ξ through space. Invariant rates are the time derivatives of $\mathbf{q}$ with respect to this body-fixed frame. They are not unique:

$$\frac{D\mathbf{q}}{Dt}=\frac{\partial\mathbf{q}^i}{\partial t}\mathbf{a}_i$$

or (8.18)

$$\frac{D\mathbf{q}}{Dt}=\frac{\partial\mathbf{q}_i}{\partial t}\mathbf{a}^i\ .$$

The derivatives of the base vectors are ignored. Now, the substantial derivative of $\mathbf{q}$ following the motion is

$$\frac{d\mathbf{q}(\mathbf{x}(\xi,t),t)}{dt}=\frac{\partial\mathbf{q}}{\partial t}+\mathbf{u}\cdot\frac{\partial\mathbf{q}}{\partial\mathbf{x}}\stackrel{\text{def}}{=}\left.\frac{\partial\mathbf{q}}{\partial t}\right|_{\xi}\ .$$

Thus we may differentiate the two representations of $\mathbf{q}$, using

$$\frac{d\mathbf{a}^i}{dt}=\mathbf{L}\mathbf{a}^i,\quad \frac{d\mathbf{a}_i}{dt}=-\mathbf{L}^T\mathbf{a}_i\ ,$$

where $\mathbf{L}=\nabla\mathbf{u}$, to get

$$\frac{d\mathbf{q}}{dt}=\frac{D\mathbf{q}}{Dt}+\mathbf{L}\mathbf{q}\quad\text{contravariant}$$

and (8.19)

$$\frac{d\mathbf{q}}{dt}=\frac{D\mathbf{q}}{Dt}-\mathbf{L}^T\mathbf{q} \text{ covariant .}$$

The rates D/Dt are called convective derivatives, and they are properly invariant. Any linear combination of these derivatives is also properly invariant, and the form given by Fox is the linear combination for which

$$\mathbf{W}=\tfrac{1}{2}(\mathbf{L}-\mathbf{L}^T) .$$

The problem of invariant derivatives has many different solutions.

1969b, N. Fox, J. Inst. Math. Its Appl. **5**, 373.

This paper is like the previous one by the same author, with an extended derivation using thermodynamics and invariance to obtain some restrictions on the constitutive equations for the title problem in solids and elastic fluids. The resulting theory is nonlinear and implicit, with many unknown functions of the dependent variables. The paper is motivated by problems in helium II; to treat these, the author linearizes his equations, and Cattaneo's equation comes out. He applies his equations to the Fountain effect and second sound. The theory for helium does not appear to be successful and is superseded by a later paper, Atkin, Fox, and Vasey (1975).

1969, M. J. Maurer, J. Appl. Phys. **40**, 5123.

Maurer points out that the theoretical foundation for application of Cattaneo's model (1.2) to solids should follow along lines laid down by Tavernier (1962), who used the linearized form of Boltzmann's transport equation with quantum-mechanical effects neglected [see also Guyer and Krumhansl (1964)]. In the paper, a time-dependent relaxation model for the heat flux in metals is derived from the quantum-mechanical form of the Boltzmann transport equation. The phonons are assumed to be in thermal equilibrium at all times, and the Lorenz approximation is used to treat electron-phonon interactions. Maurer comes up with Eq. (8.2) as the governing equation once again. He gets an estimate of 10^{-14} sec for the relaxation time in monovalent metals.

1969, I. Müller, Arch. Ration. Mech. Anal. **34**, 259.

Müller derives Cattaneo's equation from a formulation of the thermodynamics of irreversible processes in the relativistic case.

1970, D. E. Amos and P. J. Chen, ASME J. Appl. Mech. **37**, 1145.

These authors solve Eq. (6.5), using Laplace transforms, when the boundary $x=0$ of the semi-infinite plate is suddenly heated to $f(t)$. They do a short-time expansion under the integral to find that near the shock front

$$\theta(x,t)\sim f(x-t/c)\theta\left[x,\frac{x}{c}\right], \tag{8.20}$$

where $\theta(x,x/c)$ is given by Eq. (7.11).

1970, D. B. Bogy and P. M. Naghdi, J. Math. Phys. **11**, 917.

Bogy and Naghdi find that within the framework of rate-dependent constitutive assumptions for the temperature in a rigid conductor, thermal waves can occur in the finite theory, but not in the corresponding linearized theory.

1970, B. Bertman and D. J. Sandiford, Sci. Am. **222**, 92.

This is a popularized summary of research on heat waves in dielectric crystals at low temperatures. The article could be used as a starting place for getting acquainted with the ideas of quantized lattice vibrations (phonons) and normal and umklapp processes.

1970, P. J. Chen and M. E. Gurtin, Z. Angew. Math. Phys. **21**, 232.

Here Chen and Gurtin extend part of the theory of Gurtin and Pipkin (1968) to deformable media. They show that there exist two speeds of propagation for acceleration waves: the "first sound speed" is mechanical in nature and lies near the isothermal and isentropic sound speeds of the material, while the "second sound speed" is associated with a predominantly thermal wave. Their results generalize some of Achenbach's (1968) to the nonlinear case and to a more general constitutive equation. Their analysis (and Achenbach's) are restricted to one space dimension. They say that "The extension to three space dimensions is, aside from notational difficulties, entirely elementary."

1970, R. J. Hardy, Phys. Rev. B **2**, 1193.

From the author's summary,

> It has been suggested that two types of second sound, "drifting" and "driftless," are possible in dielectric crystals. The conditions for the existence of these two types of second sound are obtained both from a heuristic analysis of the problem and from an exact solution of the complete linearized Boltzmann equation. The exact solution is given in terms of the eigenvalues and eigenvectors of the collision matrix, with the effects of normal processes, umklapp processes, and imperfections included. It is shown that to get drifting second-sound normal-process scattering must dominate so that crystal momentum is approximately conserved while to get driftless second sound, the scattering must be such that a uniform energy flux will decay exponentially. These conditions for the two types of second sound are not mutually exclusive. It is found that normal-process scattering

need not dominate for second sound to exist, but that only when it does dominate, is second sound likely to be observable. The relaxation times for both types of second sound are shown to be the same and equal to the reciprocal of smallest nonzero eigenvalue of the collision matrix. An expression is given for a lower limit on this relaxation time.

According to Hardy, second sound will be said to exist when an accurate description of variations of the local temperature requires the use of the telegraph equation (1.7), where τ and c are the relaxation time and propagation velocity of second sound, respectively. In fact, Hardy's analysis identifies three distinct propagation speeds rather than the two mentioned in his summary. He raises the possibility of even other types of second sound:

> If none of the conditions for the existence of second sound discussed here are satisfied for a particular material and temperature range, it does not follow that the applicability of the diffusion equation for heat extends to arbitrarily rapidly varying processes. It means only that the range of applicability of the diffusion equation cannot necessarily be extended by simply adding on a term which changes it to a damped wave equation. Nothing in the present discussion excludes the possibility of there being even more types of second sounds than the three suggested here.

1970, H. E. Jackson, C. T. Walker, and T. F. McNelly, Phys. Rev. Lett. **25**, 26.

1970, H. W. Lord and A. A. Lopez, Acta Mech. **10**, 85.

This paper discusses the expected speed of second sound without considering the effect of thermal expansion, which is treated by Pao and Banerjee (1973).

1970a, M. McCarthy, Int. J. Eng. Sci. **8**, 467.

McCarthy develops an abstract theory of materials with memory in which the response functionals are assumed to depend on the histories of the deformation gradient temperature and integrated history of the temperature gradient.

1970b, M. McCarthy, Proceeding of Vibration Problems Warsaw **11** (2), 123.

Here McCarthy applies his equations to a one-dimensional problem of propagation of first-order waves. The results are essentially the same as those presented by Chen and Gurtin (1970).

1970, J. Meixner, Arch. Ration. Mech. Anal. **39**, 108.

The problem of heat transmission is treated using the author's formulation of irreversible thermodynamics. He derives Eq. (5.3) of Gurtin and Pipkin and claims (p. 120) that his theory shows that "... $Q(s)$ is an even positive definite function with mean value zero."

1970, T. F. McNelly, S. J. Rogers, D. J. Channin, R. J. Rollefson, W. M. Goubau, G. E. Schmidt, J. A. Krumhansl, and R. O. Pohl, Phys. Rev. Lett. **24**, 100.

These authors present data for very pure NaF which show the behavior of incipient second sound. They also report results of experiments on NaF, NaI, and LiF crystals of modest chemical or isotropic purity in which heat propagates as diffusion governed by Fourier's law or in the form of longitudinal and transverse elastic waves (ballistic propagation), probably arising as waves of thermal contraction or expansion induced by pulsed heating [cf. Tsai and MacDonald (1976)]. They say that as τ_N is increased (decreasing temperature), the pulse should continue to speed up and eventually disappear into ballistic propagation. Perhaps this is not correct because their Eq. (4.3) indicates dominating diffusive effects of normal processes when τ_N is large.

1971, S. H. Chan, M. J. D. Low, and W. K. Mueller, Am. Inst. Chem. Eng. J. **17**, 1498.

These authors solve a problem using the telegraph equation. They claim that thermal waves in solids may be of importance in catalysis.

1971, I. Müller, Arch. Ration. Mech. Anal. **41**, 319.

Müller develops a thermodynamic theory for thermoelastic materials which leads to finite speeds of propagation of temperature disturbances and a symmetric heat conductivity tensor. He derives a telegraph equation (4.36) for the heat conducted in a body of uniform density at rest.

1971, J. W. Nunziato, Q. Appl. Math. **29**, 187.

The equations of Gurtin and Pipkin (1968) are generalized by allowing the heat flux to depend on the present value of the temperature gradient, as well as its history, as in Eq. (6.4). No physical or philosophical argument is presented for this interesting generalization. Nunziato proves that solutions of the initial history problems are unique when, in our terms,

$$[k_1>0,\ \gamma>0,\ F(0)>0]$$

or

$$[k_1=0,\ \gamma>0,\ Q(0)>0,\ F(0)\geq 0]\ .$$

He derives our equations (7.1)–(7.7) and he gets the formulas for the speed and attenuation of jumps in $\partial\theta/\partial t$, as in Chen (1969).

1971, S. J. Rogers, Phys. Rev. B **3**, 1440.

From the author's summary,

> The propagation of short thermal pulses has been studied in very pure samples of NaF, Li^7F, and NaI in various crystallographic directions. In each of these crystals the flow of heat at high temperatures is by diffusion, and at the lowest temperatures, by the direct flight of phonons from heater to detector. In the ballistic region, the elastic anisotropy gives rise to a channeling of mode energy into certain preferred directions. Over a limited intermediate temperature range, the effect of normal-process scattering on the propagated heat pulse has been observed in NaF and Li^7F: In the best NaF crystals the pulse velocity approaches the expected second-sound velocity. The observations can be explained satisfactorily in terms of the hydrodynamics of a weakly interacting phonon gas. Computer solutions generated to fit the observed thermal pulse shapes suggest that in NaF, the mean free path for normal-process scattering can be represented by $l_N = 1.42 \times 10^3 T^{-3.71}$ cm in the temperature range 10–20°K.

The hydrodynamic equations just mentioned are Eqs. (4.1) and (4.2) of Guyer and Krumhansl. Rogers remarks that the telegraph equation, which arises when $\tau_N = 0$, cannot describe the transition with decreasing phonon-phonon interaction from the second to the first sound regime. He says that the τ_N terms were essential for his experiments.

1971, D. H. Tsai, "An atomistic theory of shock compression of a perfect crystalline solid," in "Accurate characterization of the high pressure environment," NBS Spec. Publ. 326, edited by E. C. Lloyd (Natl. Bur. Stand., Washington, D.C.), pp. 105–123.

Tsai reports on a molecular dynamic calculation [see Tsai and MacDonald (1973)] of the propagation of a strong shock wave in a two-dimensional lattice. It was shown that an energy relaxation process occurred behind the shock front in such a way that the thermally relaxed region propagated in a wavelike mode, with a velocity less than the shock velocity. This led to the speculation that second sound was being observed under high-stress, high-temperature conditions.

1972, J. M. Finn and L. T. Wheeler, Z. Angew. Math. Phys. **23**, 922.

Finn and Wheeler thought it desirable to give a proof that the Gurtin and Pipkin equation allows wave propagation. They prove that initial history problems are unique when $k_1 = 0$, $\gamma > 0$, $Q(0) > 0$, $Q'(0) < 0$, $F(0) > 0$.

1972, A. E. Green and K. A. Lindsay, J. Elasticity **2**, 1.

Another generalization of thermoelasticity, which leads to a symmetric heat conduction tensor and to Cattaneo's law (in the isotropic case), is derived using a generalization of an entropy production inequality of Müller (1971).

1972, V. Narayanamurti and R. C. Dynes, Phys. Rev. Lett. **28**, 1461.

These authors measured the pulse speed of temperature waves. For example, at $\theta = 3.4$ K, $c_2 = (7.8 \pm 0.5) \times 10^4$ cm/sec.

1972, A. Nayfeh and Nemat-Nasser, Z. Angew. Math. Phys. **23**, 50.

Nayfeh and Nemat-Nassar study wave motions in a semi-infinite isotropic elastic body under a step change in the temperature and heat flux at $x = 0$, when the heat flux is given by Cattaneo's law. They obtain a solution using the Cagniard-DeHoop method, which is a method for manipulating Laplace transform into transforms of known functions.

1972, Y. Taitel, Int. J. Heat Mass Transfer **15**, 369.

Taitel solves the telegraph equation in a thin layer subjected to a step change of temperature on both sides. The transient temperature may momentarily, exceed the boundary temperature, as well as the initial temperature of this layer. Taitel expresses surprise at this well known and perfectly acceptable feature of amplitude reinforcement from left and right traveling waves.

1973, C. E. Beevers, Acta. Mech. **17**, 55.

Beevers assumes a Cattaneo's law as modified for invariance by Fox (1969), who replaced the partial time derivative with a corotational one. He writes a general nonlinear system of equations governing thermoelastic dilatational waves. The linearized equations he then obtains are close to but not the same as those of Lord and Shulman. He then uses a stability argument based on ideas about wave hierarchies introduced by Whitham. He shows that some materials will be unstable by this criterion if the theory of Lord and Shulman is used, but all materials in his theory are stable. He also considers strong dilatational shock waves and derived equations for the extended heat conduction law.

1973, M. J. Maurer and H. A. Thompson, J. Heat Transfer **95**, 284.

Maurer and Thompson use Cattaneo's equation (1.2) to calculate thermal stresses in a solid subjected to a sudden change in heat flux. They correct a misleading discussion by Brazel and Nolan (1967) concerning the use of Catteneo's equation (1.2) to derive boundary conditions for this problem. They show that a correct model leads to high momentary temperature due to a step jump in heat flux.

1973, Y. H. Pao and D. K. Banerjee, Lett. Appl. Eng. Sci. **1**, 35.

Pao and Banerjee derive a linearized, anisotropic theory of thermoelasticity. Four modes and speeds are found for waves in dielectric crystals—two quasilongitudinal and two quasitransverse modes. These waves can be identified either in terms of strains or in terms of change of temperature. The second quasilongitudinal mode, which has no counterpart in the isothermal or adiabatic theory of acoustics, is the second sound. A one-dimensional analysis for heat pulses in NaF crystals was carried out, and the results agree favorably with experimental observation.

1973, R. Nachlinger and L. Wheeler, Q. Appl. Math. **31**, 267.

These authors prove that solutions of the initial-value problem for Nunziato's equation (6.5) are unique when $k_1>0$, $\gamma>0$ or $k_1=0$, $\gamma>0$, $Q(0)>0$ (cf. Nunziato, 1971).

1973, J. W. Nunziato, SIAM J. Appl. Math. **25**, 1.

Nunziato here improves the result in his 1971 paper, proving uniqueness when $k_1=0$, $\gamma>0$, $Q(0)>0$.

1973, D. H. Tsai and R. A. MacDonald, J. Phys. C **6**, L171.

The formulation of the three-dimensional problem and the method of solution were basically the same as in the two-dimensional problem studied by Tsai (1971). The equations of motion of the individual atoms defining the lattice were solved on computers. The equations of motion were obtained from an interatomic potential, which was fitted to the elastic properties of α iron (body-cubic centered) and to the data of the pressure-induced phase transformation from α iron to ε iron (Chang, 1968). The potential was strongly anharmonic.

Tsai and MacDonald use a semi-infinite lattice $z\geq 0$ divided into period filaments in the form of semi-infinite bars of square cross section, formed by laying cubical blocks $(5\times5\times5)$ of unit cells. Each cell has five atoms, four in the corners, one in the center. Filaments are joined together in x,y planes by cyclic or mirror boundary conditions. One-, two-, or three-dimensional problems can be studied on this lattice. The dimensionality is altered by setting the y motion to zero for a two-dimensional system and both x and y motion to zero for a one-dimensional equation.

The shock wave is driven into a semi-infinite lattice by the collision of this lattice with its image across $z=0$, as the two move toward each other. Average kinetic and potential energies, stress components, and density of lattice planes are obtained from the numerical analysis. Results are presented for average kinetic energy (proportional to the kinetic temperature) and the longitudinal component of the stress S_{ZZ}. Both quantities experience a sharp rise at the start and propagate as waves. Behind the shock front is a region of thermal relaxation, which increases with time because it propagates more slowly than the longitudinal wave of S_{ZZ} at the front. The authors write,

> Our view of the mechanism of thermal relaxation in the shock-compressed solid is as follows: The shock front extends over about five lattice planes only. Its steepness is due to anharmonicity of the interatomic potential. Because of this steepness, only the very-highest-frequency oscillations are excited, and these oscillations must share their energy with oscillations at other frequencies in order to reach thermal equilibrium. This is the thermal relaxation process we observe. In a dispersive lattice, the high-frequency oscillations propagate at a lower velocity than the shock front. Since the energy content of the thermally equilibrated region is concentrated in the higher frequencies, the thermally equilibrated region must trail farther and farther behind the shock front as time increases. Thus dispersion is the underlying mechanism for our temperature wave. Since dispersion is a property of the lattice, we conclude that our results will hold for all time and are not transient effects due to the starting condition.
>
> We believe that our results constitute a natural extension of the experimental and theoretical results of second sound in crystals from the conventional low-temperature, low-pressure régime to the high-pressure, high-temperature régime. Both cases deal with momentum-conserving interactions of lattice waves and with propagation of thermal energy in a wavelike mode. We consider the discussion of low-temperature second sound in terms of phonons to be a matter of convenience. Under our conditions of high temperature, large anharmonicity, it is more convenient to discuss the problem in terms of simple classical-mechanical concepts. Another point of contrast is that dispersion may be neglected in the low-temperature case since only very-low-frequency phonons are excited, whereas in the case of shock compression, dispersion is all-important. We conclude that second sound is a more general property of a solid than the earlier low-temperature studies would suggest; it arises from the close coupling between atoms.

1974, D. K. Banerjee and Y. H. Pao, J. Acout. Soc. Am. **56**, 1444.

From the author's summary,

> As motivated by the recent discovery of heat pulses propagating in dielectric crystals at low temperature, a continuum theory of thermoelasticity, which is modified to include the effect of thermal phonon relaxation, is applied to investigate the propagation of plane harmonic waves in unbounded anisotropic solids. Four characteristic wave speeds are found, three being analogous to those of isothermal or adiabatic elastic waves; the fourth wave, which is predominantly a temperature disturbance, corresponds to the heat pulses, known also as the second sound. Velocity, slowness, and wave surfaces of the thermoelastic waves are analyzed and are illustrated with numerical and graphical results for NaF and solid helium crystals. A new definition of the group velocity for waves in a dissipative and dispersive anisotropic medium is proposed and is calculated and compared with the energy transport speed of thermoelastic waves.

This work is based in part on a generalization of the Cattaneo equation appropriate for anisotropic solids,

$$\mathbf{T}\frac{\partial \mathbf{q}}{\partial t}+\mathbf{q}=-\mathbf{k}\nabla\theta ,$$

where $\mathbf{T}$ and $\mathbf{k}$ are relaxation time and conductivity tensors of second order (cf. Tavernier, 1962). The analysis is illustrated by calculating and constructing velocity, slowness, and wave surfaces for two crystals: (1) sodium fluoride, which represents the cubic class, and (2) solid helium four, which typifies the hexagonal class of crystals.

1974, V. A. Bubnov, Int. J. Heat Mass Transfer **19**, 175.

Bubnov studies the motion of isotherms in a conducting body, deriving a formula for the speed of isotherm propagation along the normal. The derivation follows along lines evidently introduced by Predvoditelev (1970), who represents the manifold of thermal states by temperature $\theta(\mathbf{x},t)$ and thermal conductivity $k(\mathbf{x},t)$ surfaces. Some mathematical operation with these concepts leads to a nonlinear wave equation. Bubnov assumes a relationship between the propagation speed and thermal diffusivity and he studies a few special cases. The resulting equations seem to require nonconstant conductivity $k(\mathbf{x},t)$, if a constant speed of propagation is assumed. Bubnov also gives a derivation of the telegraph equation for heat transfer in gases, using Maxwell's method. The paper is a source of references to some Russian literature that appears not to have been translated.

1974, P. Chen and J. Nunziato, Z. Angew. Math. Phys. **25**, 791.

Chen and Nunziato consider the Gurtin-Pipkin theory and use the second law of thermodynamics to show that $F(0), Q(0), -Q'(0)$ are ≥ 0 and

$$k=\int_0^\infty Q(s)ds>0 .$$

1974, B. DeFacio, J. Math. Phys. **16**, 971.

This paper develops the same theory as is offered by Roetman (1975). The author evidently got the idea from Roetman in a private communication. DeFacio is very confused. He says wrongly that the work of Gurtin and Pipkin (1968) leaves open the question of obtaining heat propagation at a finite speed in linear media. The development of a singular perturbation theory for reducing the telegraph to a diffusion equation is a wasted effort, producing neither man nor beast.

1974, G. Lianis, Arch. Ration. Mech. Anal. **55**, 300.

The author builds a theory in a frame that is attached to the particle while it does not rotate with respect to the inertial space. The treatment is formal and the resulting equations abstract.

1974, F. Mainardi, "On thermal waves in the general theory of heat conduction with finite wave speeds," unpublished talk delivered at the CISM Symposium on Thermomechanics in Solids, Udine, Italy.

Mainardi was the first to use the method of ray series for hyperbolic, one-dimensional problems in the Gurtin-Pipkin (1968) theory.

1974, G. A. Maugin, J. Phys. A **7**, 465.

From the author's summary,

> A heat-flux constitutive equation is derived in three approximations from a general *functional* constitutive equation which describes heat conduction in so-called 'simple' thermodeformable media in general relativity. The three approximations correspond to materials having a so-called 'fading memory,' an 'infinitely short memory,' and materials of the 'rate-type,' respectively. The first approximation leads to an integral constitutive equation which, after inversion of the integral operator, yields a differential law that: (i) exhibits the *relaxation* process needed to guarantee a propagation of heat disturbances at a speed smaller than that of light; (ii) is essentially spatial; (iii) satisfies the requirements now imposed in continuum physics, in particular, the *principle of objectivity* as formulated by the author or the *rheological invariance* of Oldroyd. The equation obtained has the same three-dimensional limit as the spatial part of Kranys' equation for rigid heat conductors. However, Kranys' equation was not objective.

1975, R. J. Atkin, N. Fox, and M. W. Vasey, J. Elasticity **5**, 237.

These authors derive a continuum theory in which an additional vector field is introduced to represent the flow of microscopic excitations from which second sound is thought to originate. An effective conductivity actually appears in their equations [the last term in their (4.3) and the first in (4.6)]. When the effective conductivity is zero, their linearized equations reduce to those given by Lord and Shulman (1967).

1975, C. Barnabes, Nuovo Cimento B **28**, 377.

From the author's summary,

> This article is a study of elastic media in general relativity; it is based on a relativistic generalization of the functional constitutive equations of continuous media of Truesdell and Noll. We show that it is possible to give a definition of hyperelasticity in general relativity, on condition that the existence of a reference state of minimum free energy is postulated. This approach allows us also on the one hand to study the case of relativistic elasticity under high pressure (cf. certain stellar models), and on the other hand to study thermoelastic media without the paradox of infinite velocity of heat conduction.

1975, H. Beck, in *Dynamical Properties of Solids*, edited by G. K. Horton and A. A. Maradudin (North-Holland, Amsterdam), Vol. 2, p. 205.

This is a review of the work of physicists on second sound and thermal conductivity. It is probably the most complete reference for microscopic theory used to predict frequency and temperature windows for the passage of second sound. Macroscopic equations are not treated. Problems of heat transmission in ordinary materials at room or high temperatures are not considered.

1975, M. S. Kazimi and C. A. Erdman, J. Heat Transfer **97**, 615.

Kozimi and Erdman use Cattaneo's equation (1.2) to determine the interface temperature of two suddenly contacting semi-infinite bodies. This work is motivated by the correlation between the instantaneous interface of two suddenly contacting liquids and the potential for rapid development of spontaneous nucleation in the cooler liquid. The time for which non-Fourier effects are important is assessed. The estimation of time is frustrated by a lack of information about wave speeds.

1975, I. F. I. Mikhail and S. Simons, J. Phys. C **8**, 3068 (Part I); 3087 (Part II).

Mikhail and Simons undertake a theoretical study of Boltzmann's equation for a phonon gas which models dielectric crystals with negligible electronic contribution. In contrast to previous works, they do not assume a constant thermal phonon relaxation time, independent of the wave number k. They note that in the majority of phonon interaction mechanisms τ/k^{-n}=const with $n \geq 1$. They find a dispersion relation for propagation of plane waves when the wave-number dependence of thermal relaxation time for phonons is accounted for. In Part I, normal processes are neglected, $\tau_N=0$. The dependence of relaxation times on phonon wave number implies that Cattaneo's equation or the Eqs. (4.1) and (4.2) of Guyer and Krumhansl cannot yield a dispersion relation for harmonic waves that holds over a broad range of wave frequency.

Of particular interest are the results of the analysis of plane waves in Sec. VII of this paper. The dispersion relations given there might hold over a broad range of frequency by suitable choices of the memory kernel. In this approach, we would abandon single relaxation times and look for a spectrum, which may depend on phonon wave length, perhaps only weakly. No work has been done following this line of thought.

1975, E. L. Roetman, Int. J. Eng. Sci. **13**, 699.

Roetman writes down the equations for continuum mechanics when the constitutive equation for heat is proportional to the gradient of the pressure, as well as the temperature, and he arrives at a hyperbolic equation for the temperature. No argument is presented for why the heat flux should depend on the pressure.

1976, T. T. Kao, AIAA Journal **14**, 818.

One-dimensional propagation of coupled stress and temperature waves in a Cattaneo-type conducting deformable semi-infinite solid, subject to a sudden jump of heat flux at the boundary $x=0$ at $t=0$, is solved with Laplace transforms. The concern is that "... under sufficiently high flux conditions, this jump in surface temperature may result in very severe thermal stress at the surface" (cf. Maurer and Thompson, 1973, and Brazel and Nolan, 1967). There are two stress waves and a temperature wave. The fast wave and the thermal wave travel together. Since some of the thermal and mechanical wave parameters are independent, it is not possible to order the wave speeds. Strangely, Kao chooses the sound wave as the slower one.

1976, G. Lebon and J. Lambermont, J. Méce. **15**, 579.

Lebon and Lambermont obtain Cattaneo's equation from a thermodynamic argument in which the entropy is assumed to depend on the temperature, using the method of Onsager's nonequilibrium thermodynamics.

1976, A. V. Luikov, V. A. Bubnov, and I. A. Soloviev, Int. J. Heat Mass Transfer **19**, 245.

This group discusses the theory set out in the paper by Bubnov (1974), showing how it leads to variants of the telegraph equation that are well posed as initial-value problems.

1976, R. A. MacDonald and D. H. Tsai, Thermal Conductivity **14**, 145.

Initially, the lattice is in thermal equilibrium. There is a Maxwellian distribution of atomic velocities with an equipartition of kinetic energy in the x,y,z degrees of freedom and when the energy density constant, etc., lattice is said to be in thermal equilibrium. The first ten lattice planes are heated quickly to a temperature T_h and maintained. Waves of stress and density propagate into the lattice with the speed of sound. These waves are generated by a thermal expansion of the lattice due to sudden heating of the ten lattice planes (thermoelastic coupling). There is also evidence for second sound, but it is heavily damped. The temperature profiles are for the kinetic temperature of a plane defined as $m\langle v^2\rangle/2k$ where m is the mass of an atom, $\langle v^2\rangle$ the average of the velocity squared over the plane, and k is Boltzmann's constant. MacDonald and Tsai fit diffusive curves computed by finite differences from a theory using Fourier's law to the numerically computed data, and they obtain a value of thermal conductivity of about $\frac{1}{3}$ of the measured value for an iron alloy with parameters closest to those used in the experiment.

It would be interesting to fit their numerical results to a non-Fourier model, say Cattaneo's model, with two parameters, conductivity and a relaxation time.

1976, D. H. Tsai and R. A. MacDonald, Phys. Rev. B **14**, 4714.

To form a heat pulse, energy is added to heat the first ten planes rapidly from 0 to 800 K, hold the high temperature for a time, then remove heat rapidly to a fixed temperature. The duration of this pulse is 40τ where $\tau = O(10^{-13}$ sec). Various waves shown in Fig. 5 propagate. They find that the pulse propagates into the lattice as a combination of stress waves and heat waves superimposed on a diffusive background. The second sound wave is a composite of several waves.

The results shown and explained in the caption are for very short times, $\tau = O(10^{-13}$ sec). Tsai and MacDonald later compare their results for high temperatures with the experiments of McNelly *et al.* (1970) on NaF crystals at low temperatures and say (MacDonald and Tsai, 1978, p. 18),

> That our results for an intense heat pulse can be scaled (by factors of 10^6 in both time and distance) to correspond to the experimental results under low temperature implies that the relative damping of the waves must also scale in some manner. In both cases damping has been minimized: in our case by the short time of calculation and in the experiment by careful choice of conditions so that momentum-conserving scattering processes (N processes) dominate momentum-nonconserving processes (D processes). In neither case has damping been investigated. The fact that we observe second sound and diffusion

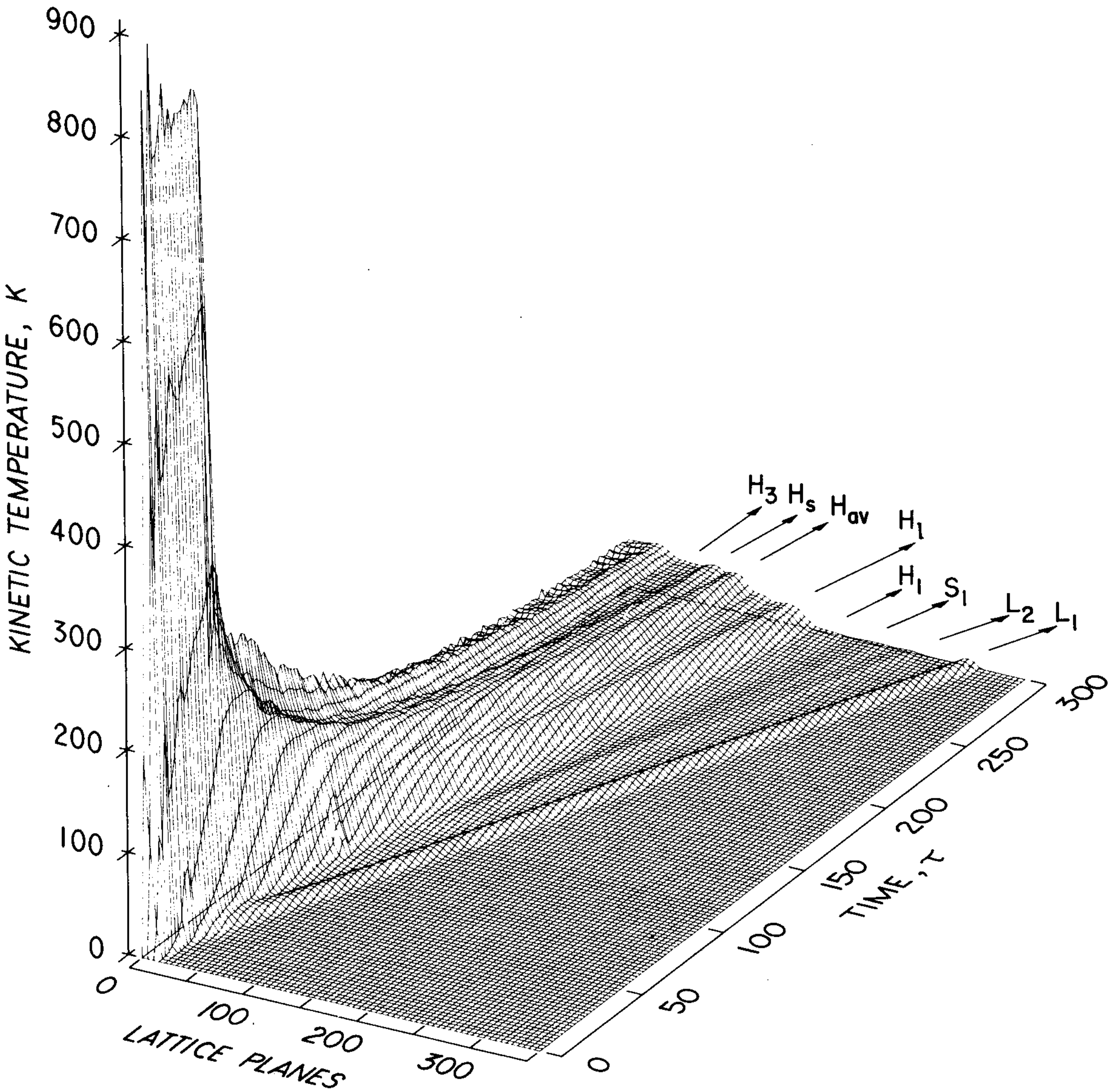

FIG. 5. Evolution of kinetic temperature profile K as a function of lattice plane number and time τ for $T_i = 0$ K, mirror boundary conditions, and three-dimensional lattice. L_1, L_2, S_1, H_1, H_l, H_{av}, H_s, and H_3 label features in the profile moving with constant velocity. From Tsai and MacDonald, 1976.

implies that our time observation is near the limit of the frequency window where relaxation times for N and U processes are comparable. This is apparently the case also in the experiments where a diffusive temperature rise is observed.

In Fig. 5, L_1 is a longitudinal stress wave generated by thermal expansion by heating the first ten planes. L_2 is the corresponding wave of compression generated by removing heat. S_1 is a transverse stress wave. The stress-induced pulses are not in local thermal equilibrium; equipartition has not been achieved (see MacDonald and Tsai, 1976, for the definition of thermal equilibrium), but they generate their own temperature waves H_l, H_{av}, H_s. These waves are bounded by the straight lines H_1 and H_3, whose slope gives wave speeds. H_l propagates with velocity $C_l/\sqrt{3}$, where C_l is the speed of the longitudinal stress waves L_1 and L_2, and $C_l/\sqrt{3}$ is also the speed of the wave front H_1. H_s propagates the speed $C_s/\sqrt{3}$, where C_s is the speed of shear waves. H_{av} propagates with velocity $C_{av}/\sqrt{3}$, where

$$C_{av}^2 = \sum_j C_j^{-3} \Big/ \sum_j C_j^{-5} \tag{8.21}$$

is an average of the longitudinal and transverse velocities, as discussed in the theory of second sound in a Debye solid generated by a heat pulse (Sussman and Thellung, 1963). H_3 propagates with a velocity lower than H_s. This broadening of the pulse may be due to diffusion, as in the experiments of Ackerman and Guyer (1968).

The correspondence between the theory and calculations of this paper for high temperatures at short times and, to some degree, the experiments on second sound in helium II and dielectric crystals at very low temperatures and longer time, is astonishing. This suggests that some ideas of time and temperature equivalence, which are well known in polymer physics, may also be relevant.

1977, M. Kranys, J. Phys. A **10**, 1847.

A phenomenological, general-relativistic theory of dissipative elastic solids whose equations form a hyperbolic system is proposed. The nonstationary transport equations for dissipative fluxes containing new cross-effect terms, as required by compatibility with irreversible thermodynamics, have been adopted. The complete system of special-relativistic propagation modes of an elastic solid is determined from the linearized equations. There are four mutually distinct nontrivial propagation modes, two for longitudinal waves and two for transverse waves.

1977, A. Nayfeh, AIAA Journal **15**, 957.

One-dimensional propagation of coupled stress and temperature waves in a Cattaneo-type conducting deformable solid, subject to a delta-function heat source $q\delta(x)\delta(t)$ at the boundary $x=0$ at $t=0$, is solved with Laplace transforms. The governing equations differ slightly from those used by Kao (1976), and the initial conditions are different.

1977, M. H. Sadd and J. E. Didalke, J. Heat Transfer **99**, 25.

Sadd and Didalke were the first to consider this problem, and they give a solution for a step change of temperature in terms of Laplace transforms, which shows importance at very small times locally, near the front.

1977, D. C. Wiggert, J. Heat Transfer **99**, 35.

Wiggert applies the numerical method of characteristics to find solutions of the telegraph equation. Two sample calculations are given.

1978, V. K. Agarwal, J. Elasticity **8**, 171.

Agarwal solves the problem of wave propagation for prescribed-time harmonic surface waves using the equations of Lord and Shulman (1967), on the one hand, and Green and Lindsay (1972), on the other. The two theories agree when some parameters are put to zero.

1978, R. A. MacDonald and D. H. Tsai, Phys. Rep. **46**, 1.

This is a review paper covering results on stress waves, thermoelastic coupling, heat waves, and thermal conductivity, which have been discussed in the earlier papers. In addition, the authors note that

> In one dimension it is shown that energy sharing between modes of vibration is difficult; therefore it is doubtful that the soliton concept is a useful one in nonlinear problems where thermal relaxation is involved. In two and three dimensions, energy sharing occurs readily.

1978, R. K. Miller, J. Math. Anal. Appl. **66**, 313.

Miller obtains existence, stability, and asymptotic stability results for Nuziato's equation under certain technical conditions. Basically, he establishes that history value problems for Nunziato's equations always have a unique generalized distribution solution that depends continuously on the initial history and on heat sources.

1978, S. Simons, Phys. Lett. A **66**, 453.

Simons shows that Catteneo's equations may be incorrect when the heat carrier's relaxation time is wave-number dependent. To do this, he writes the solution for a plane wave as a Fourier transform and finds an expression for the heat flux from the energy balance. Then he uses dispersion relations derived by Mikhail and Simons (1975,I) to invert the transform and to obtain heat-flow laws, for long waves. This approximate procedure never gives rise to Cattaneo's law or the associated telegraph equation. He derives a number of curious partial differential equations corresponding to different assumptions about the wave-number dependence of the phonon relaxation times.

1978a, R. Swenson, Am. J. Phys. **46**, 76.

Swenson modifies the heat-flux law so that it depends on the history of the temperature gradient (2.2). There is basically nothing new in this paper, and the author did not know the works that preceded his (see Agarwal, 1981, for a critical review).

1978b, R. Swenson, J. Non-Equil. Thermodyn. **3**, 39.

1979, V. K. Agarwal, Acta Mech. **31**, 185.

The propagation and stability of harmonically time-dependent thermoelastic plane waves of assigned frequency in the theory of Green and Lindsay (1972) are treated and compared with theory of Lord and Shulman (1967). The two theories are the same when some parameters have special values. Plane thermoelastic waves in the theory of Green and Lindsay are always stable, according to a criterion given by Whitham and applied previously by Beevers (1973) to the theory of Lord and Shulman.

1980a, A. Morro, Rend. Semin. Mat. Univ. Padova **63**, 169.

1980b, A. Morro, Arch. Mech. Warszawa **32**, 145.

1980, G. Seifert, Q. Appl. Math. **38**, 246.

Seifert studied a one-dimensional version of the problem treated by Miller (1978) and obtained asymptotic stability for the zero solution when there are no sources and $k>0$, $F(0)>0$ in Nunziato's model.

1981, V. K. Agarwal, Am. J. Phys. **49**, 503.

This is a short general review of the literature meant to correct omissions of Swenson (1978a, 1978b). Agarwal says, "There are several ways of obtaining the finite propagation speed for thermal disturbances in isotropic, homogeneous, and rigid solids. Many theories exist, but as yet there is no consensus as to which is the theory."

1982, B. D. Coleman, M. Fabrizio, and D. R. Owen, Arch. Ration. Mech. Anal. **80**, 135.

Here Coleman, Fabrizio, and Owen use the same Cattaneo law [Eq. (8.21)] as do Banerjee and Pao (1974), but they allow the matrices of relaxation times and conductivities to depend on temperature. Using their formulation of thermodynamics and some approximations (pp. 141 and 142) for the thermodynamic variables in which powers of the invariants $\mathbf{q}\cdot\mathbf{q}$ greater than one are neglected they find that the time derivative of the internal energy is given by

$$\dot{e}=(\gamma+\mathbf{q}\cdot\mathbf{A}'\mathbf{q})\dot{\theta}+2\mathbf{q}\cdot\mathbf{A}\dot{\mathbf{q}}_t\ , \tag{8.22}$$

where γ, $\mathbf{A}$, and $\mathbf{A}'=d\,\mathbf{A}/d\theta$ depend on θ,

$$\mathbf{A}(\theta)=-\frac{\theta^2}{2}\frac{d}{d\theta}\left[\frac{\mathbf{k}^{-1}(\theta)\mathbf{T}(\theta)}{\theta^2}\right].$$

The nonlinear part is determined by the temperature dependence of the parameters of Cattaneo's law (8.21). A system of four first-order quasilinear partial differential equations [given as Eq. (10.16) by Coleman, Fabrizio, and Owen, 1986] in the three components of $\mathbf{q}$ and θ is implied by the energy balance [Eqs. (1.3)and (8.22)],

$$\begin{gathered}\mathbf{T}(\theta)\dot{\mathbf{q}}+\mathbf{q}+\mathbf{k}(\theta)\nabla\theta=0\ ,\\ \mathrm{div}\mathbf{q}+[\gamma(\theta)+\mathbf{q}\cdot\mathbf{A}'(\theta)]\dot{\theta}+2\mathbf{q}\cdot\mathbf{A}(\theta)\dot{\mathbf{q}}=0\ .\end{gathered} \tag{8.23}$$

Coleman and Owen (1983) studied the one-dimensional version of Eq. (8.23) to determine when the system gives rise to hyperbolicity. This analysis gives rise to formulas for the slope dx/dt of characteristics in the (x,t) plane, giving the nonlinear wave speed of hyperbolic waves. From this formula, they conclude that second sound should propagate faster against the heat flow than with it.

In all of the papers by Coleman *et al.*, deformations are set to zero, as is appropriate for rigid bodies at rest. They say (1982) that

> The reader may observe that the form of the relation ... is not invariant under a time-dependent change of frame. This lack of frame invariance is shared by several of the constitutive relations we shall discuss here but is not important for the problems we treat. Each constitutive relation we shall present requires only slight modification to be rendered frame invariant. For example, the equation
>
> $$\tau(\dot{\mathbf{q}}-\mathbf{W}\mathbf{q})+\mathbf{q}=-\kappa\mathbf{g}$$
>
> is invariant under all time-dependent changes of frame when $\mathbf{W}$ is set equal to either the velocity gradient or its skew part, the vorticity tensor. Our discussion will be confined to rigid bodies, and hence any motion of the materials to be described must be such that the velocity gradient *is* a skew tensor.

It could be argued that in the case of second sound, a dielectric crystal at temperatures near the one for which conductivity is maximum should not be modeled as a rigid body, but as a phonon gas. This point of view has been adopted in many studies, and a good discussion of it can be found in the (1966b) paper of Guyer and Krumhansl. In fact, Guyer and Krumhansl (1966a, p. 773) note that the fluid velocity $\mathbf{u}$ of the phonon gas is related to the heat flux

$$\mathbf{q}=\frac{\gamma T_0}{3}\mathbf{u}\ , \tag{8.24}$$

where T_0 is an average temperature in the range of temperatures in which second sound will propagate. In this case, the appropriate nonlinear generalization of Cattaneo's law is

$$\mathbf{T}(\theta)\frac{D\mathbf{q}}{Dt}+\mathbf{q}=-\mathbf{k}(\theta)\nabla\theta\ , \tag{8.25}$$

where

$$\frac{D\mathbf{q}}{Dt}=\frac{\partial \mathbf{q}}{\partial t}+(\mathbf{u}\cdot\nabla)\mathbf{q}+\mathbf{W}[\mathbf{u}]\mathbf{q}\ .$$

Here $\mathbf{W}[\mathbf{u}]$ is the velocity gradient or the negative of the transpose of this gradient [see our summary of Fox (1969a)] or any linear combination of these. Each invariant derivative gives rise to a different constitutive law, and, at present, we know of no criterion of choice. If we use Eq. (8.24), then

$$\frac{D\mathbf{q}}{Dt}=\frac{\partial \mathbf{q}}{\partial t}+\frac{3}{\gamma T_0}(\mathbf{q}\cdot\nabla)\mathbf{q}+\frac{3}{\gamma T_0}\mathbf{W}[\mathbf{q}]\mathbf{q}\ . \tag{8.26}$$

This again gives rise to a quasilinear system.

1982, L. M. deSocio and G. Gualtieri, Q. Appl. Math. **41**, 253.

These authors do not refer to the earlier work of Sadd and Didalke (1977) and appear to have come to this problem independently. They give an approximate solution for a sudden change of the heat flux, such that melting occurs.

1983, D. W. Barclay, T. B. Moodie, and R. J. Tait, Int. J. Eng. Sci. **21**, 663.

The authors demonstrate that wave-front expansions for the analysis of transient phenomena are far from adequate back of the wave front. Padé approximations are used to try to extend the series solution.

1983, B. D. Coleman and D. R. Owen, Comput. Math. Appl. **9**, 527.

1983, T. B. Moodie and R. J. Tait, Acta Mech. **50**, 97.

The problem of a step change of temperature at the boundary of a semi-infinite rigid solid is solved for the linearized Gurtin-Pipkin theory using a ray series approach.

1983, B. Vick and M. N. Özisk, J. Heat Transfer **105**, 902.

Vick and Özisk consider heat propagation in a semi-infinite medium with volumetric energy sources using the Cattaneo law. They say that

> When a concentrated pulse of energy is released, the temperature and the heat flux in the wave front become severe. For situations involving very short times or very low temperatures, the classical heat diffusion theory significantly underestimates the magnitude of the temperature and heat flux in this thermal front . . . .

They reduce their equations to parameter-free form and, in this way, avoid the problem of choosing a relaxation time.

1985, R. P. Sawatzky and T. B. Moodie, Acta Mech. **56**, 165.

The problem of one-dimensional wave propagation in deformable elastic solids on a half-space is solved by ray series, using the linearized theory of Chen and Gurtin (1970).

1985, A. Solomon, V. Alexiades, D. Wilson, and J. Drake, Q. Appl. Math. **43**, 295.

Solomon *et al.* derive an equation for the motion of the interface containing a term absent in earlier work, which they claim is in error. This missing term appears to be important, as it balances against the speed of the small waves. They pose, but do not answer, the following questions.

(a) What is the form of a well-posed problem?

(b) On what basis can we assign a value to the time derivative of the temperature at the initial time, as required for solving a hyperbolic equation of second order?

(c) What is the nature of the "temperature" that obeys the telegraph equation?

(d) What happens if the phase-change front moves at a speed greater than the characteristic signal speed? Is this at all possible?

(e) Is the model of physical relevance?

1986, G. V. Caffarelli and E. G. Virga, Boll. Un. Mat. Ital., Ser. 6, **5A**, 33.

1986, B. D. Coleman, W. J. Hrusa, and D. R. Owen, Arch. Ration. Mech. Anal. **94**, 267.

This paper is about the global existence and asymptotic behavior of continuously differentiable solutions of Eq. (8.23) for an appropriate class of smooth initial data.

1986, B. D. Coleman, M. Fabrizio, and D. R. Owen, in *New Perspectives in Thermodynamics,* edited by J. Serrin (Springer, Berlin), p. 171.

1988, B. D. Coleman and D. Newman, Phys. Rev. B **37**, 1492.

From the authors' summary,

> The phenomenological relations usually employed to describe second sound in pure nonmetallic solids at temperatures θ near that at which the thermal conductivity attains its maximum value were recently found to imply a quadratic dependence of the internal energy density e on the magnitude of the heat flux q, i.e., $e_0=e(\theta)+a(\theta)q^2$. The coefficient $a(\theta)$ can be calculated from measurements of the temperature dependence of the speed $\hat{U}(\theta)$ of second-sound pulses in media for which the unperturbed temperature field is uniform. The studies of second-sound pulses in NaF crystals by Jackson, Walker, and McNelly and in Bi crystals by Narayanamurti and Dynes yield $a(\theta)>0$ and $da(\theta)/d\theta<0$. The theory of pulse propagation along temperature gradients is examined here in detail. For $a(\theta)>0$ the theory implies that a small pulse propagating in a body conducting heat will travel more slowly in the direction

of heat flow than in the opposite direction. The magnitude of the effect is estimated for NaF and Bi crystals.

1987, J. M. Greenberg, IMA J. Appl. Math. **38**, 1.

The questions raised by Solomon *et al.* (1985) motivate this paper. Greenberg notes that

> Most formulations of melt problems for hyperbolic models insist on continuity of the temperature across the melt interface, and a number of investigators have observed that this insistence leads to mathematical difficulties. In this paper an alternative model is explored, where continuity of the temperature at the melt interface is not imposed. Instead, we insist that the relaxation process describing the relation between heat flux and temperature gradient be interpreted as a conservation equation that must hold across a melt interface. With this formulation, we are led to a solvable problem with desirable asymptotic properties.

It may be possible to obtain the continuity of temperature together with hyperbolic models like Gurtin's and Pipkin's, using a singular kernel of the Renardy type (Sec. VII).

IX. CONCLUDING REMARKS

The ideas of this paper form a conceptual framework for the discussion of heat waves in materials of heterogeneous microstructure, each with its own time of relaxation, as in the case of electronic and lattice contributions to heat transmission in metals. If most of the heat is transported by the more slowly relaxing structures, the fast relaxation will give rise to small, effective conductivity, singularly perturbing wave propagation. The slow relaxation is a time unit for an internal clock against which fast relaxations are timed. This point of view shows how an effective conductivity leading to diffusion arises out of rapidly damped waves.

The internal energy, like the heat flux, may be viewed in a "viscoelastic" setting with the present value of the energy determined by a weighted integral of the history of the temperature. Consistent with this point of view is the notion that the heat capacity arises as an effective value from the decay of fast modes. Next to nothing (maybe nothing) is known about the history dependence of the internal energy.

The finite speed of heat waves is a satisfactory resolution of the paradox of infinite speed for diffusion. But, as a practical matter, huge speeds and rapid relaxation restore diffusion even on the scale of the response time of modern oscilloscopes. Heat waves have been observed, but only in the relatively exotic situations of second sound in helium II and in certain dielectric crystals at low temperatures.

In principle, finite wave speeds can be measured in any material at times shorter than the effective time of relaxation into diffusion. The molecular dynamic calculations of MacDonald and Tsai suggest a time-temperature principle, well known in polymer physics, in which long times at low temperatures are equivalent to short times at high temperatures. Unfortunately, these short times are found to be too short to measure with the techniques we know today.

ACKNOWLEDGMENTS

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Bubnov, B. A., 1974, "Wave concepts in the theory of heat," Int. J. Heat Mass Transfer **19**, 175.

Caffarelli, G. V., and E. G. Virga, 1986, "The classical heat equation compared with a hyperbolic one: A pointwise estimate for the difference between their solutions," Boll. Un. Mat. Ital., Ser. 6, **5A**, 33.

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