Robust Analysis and Synthesis for Linear Parameter Varying Systems

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Research Areas

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Parul Singh
Shu Wang
Wind Energy

Bin Hu
Inchara Lakshminarayan
Raghu Venkataraman
Safety Critical Systems

Masanori Honda
Hard Disk Drives

Robust Control Design and Analysis

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Research Areas: Aeroservoelasticity

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Harald Pfifer
Julian Theis

Outline

- Linear Parameter Varying (LPV) Systems
- Applications
  - Flexible Aircraft
  - Wind Farms
- Theory for LPV Systems
  - Robustness Analysis
  - Model Reduction

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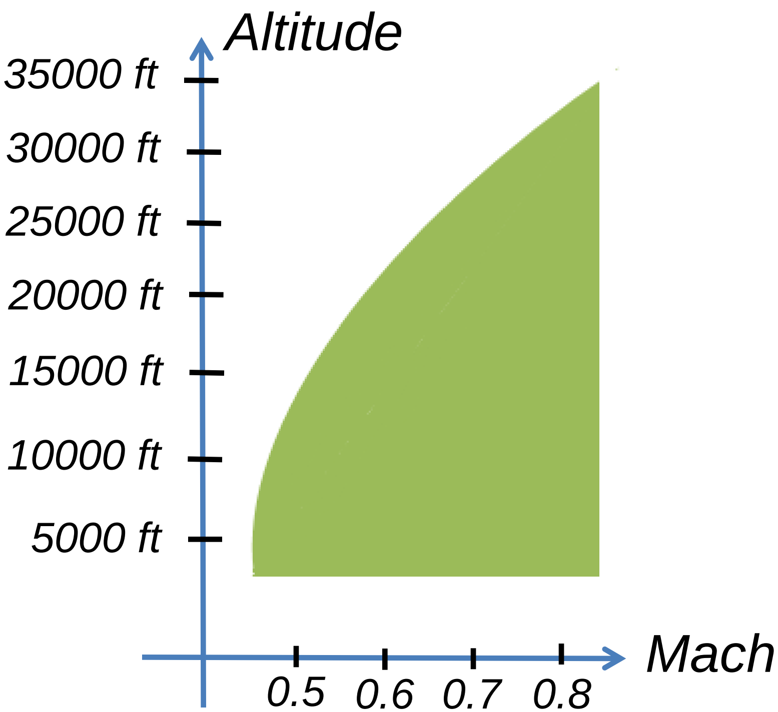
Modeling for Aircraft Control

u(t)

Rockwell B-1 Lancer (Photo: US Air Force)

y(t)



Flight Envelope

Nonlinear ODE

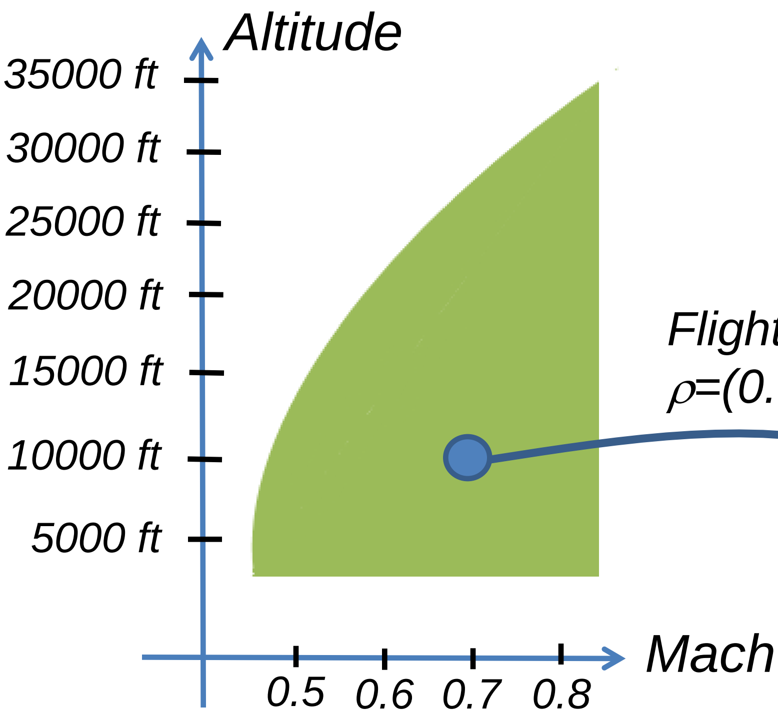
$$\dot{x}(t) = f(x(t), u(t))$$
$$y(t) = h(x(t), u(t))$$

Modeling for Aircraft Control

$u(t)$

Rockwell B-1 Lancer (Photo: US Air Force)

$y(t)$



Flight Envelope

Flight Condition
ρ=(0.7,10000ft)

Nonlinear ODE

$$\dot{x}(t) = f(x(t), u(t))$$
$$y(t) = h(x(t), u(t))$$

Equilibrium Condition

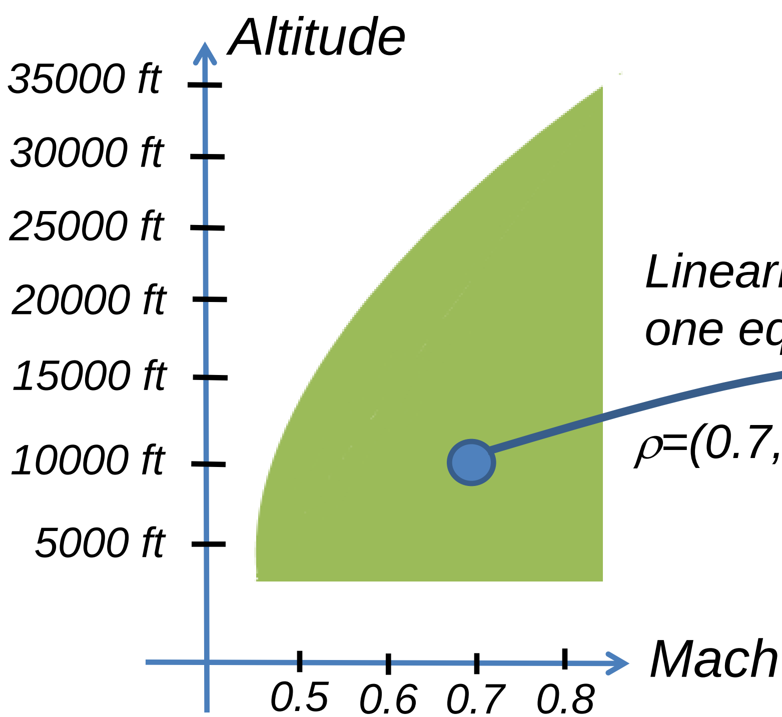
$$0 = f(\bar{x}, \bar{u})$$
$$\bar{y} = h(\bar{x}, \bar{u})$$

Modeling for Aircraft Control

u(t)

Rockwell B-1 Lancer (Photo: US Air Force)

y(t)



Flight Envelope

Linearize near one equilibrium

ρ=(0.7,10000ft)

Linear Time Invariant (LTI)

$$\dot{\delta}_x(t) = A\ \delta_x(t) + B\ \delta_u(t)$$
$$\delta_y(t) = C\ \delta_x(t) + D\ \delta_u(t)$$

where

$$\delta_x(t) := x(t) - \bar{x}$$

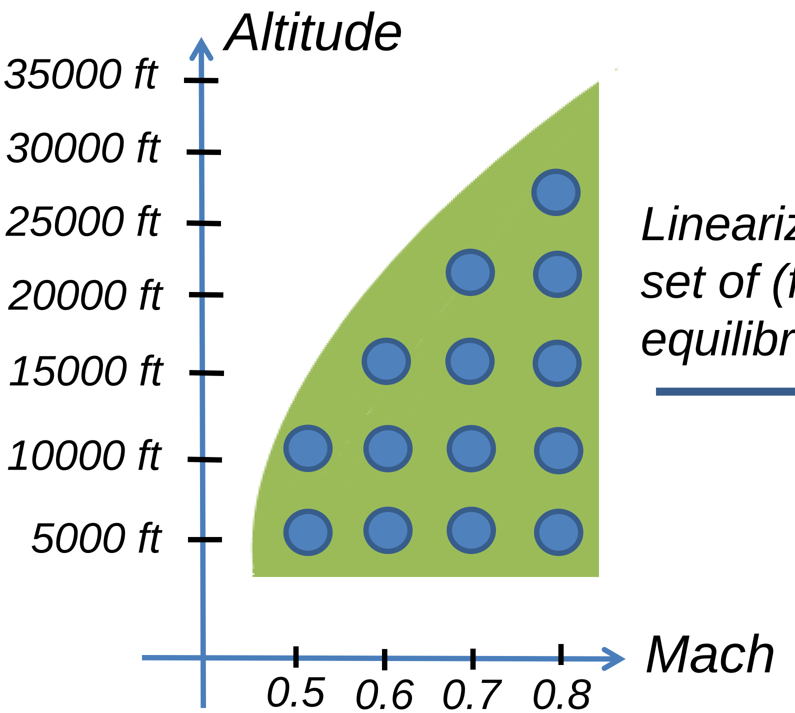
Use for linear control design

Modeling for Aircraft Control

$u(t)$ →

Rockwell B-1 Lancer (Photo: US Air Force)

→ $y(t)$



Flight Envelope

Linearize near set of (fixed) equilibria →

Parameterized LTI

$$\dot{\delta}_x(t) = A(\rho)\ \delta_x(t) + B(\rho)\ \delta_u(t)$$
$$\delta_y(t) = C(\rho)\ \delta_x(t) + D(\rho)\ \delta_u(t)$$

where

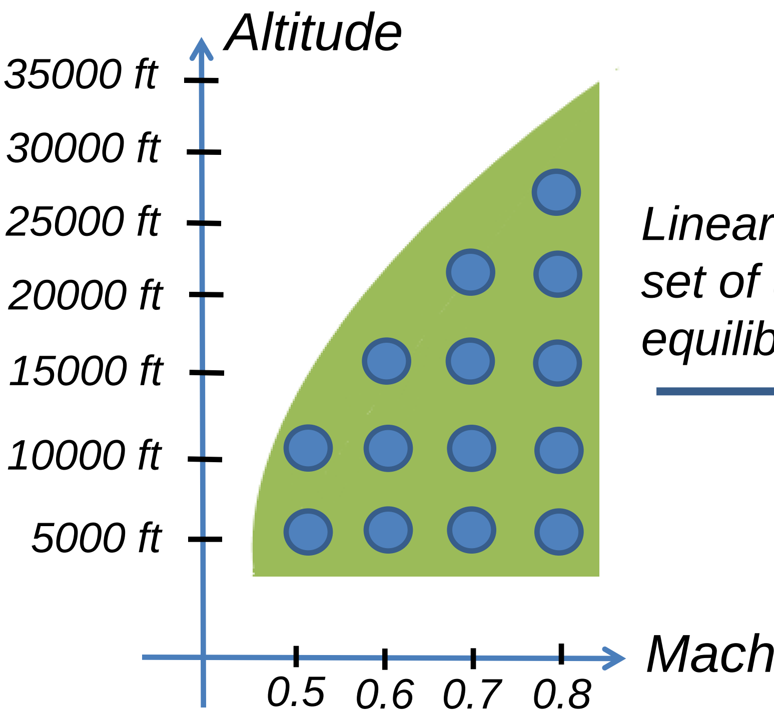
$$\delta_x(t) := x(t) - \bar{x}(\rho)$$

Modeling for Aircraft Control

u(t)

Rockwell B-1 Lancer (Photo: US Air Force)

y(t)



Flight Envelope

Linearize near set of (fixed) equilibria

Parameterized LTI

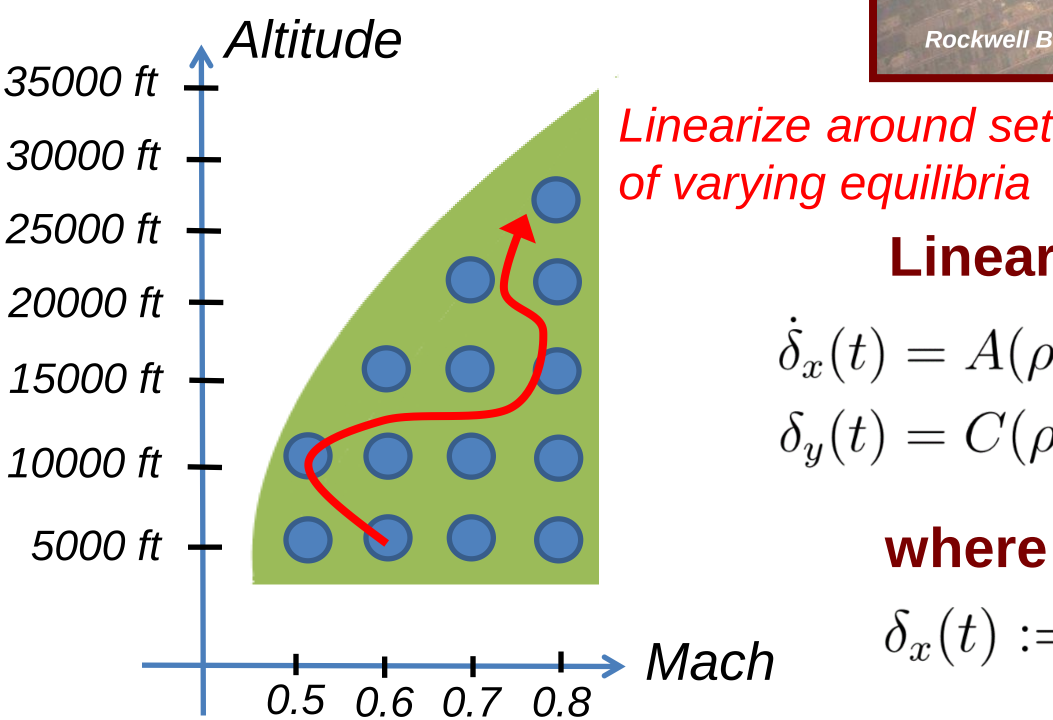
$$\dot{\delta}_x(t) = A(\rho)\ \delta_x(t) + B(\rho)\ \delta_u(t)$$
$$\delta_y(t) = C(\rho)\ \delta_x(t) + D(\rho)\ \delta_u(t)$$

Gain-Scheduling

Design controllers at many flight conditions and "stitch" together.

Modeling for Aircraft Control

Rockwell B-1 Lancer (Photo: US Air Force)



Flight Envelope

Linear Parameter Varying (LPV)

$$\dot{\delta}_x(t) = A(\rho(t))\ \delta_x(t) + B(\rho(t))\ \delta_u(t) - \dot{\bar{x}}(\rho(t))$$
$$\delta_y(t) = C(\rho(t))\ \delta_x(t) + D(\rho(t))\ \delta_u(t)$$

where

$$\delta_x(t) := x(t) - \bar{x}(\rho(t))$$

Outline

- Linear Parameter Varying (LPV) Systems
- **Applications**
  - **Flexible Aircraft**
  - **Wind Farms**
- Theory for LPV Systems
  - Robustness Analysis
  - Model Reduction

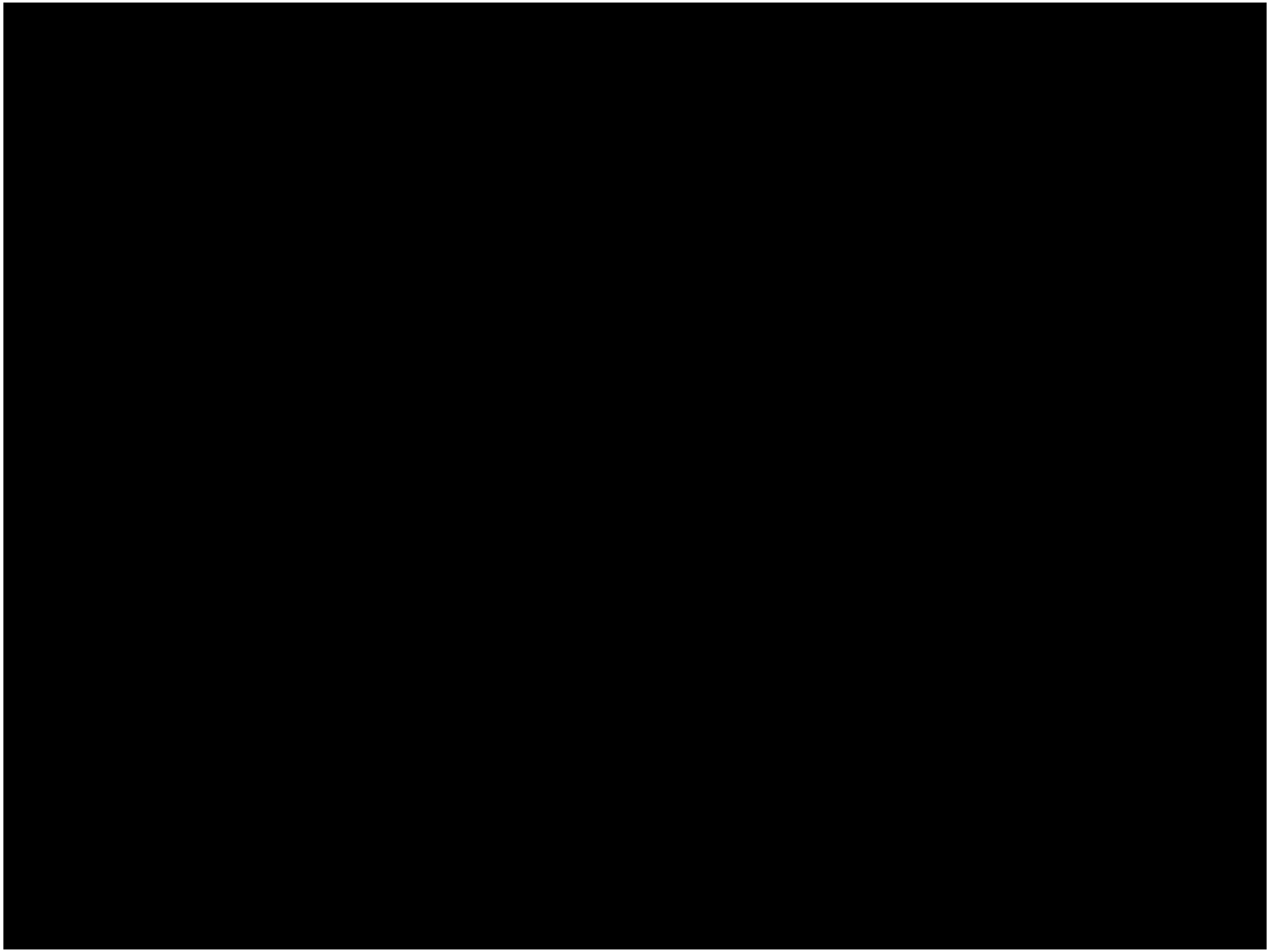
Aeroservoelasticity (ASE)

Efficient aircraft design

- Lightweight structures
- High aspect ratios

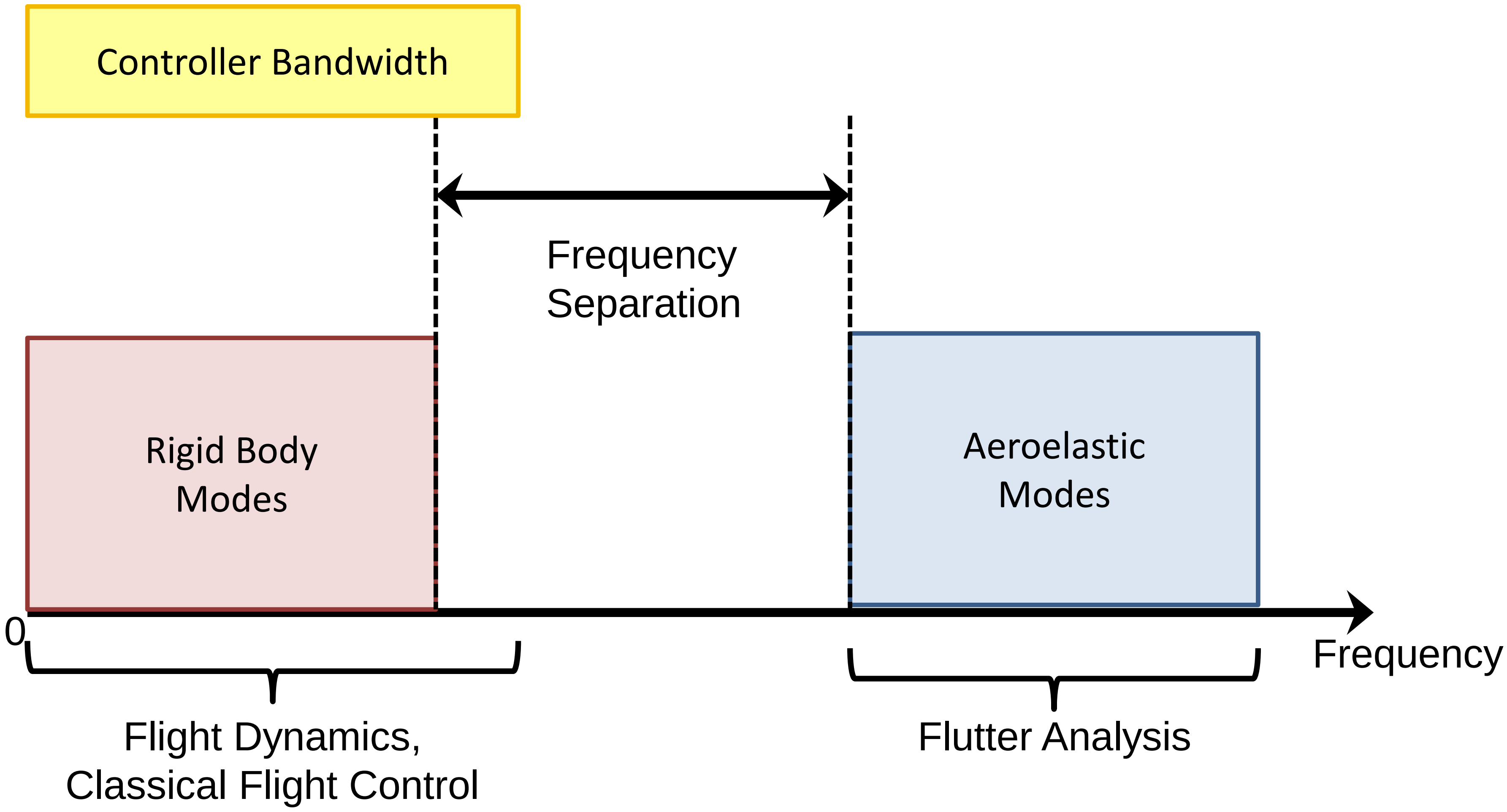
Source: www.flightglobal.com

Flutter

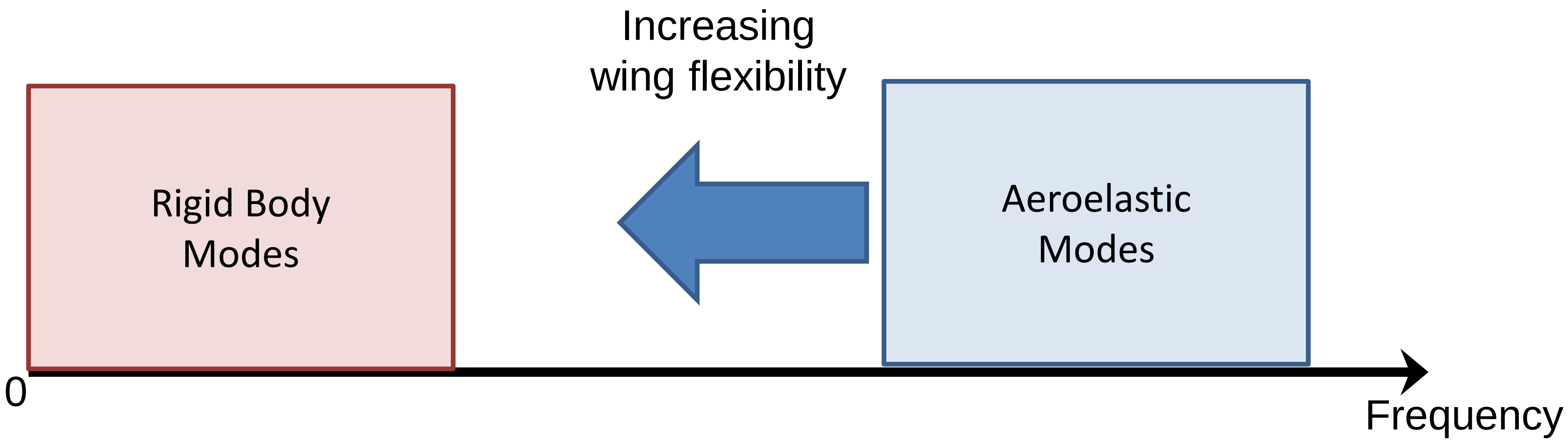


Source: NASA Dryden Flight Research

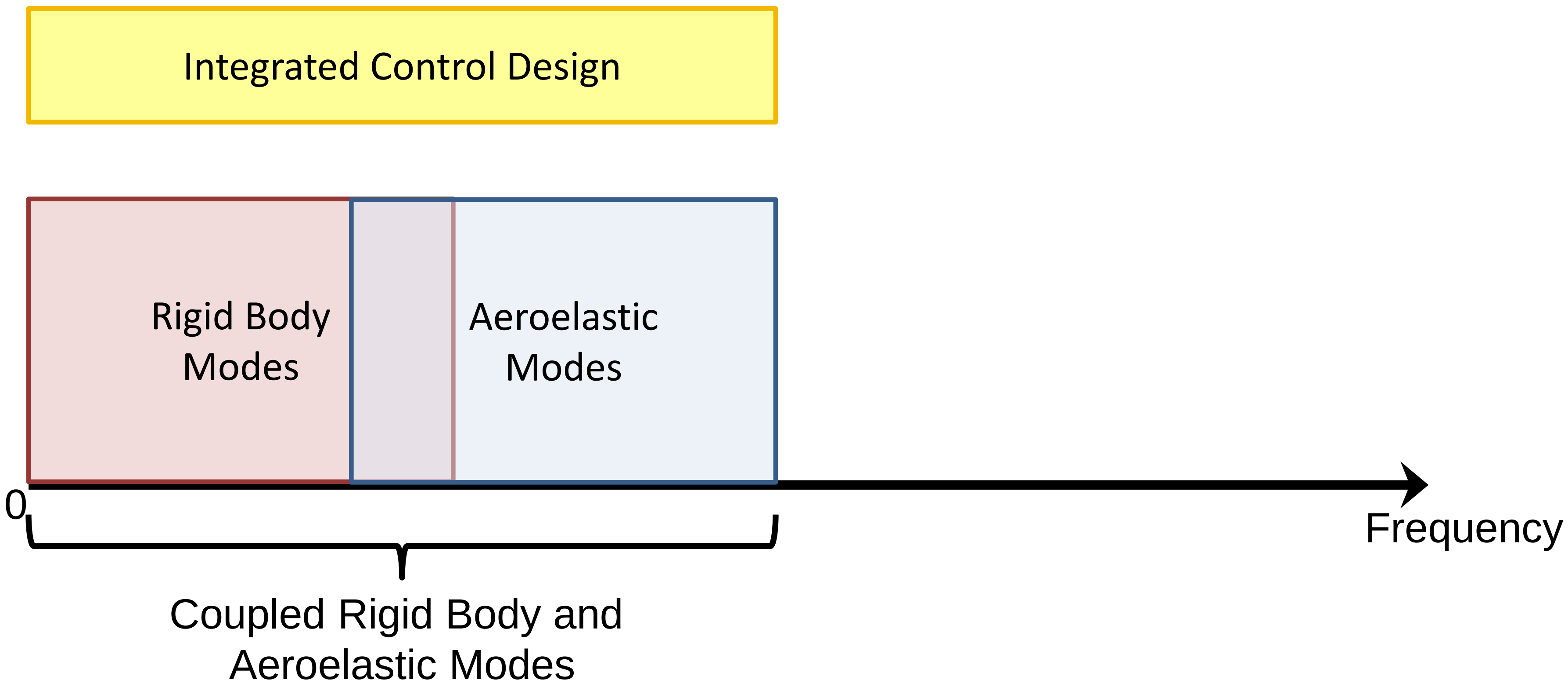
Classical Approach



Flexible Aircraft Challenges



Flexible Aircraft Challenges



Body Freedom Flutter

Performance Adaptive Aeroelastic Wing (PAAW)

- Goal: Suppress flutter, control wing shape and alter shape to optimize performance
  - Funding: NASA NRA NNX14AL36A
  - Technical Monitor: Dr. John Bosworth
  - Two years of testing at UMN followed by two years of testing on NASA's X-56 Aircraft

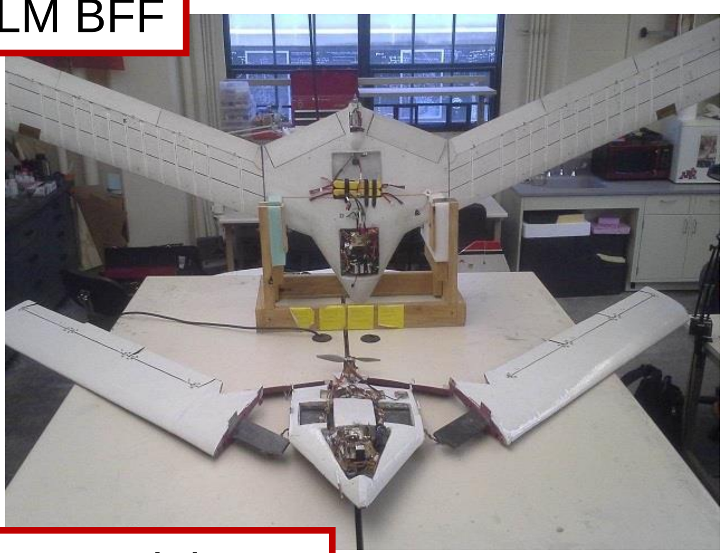
Aurora Flight Sciences

CMSoft

Schmidt & Associates

Virginia Tech Invent the Future

LM BFF

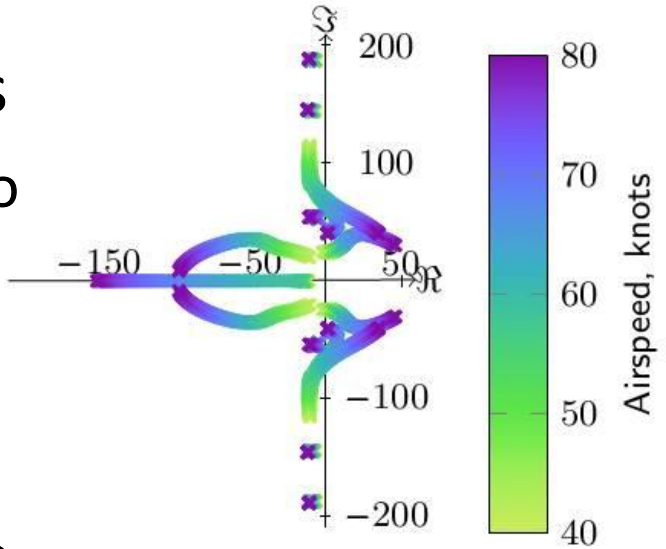


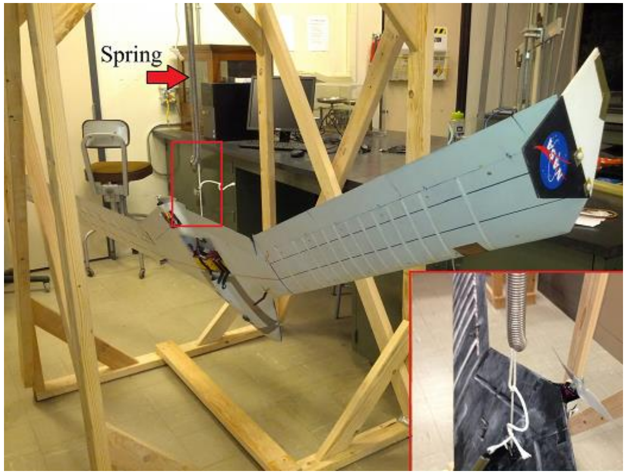
UMN Mini-Mutt

LM/NASA X-56

Modeling and Control for Flex Aircraft

1. Parameter Dependent Dynamics
   - Models depend on airspeed due to structural/aero interactions
   - LPV is a natural framework.
2. Model Reduction
   - High fidelity CFD/CSD models have many (millions) of states.
3. Model Uncertainty
   - Use of simplified low order models OR reduced high fidelity models
   - Unsteady aero, mass/inertia & structural parameters



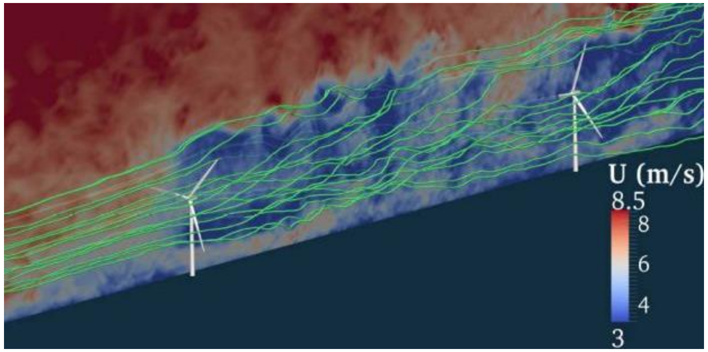


Modeling and Control for Wind Farms

1. Parameter Dependent Dynamics
   - Models depend on windspeed due to structural/aero interactions
   - LPV is a natural framework.
2. Model Reduction
   - High fidelity CFD/CSD models have many (millions) of states.
3. Model Uncertainty
   - Use of simplified low order models OR reduced high fidelity models

Eolos: http://www.eolos.umn.edu/

Saint Anthony Falls: http://www.safl.umn.edu/



Simulator for Wind Farm Applications, Churchfield & Lee
http://wind.nrel.gov/designcodes/simulators/SOWFA

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  - **Robustness Analysis**
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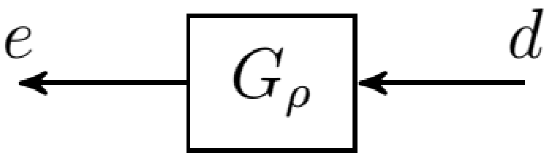
LPV Analysis



Gridded LPV System

$$\dot{x}(t) = A(\rho(t))\ x(t) + B(\rho(t))\ d(t)$$
$$e(t) = C(\rho(t))\ x(t) + D(\rho(t))\ d(t)$$

$$\rho \in \mathcal{A} := \text{Set of allowable trajectories}$$

Induced L_2 Gain

$$\sup_{\rho \in \mathcal{A}} \|G_\rho\|_{2\to 2} = \sup_{\rho \in \mathcal{A}} \sup_{0 \neq d \in L_2} \frac{\|e\|_2}{\|d\|_2}$$

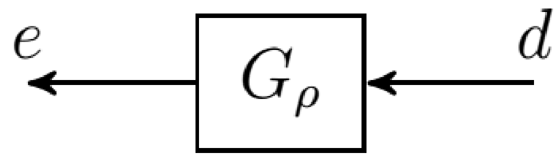
(Standard) Dissipation Inequality Condition

Theorem

If there exists $V(x,\rho) \geq 0$ such that

$$\dot{V} + e^T e \leq \gamma^2 d^T d$$

then $\sup_{\rho \in \mathcal{A}} \|G_\rho\|_{2\to 2} \leq \gamma$.



Proof: Integrate the dissipation inequality

$$\underbrace{V(x(T))}_{\geq 0} + \underbrace{V(x(0))}_{=0} + \int_0^T e(t)^T e(t) dt \leq \gamma^2 \int_0^T d(t)^T d(t) dt \quad \blacksquare$$

Comments

- Dissipation inequality can be expressed/solved using LMIs.
  - Finite dimensional LMIs for LFT/Polytopic LPV systems
  - Parameterized LMIs for Gridded LPV (requires basis functions, gridding, etc)
- **Condition is IFF for LTI systems but only sufficient for LPV**

Uncertainty Modeling

- **Goal:** Assess the impact of model uncertainty/nonlinearities
- **Approach:** Separate nominal dynamics from perturbations
  - Pert. can be parametric, LTI dynamic, and/or nonlinearities (e.g. saturation).

$$\dot{x} = (a + \Delta a)x + f(x) + d$$

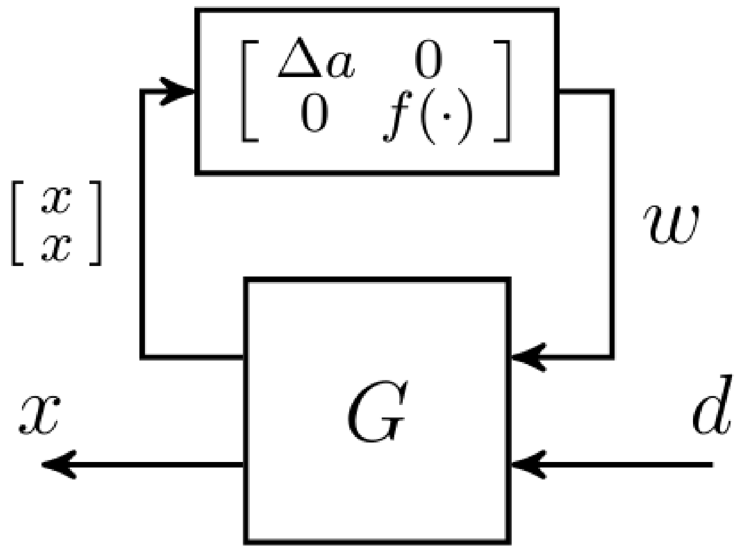
Nominal LTI, G

$$\dot{x} = ax + w_1 + w_2 + d$$

$$w_1 = \Delta a \cdot x$$

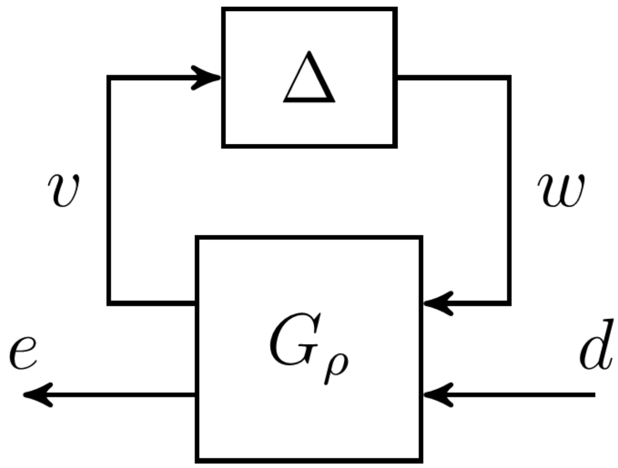
$$w_2 = f(x)$$

Perturbation, Δ



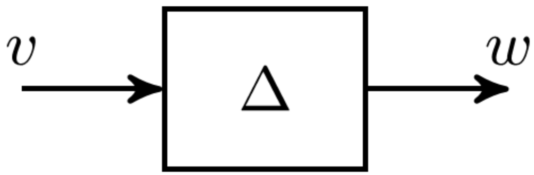
Robustness Analysis for LPV Systems

- **Goal:** Extend analysis tools to LPV



- **Approach:**
  - Use Integral Quadratic Constraints to model input/output behavior (Megretski & Rantzer, TAC 1997).
  - Extend dissipation inequality approach for robustness analysis
- **Results for Gridded Nominal system**
  - Parallels earlier results for LFT nominal system by Scherer, Veenman, Köse, Köroğlu.

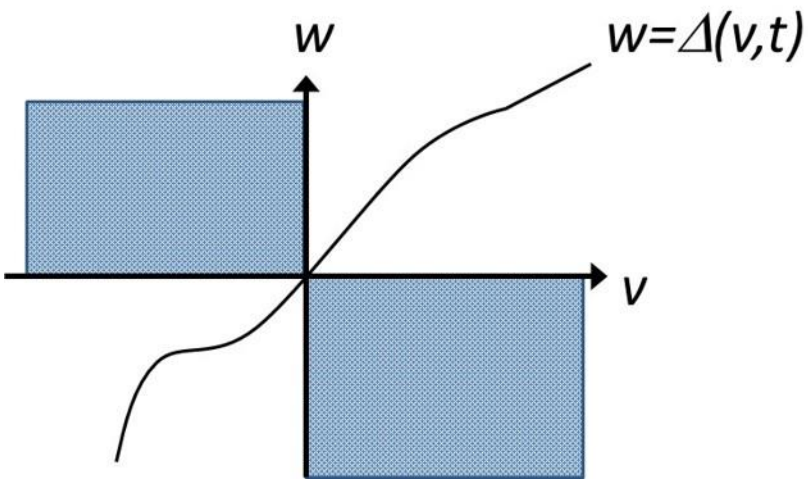
IQC Example: Passive System



$w = \Delta(v,t)$ is a passive system (pointwise in time).

$$2v(t)^T w(t) \geq 0 \ \forall t$$

$$\begin{bmatrix} v(t) \\ w(t) \end{bmatrix}^T \begin{bmatrix} 0 & I \\ I & 0 \end{bmatrix} \begin{bmatrix} v(t) \\ w(t) \end{bmatrix} \geq 0 \, \forall t$$



Pointwise Quadratic Constraint

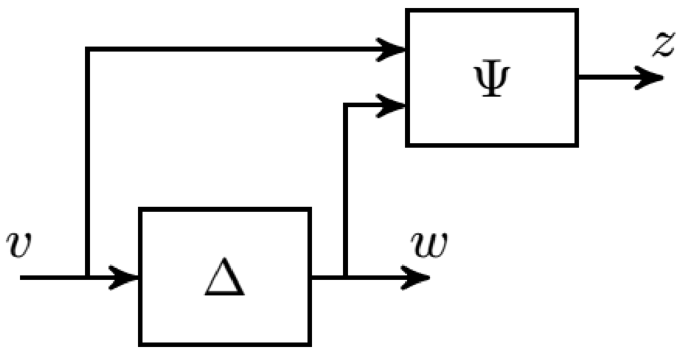
General (Time Domain) IQCs

General IQC Definition:

Let Ψ be a stable, LTI system and M a constant matrix.
Δ satisfies IQC defined by Ψ and M if

$$\int_0^T z(t)^T M z(t) dt \geq 0$$

$\forall v \in L_2[0,\infty)$, $w = \Delta(v)$, and $T \geq 0$.



Comments:

- Megretski & Rantzer ('97 TAC) has a library of IQCs for various components.
- IQCs can be equivalently specified in the freq. domain with a multiplier Π
- A non-unique factorization connects $\Pi=\Psi^* M \Psi$.
- Multiple IQCs can be used to specify behavior of Δ.

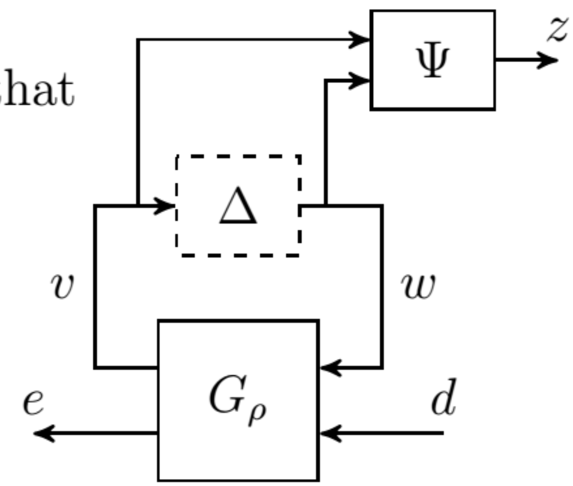
IQC Dissipation Inequality Condition

Theorem

If $\Delta \in IQC(\Psi, M)$ and there exists $V(x,\rho) \geq 0$ such that

$$\dot{V} + z^T M z + e^T e \leq \gamma^2 d^T d$$

then $\sup_{\rho \in \mathcal{A}} \|G_\rho\|_{2\to 2} \leq \gamma$.



Proof: Integrate the dissipation inequality

$$\underbrace{V(x(T))}_{\geq 0} + \underbrace{V(x(0))}_{=0} + \underbrace{\int_0^T z(t)^T M z(t) dt}_{\geq 0} + \int_0^T e(t)^T e(t) dt \leq \gamma^2 \int_0^T d(t)^T d(t) dt$$

■

Comment

- Dissipation inequality can be expressed/solved as LMIs.
- Extends standard D/G scaling but requires selection of basis functions for IQC.

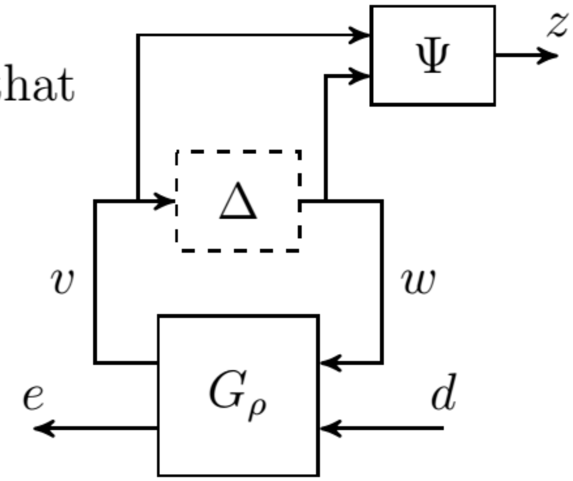
Less Conservative IQC Result

Theorem

If $\Delta \in IQC(\Psi, M)$ and there exists $V(x,\rho) \geq 0$ such that

$$\dot{V} + z^T M z + e^T e \leq \gamma^2 d^T d$$

then $\sup_{\rho \in \mathcal{A}} \|G_\rho\|_{2\to 2} \leq \gamma$.



Technical Result

- Positive semidefinite constraint on *V* and time domain IQC constraint can be dropped.
- These are replaced by a freq. domain requirement on $\Pi = \Psi^* M \Psi$.
- Some energy is "hidden" in the IQC.

Refs:

P. Seiler, Stability Analysis with Dissipation Inequalities and Integral Quadratic Constraints, IEEE TAC, 2015.

H. Pfifer & P. Seiler, Less Conservative Robustness Analysis of Linear Parameter Varying Systems Using Integral Quadratic Constraints, submitted to IJRNC, 2015.

Time-Domain Dissipation Inequality Analysis

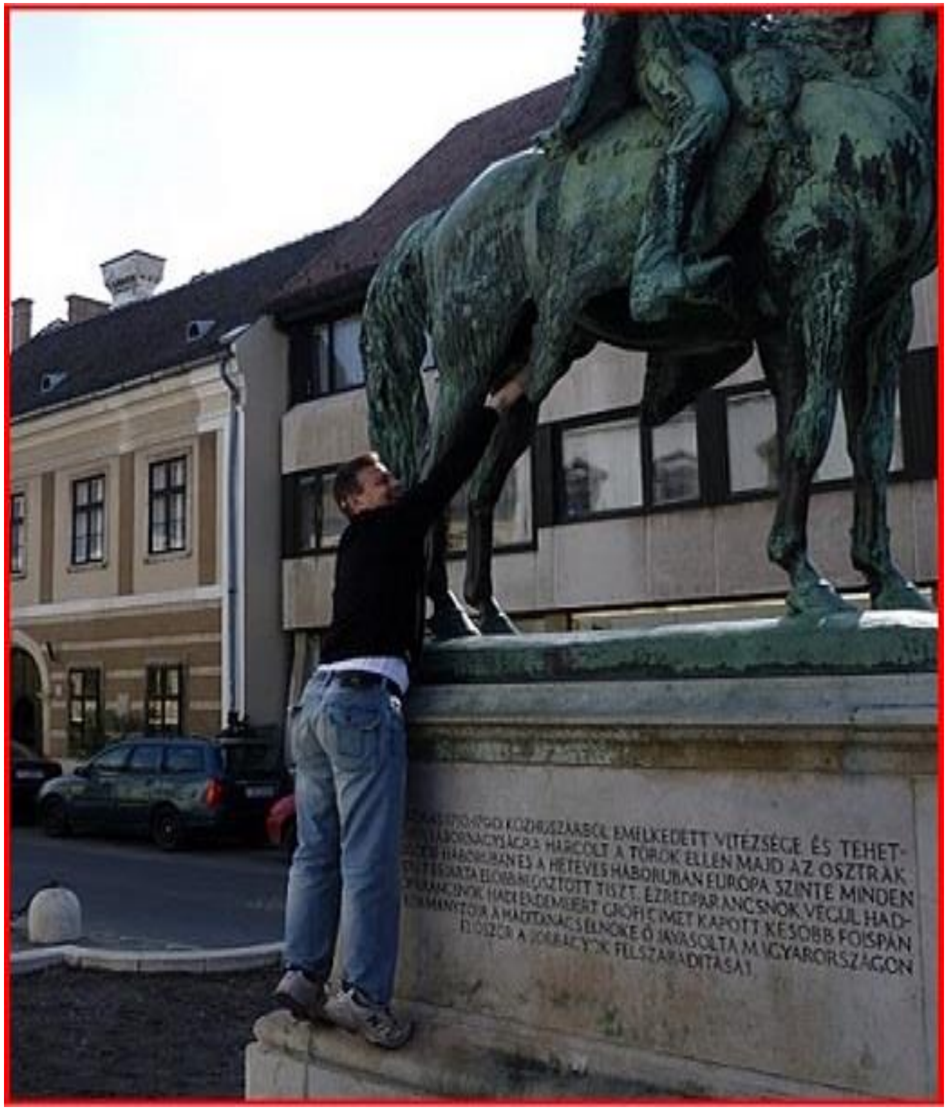
Summary: Under some technical conditions, the frequency-domain conditions in (M/R, '97 TAC) are equivalent to the time-domain dissipation inequality conditions.

Applications:

1. LPV robustness analysis (Pfifer, Seiler, IJRNC)
2. General LPV robust synthesis (Wang, Pfifer, Seiler, submitted to Aut)
3. LPV robust filtering/feedforward (Venkataraman, Seiler, in prep)
   - Robust filtering typically uses a duality argument. Extensions to the time domain?
4. Exponential rates of convergence (Hu,Seiler, submitted to TAC)
   - Motivated by optimization analysis with ρ-hard IQCs (Lessard, Recht, & Packard)
5. Nonlinear analysis using SOS techniques

Item 1 has been implemented in LPVTools. Items 2 & 3 parallel results by (Scherer, Köse, and Veenman) for LFT-type LPV systems.

Outline



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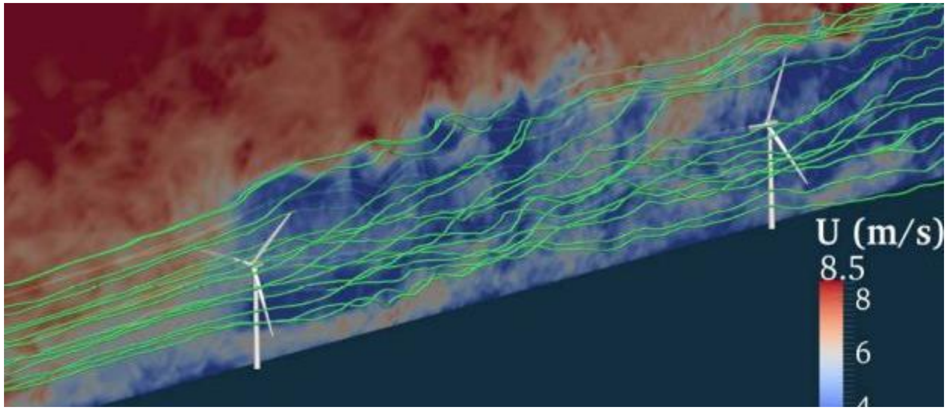
LPV Model Reduction

- Both flexible aircraft and wind farms can be modeled with high fidelity fluid/structural models.
- LPV models can be obtained via Jacobian linearization:

$$\dot{x}(t) = A(\rho(t))\ x(t) + B(\rho(t))\ d(t)$$

$$e(t) = C(\rho(t))\ x(t) + D(\rho(t))\ d(t)$$

- State dimension can be extremely large (>10^6)
- LPV analysis and synthesis is restricted to ≈50 states.
- Model reduction is required.



High Order Model Reduction

Large literature with recent results for LPV and Param. LTI

- Antoulas, Amsallem, Carlberg , Gugercin, Farhat, Kutz, Loeve, Mezic, Poussot-Vassal, Rowley, Schmid, Willcox, …

Two new results for LPV:

1. Input-Output Dynamic Mode Decomposition
   - Combine subspace ID with techniques from fluids (POD/DMD).
   - No need for adjoint models. Can reconstruct full-order state.
2. Parameter-Varying Oblique Projection
   - Petrov-Galerkin approximation with constant projection space and parameter-varying test space.
   - Constant projection maintains state consistency avoids rate dependence.

References

1A. Annoni & Seiler, A method to construct reduced-order parameter varying models, submitted to IJRNC, 2015.

1B. Annoni, Nichols, & Seiler, "Wind farm modeling and control using dynamic mode decomposition." AIAA, 2016.

1C. Singh & Seiler, Model Reduction using Frequency Domain Input-Output Dynamic Mode Decomposition, sub. to '16 ACC.

2. Theis, Seiler, & Werner, Model Order Reduction by Parameter-Varying Oblique Projection, submitted to 2016 ACC.

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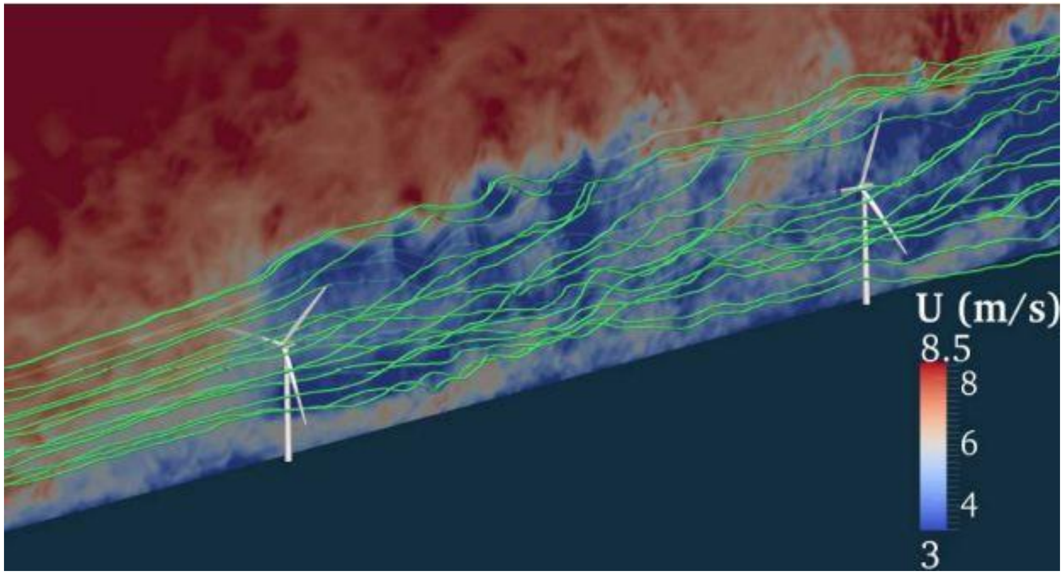
Higher-Fidelity – Large Eddy Simulation (LES)

- Simulator for On/Offshore Wind Farm Applications
- 3D unsteady spatially filtered Navier-Stokes equations
- Simulation time (wall clock): 48 hours



Churchfield, Lee
https://nwtc.nrel.gov/SOWFA

Problem Setup

Linearized discrete-time Navier-Stokes

$$x_{k+1} = Ax_k + Bu_k$$

$$y_k = Cx_k + Du_k$$

Updated velocity (x_{k+1})

Outputs (y_k)
Wind speed at nacelle
Power
Loads

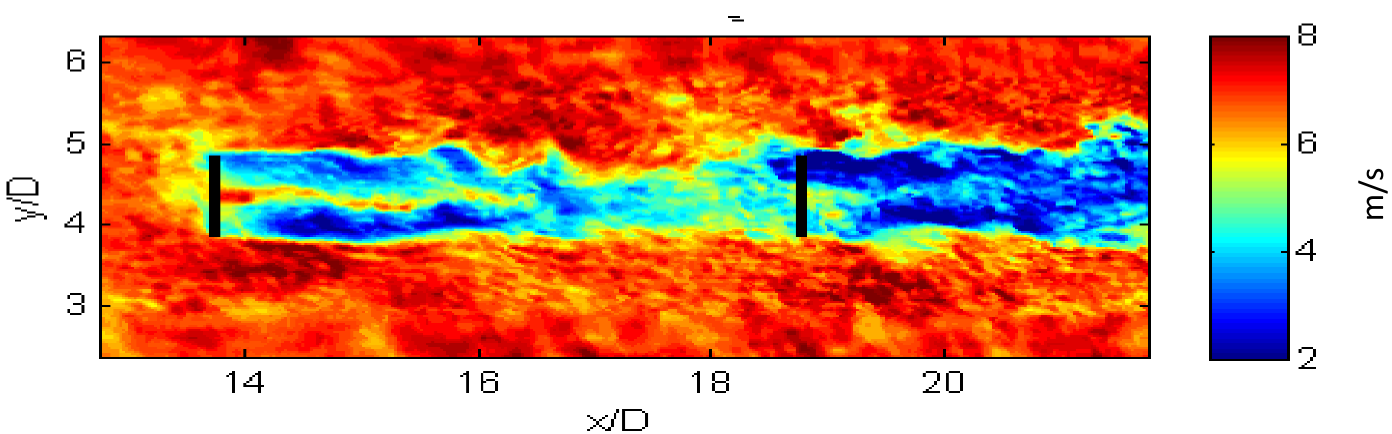
Linearized Navier-Stokes (A)

Inputs: β, τ_g (u_k)

Input operator (B)
: indicates location of turbines

Influence of input (D)

Output operator (C)

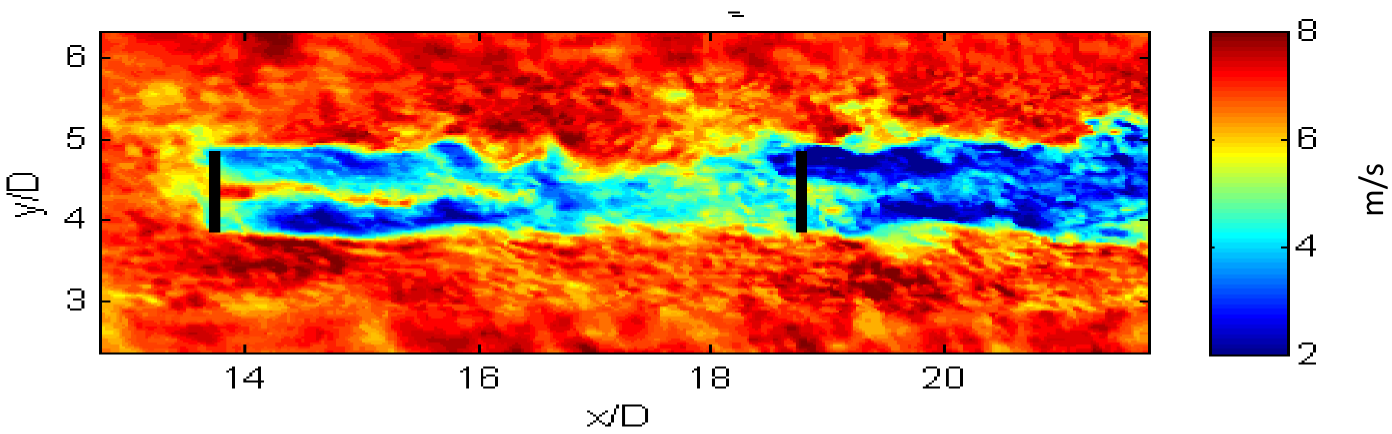


Problem Setup

Linearized discrete-time Navier-Stokes

Millions of states ← $\boxed{x_{k+1}} = Ax_k + Bu_k$

$$y_k = Cx_k + Du_k$$



Problem Setup

Linearized discrete-time Navier-Stokes

Millions of states ← x_{k+1}

$$x_{k+1} = Ax_k + Bu_k$$
$$y_k = Cx_k + Du_k$$

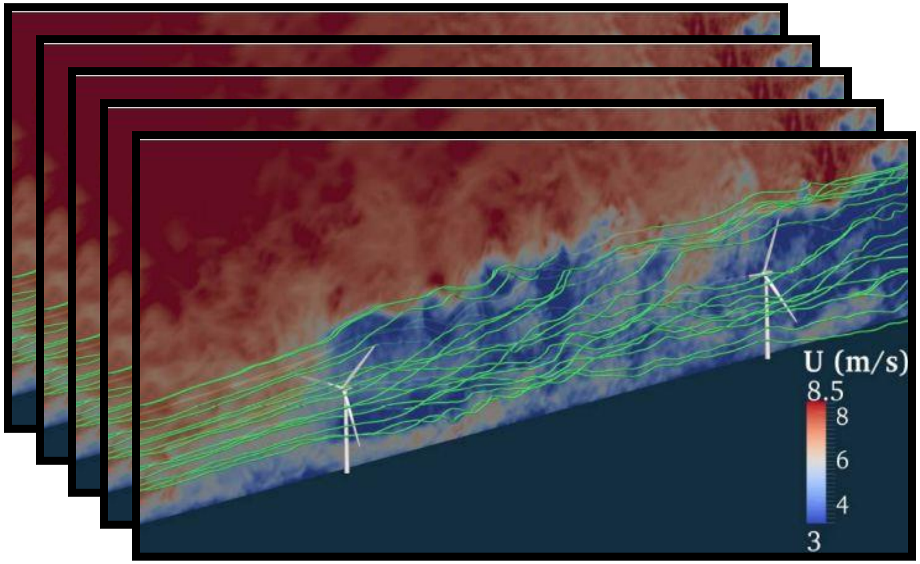
↓

10s to 100s of states ← $\tilde{x}_{k+1}$

$$\tilde{x}_{k+1} = \tilde{A}\tilde{x}_k + \tilde{B}u_k$$
$$y_k = \tilde{C}\tilde{x}_k + \tilde{D}u_k$$

Typical Approaches in Fluids

- Project onto the dominant modes of the system
  - Proper orthogonal decomposition (POD)
    - Lumley, et. al. 1967
  - Dynamic mode decomposition (DMD)
    - Schmid, Mezic, Rowley, Kutz, others



Churchfield et. al. "NWTC design codes-SOWFA"

Dynamic Mode Decomposition

- Gather snapshots from simulation or experiments
- Fit a linear operator to the snapshots

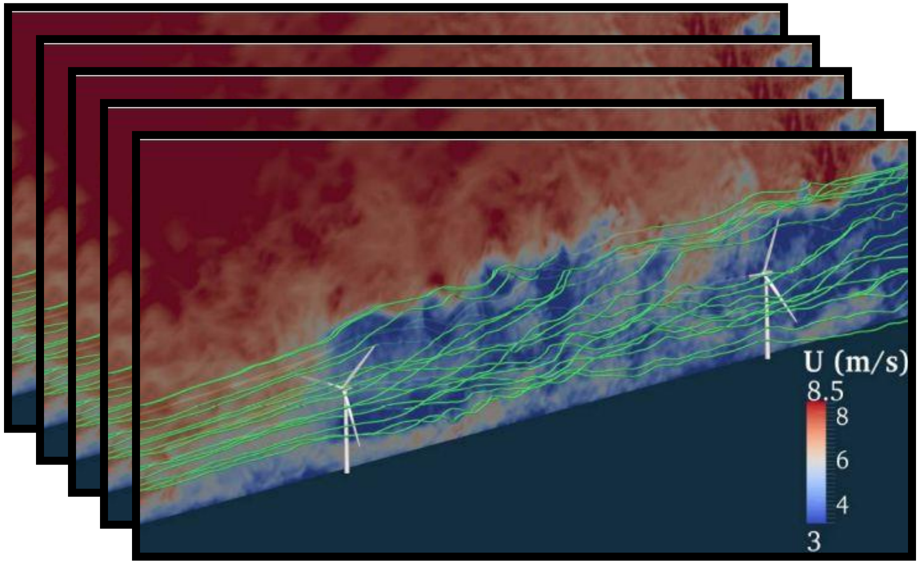
$$X_0 = [x_1, x_2, \ldots, x_m]$$
$$X_1 = [x_2, x_3, \ldots, x_{m+1}]$$

Gather snapshots

$$\longrightarrow \quad A = X_1 X_0^+$$

Fit linear operator to snapshots

Intractable for large systems



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$$X_0 = [x_1, x_2, \ldots, x_m]$$
$$X_1 = [x_2, x_3, \ldots, x_{m+1}]$$

Gather snapshots

$\longrightarrow$

$$X_0 = U \Sigma V^T$$

Compute POD Modes

$\longrightarrow$

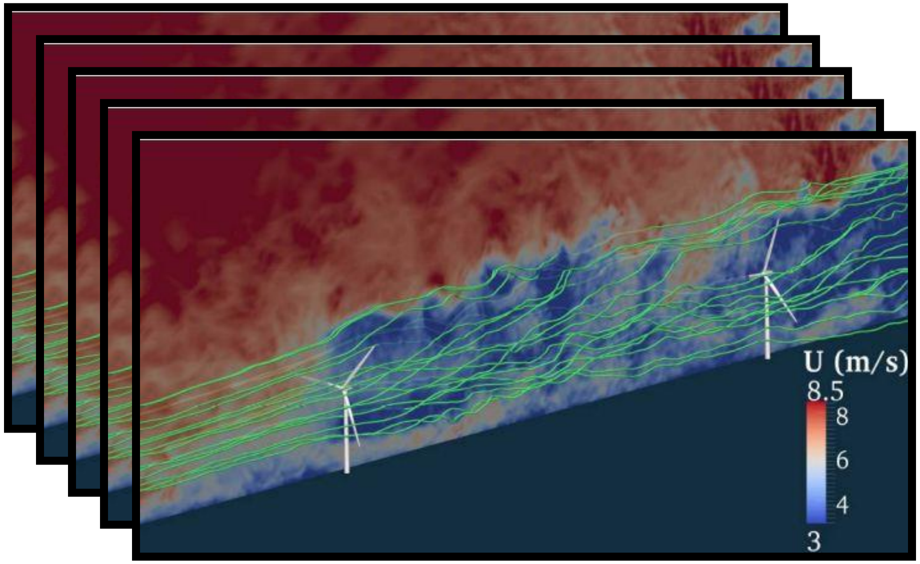
$$U_r^* X_0$$
$$U_r^* X_1$$

Project onto POD Modes

$\longrightarrow$

$$\tilde{A} = U_r^* X_1 (U_r^* X_0)^+$$

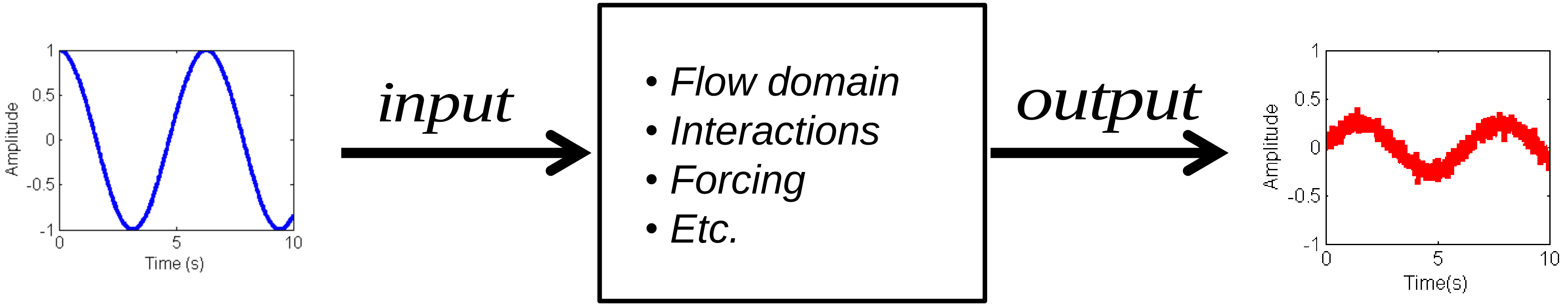
Reduced order model



Churchfield et. al. "NWTC design codes-SOWFA"

Typical Approaches in Controls

- Subspace identification
  - Fit low-order, “black-box” ODE to input/output data
  - Katayama, Larimore, Ljung, van Overschee, de Moor, Viberg, Verhaegen, others



Direct Subspace Identification (Viberg, '95)

- **Gather snapshots from simulation or experiments**
- **Measurements of inputs and outputs**
- Fit a linear operator to the snapshots

$$X_0 = [x_1, x_2, \ldots, x_{m-1}]$$
$$X_1 = [x_2, x_3, \ldots, x_m]$$

$$U_0 = [u_1, u_2, \ldots, u_{m-1}]$$
$$Y_0 = [y_1, y_2, \ldots, y_{m-1}]$$

$$X_1 = AX_0 + BU_0$$
$$Y_0 = CX_0 + DU_0$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} X_1 \\ Y_0 \end{bmatrix} \begin{bmatrix} X_0 \\ U_0 \end{bmatrix}^+$$

Intractable for large systems

IODMD

- Project state data onto a subspace

$$\begin{bmatrix} \tilde{A} & \tilde{B} \\ \tilde{C} & \tilde{D} \end{bmatrix} = \begin{bmatrix} U_r^* X_1 \\ Y_0 \end{bmatrix} \begin{bmatrix} U_r^* X_0 \\ U_0 \end{bmatrix}^+$$

POD Modes

$$X_0 = U \Sigma V^T$$

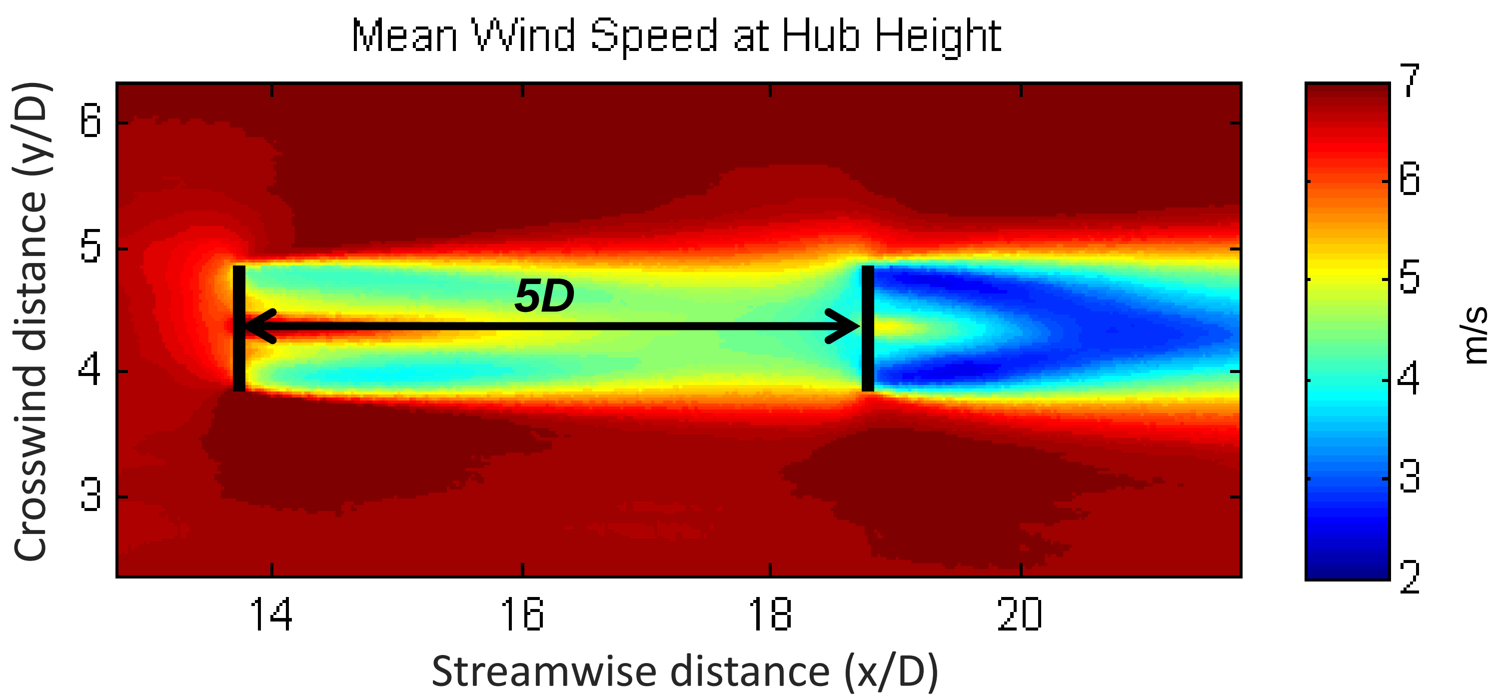
- Obtain a discrete reduced-order model of the system

$$\begin{bmatrix} \tilde{x}_{k+1} \\ y_k \end{bmatrix} = \begin{bmatrix} \tilde{A} & \tilde{B} \\ \tilde{C} & \tilde{D} \end{bmatrix} \begin{bmatrix} \tilde{x}_k \\ u_k \end{bmatrix}$$

- Blends direct subspace ID with POD/DMD
  - Handles inputs/outputs
  - Full state can be reconstructed from reduced state
  - Input forcing increases the signal to noise ratio
  - Parameter-varying version that maintains state consistency

Wind Turbine Array Setup

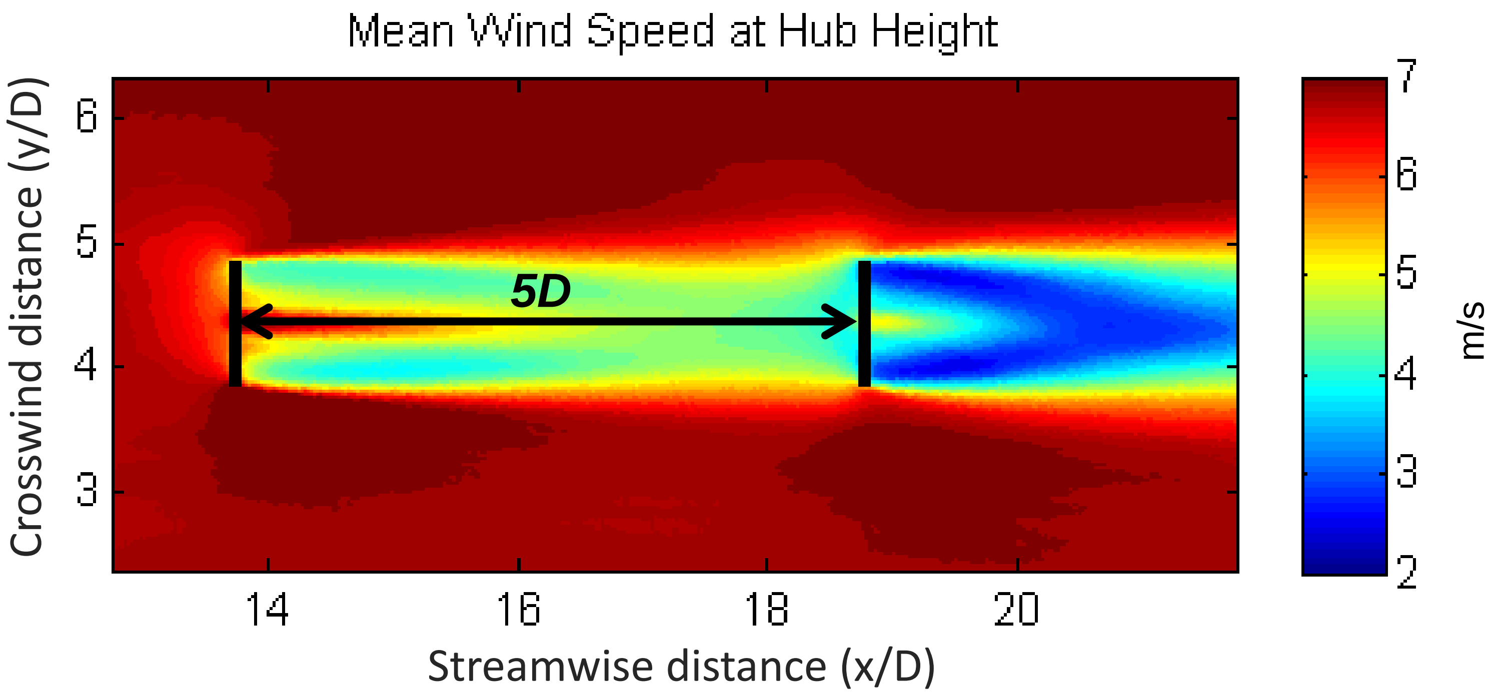
- Two turbine setup (NREL 5 MW turbines)



- D = turbine diameter (126 m)
- Neutral boundary layer
- 7 m/s with 6% turbulence

Wind Turbine Array Setup

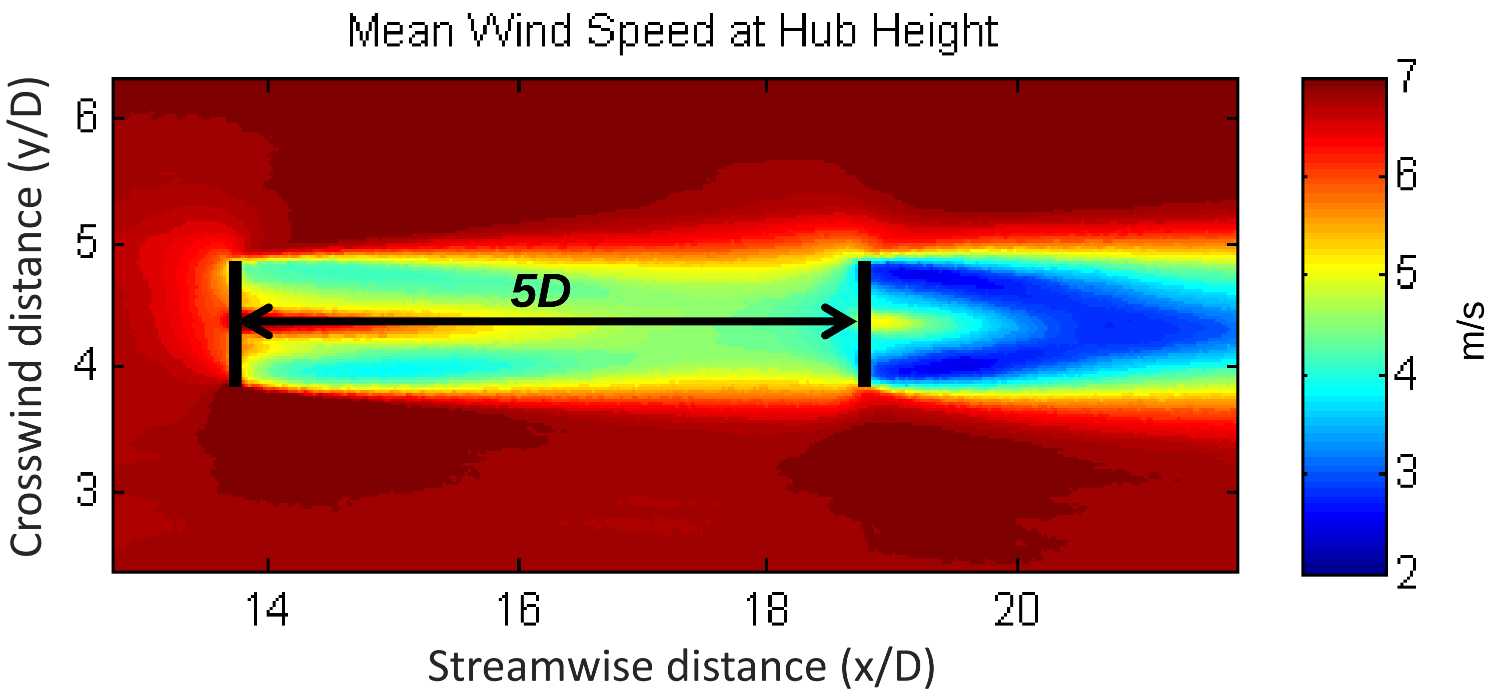
- Two turbine setup (NREL 5 MW turbines)



- Control inputs: Blade pitch angle, generator torque
- Control outputs: Power at each turbine

Wind Turbine Array Setup

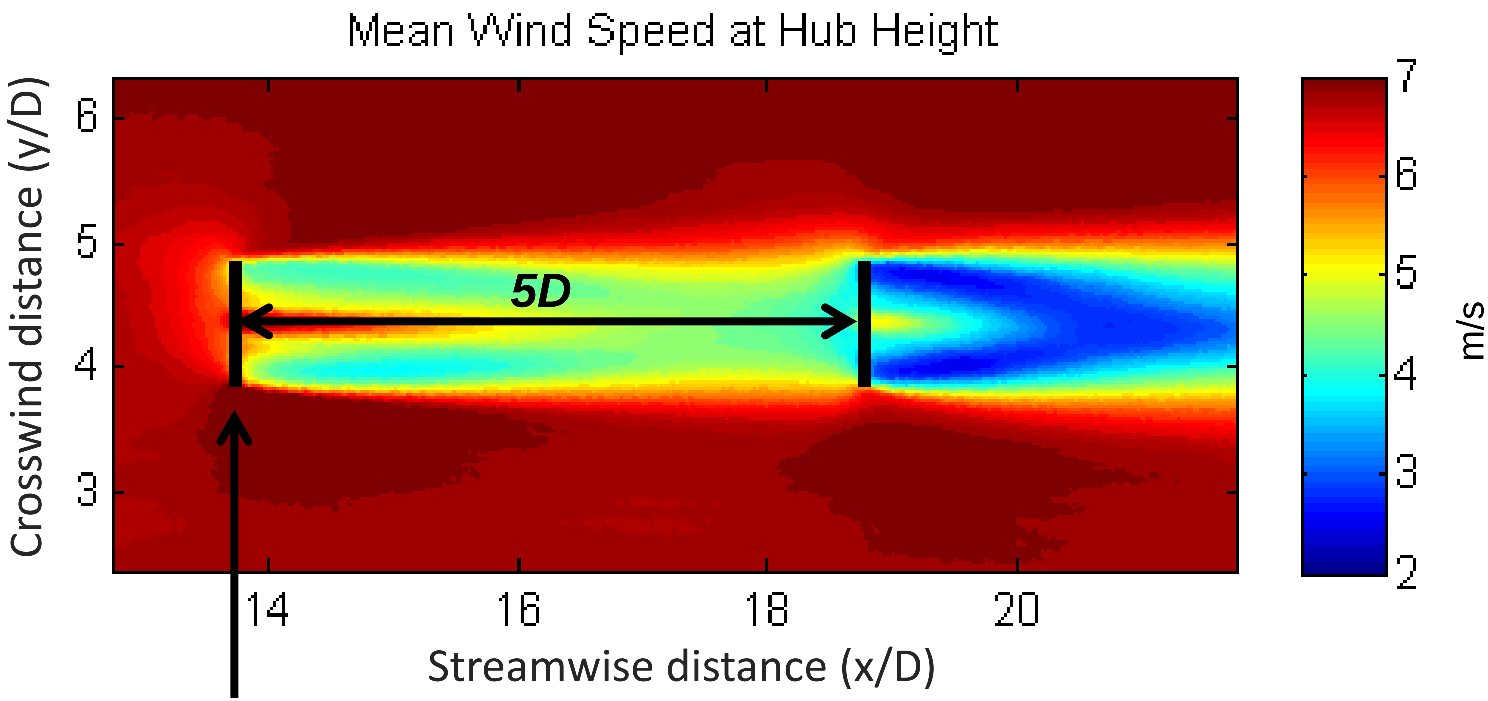
- Two turbine setup (NREL 5 MW turbines)

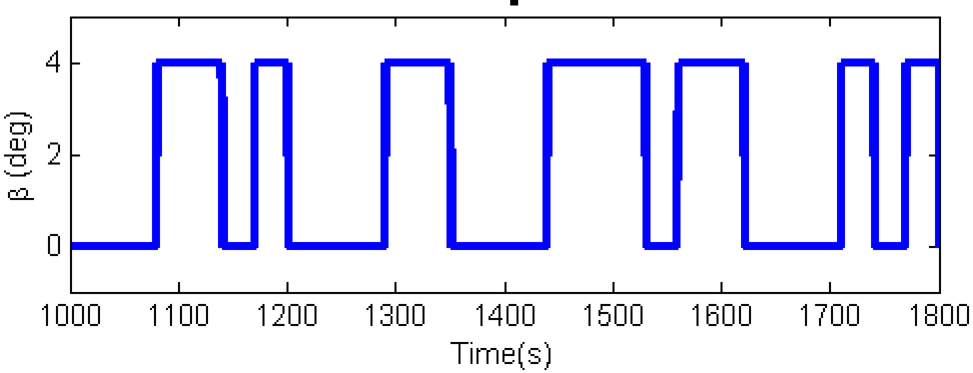


- Approximately 1.2 million grid points
  - 3 velocity components → **3.6 million states**
  - Intractable for control design

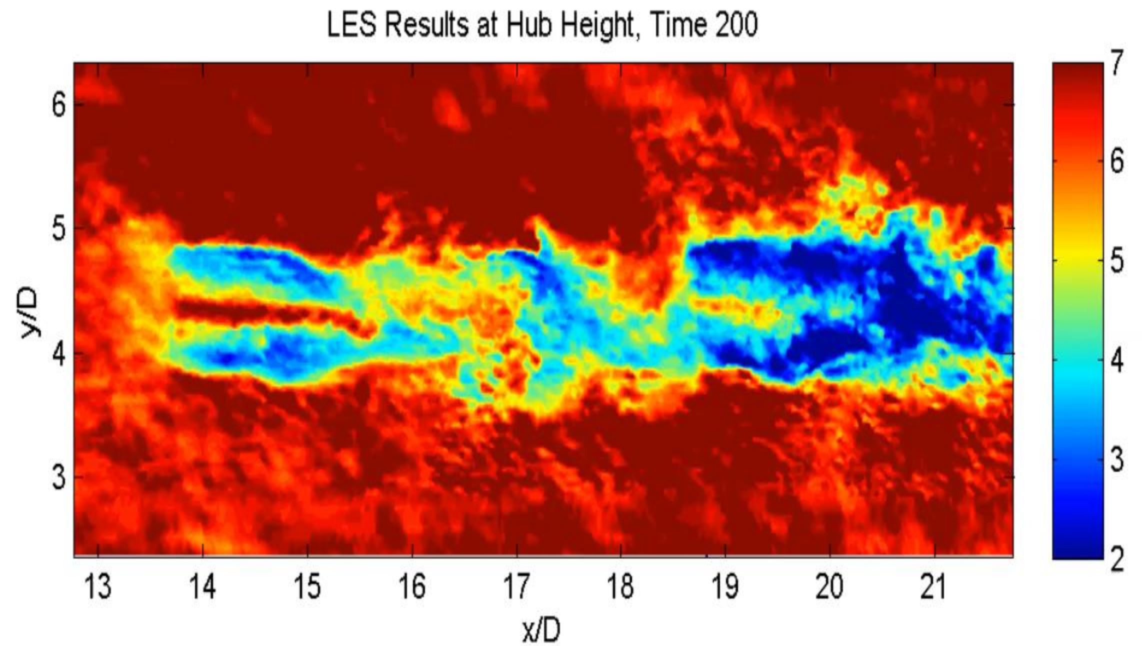
IODMD with SOWFA

- Forcing Input to first turbine

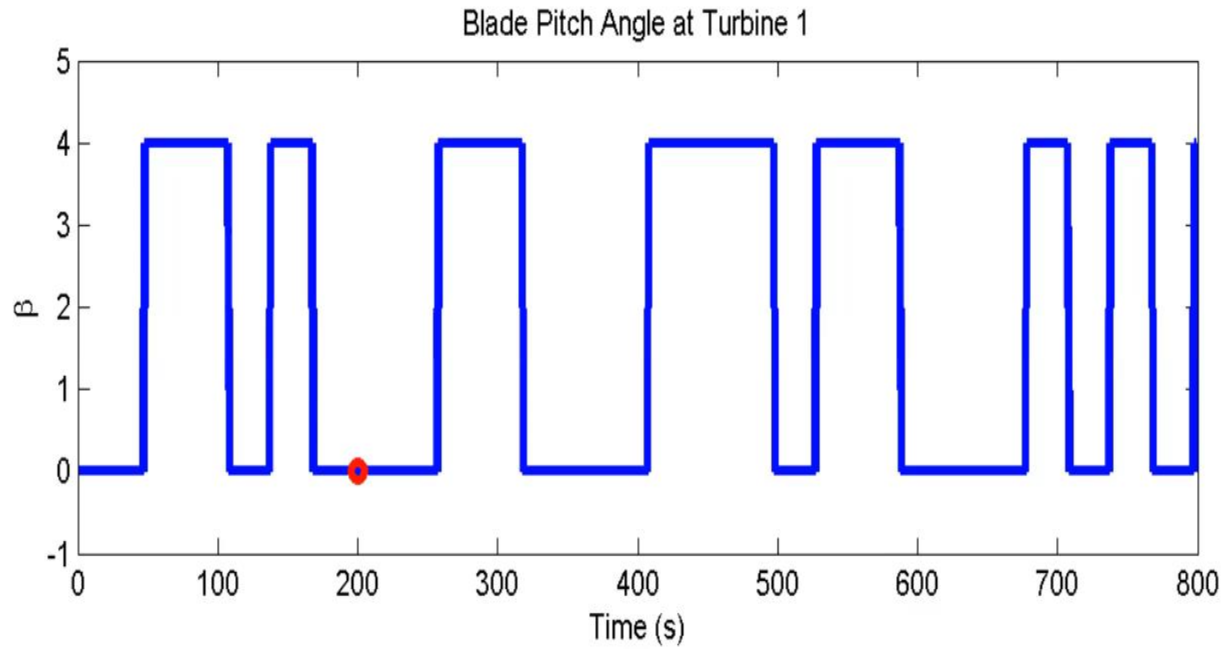




Blade pitch angle changes from 0^o to 4^o



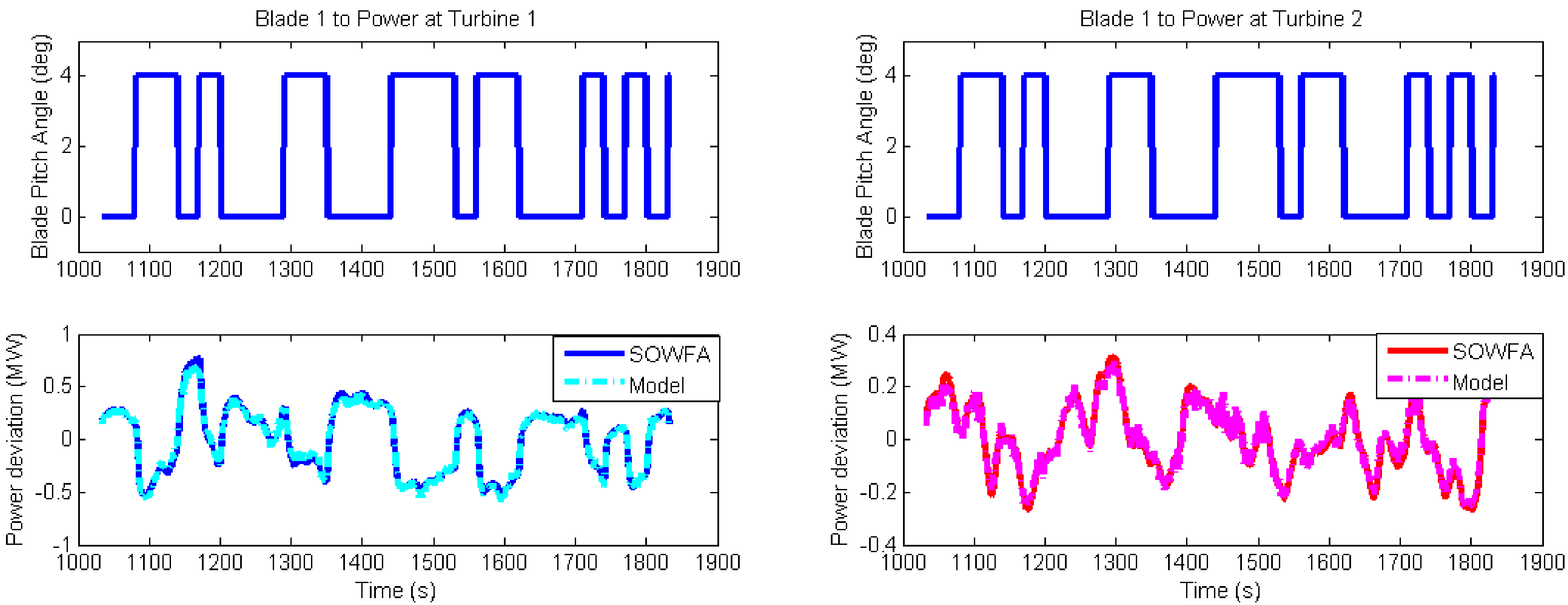
LES Results at Hub Height, Time 200
y/D
x/D



Blade Pitch Angle at Turbine 1
β
Time (s)

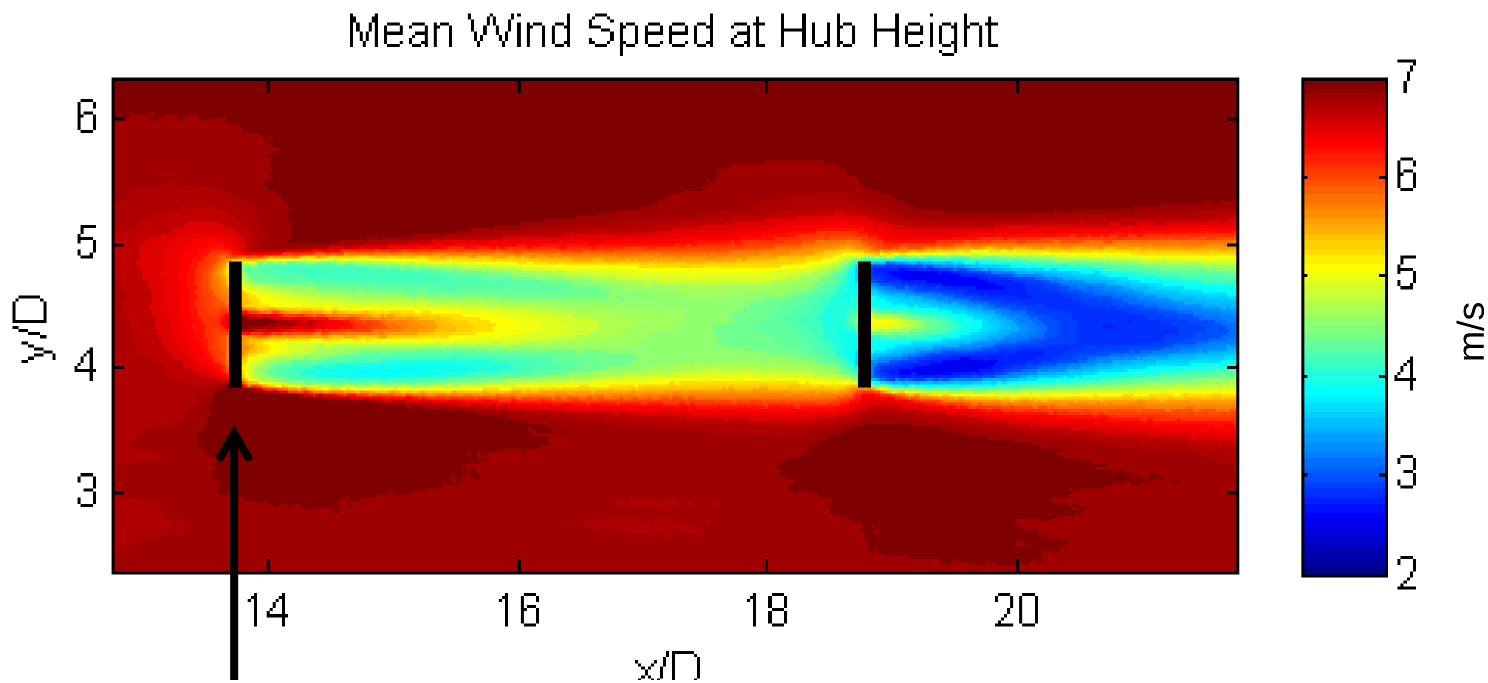
Reduced-order model

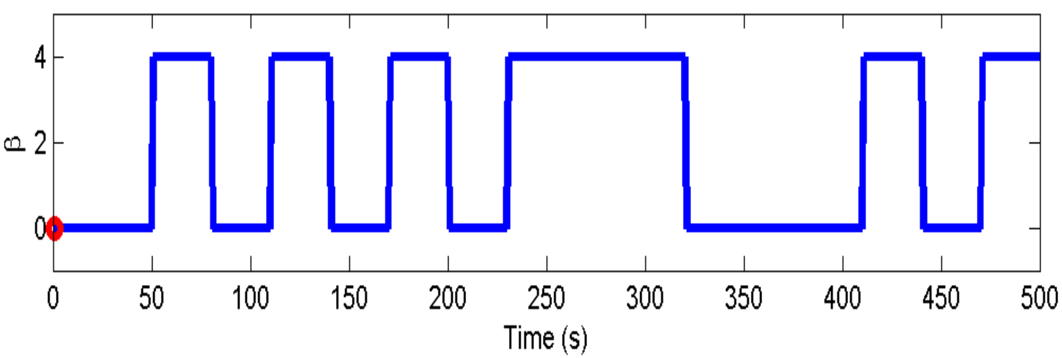
- Choose 20 modes to construct a reduced-order model
  - 3.6 million states projected onto 20 modes
  - Tall QR computations can be done on a laptop (hours)
  - Retain input-output behavior



Model applied to Validation Data

- Validation case – same setup with a different input

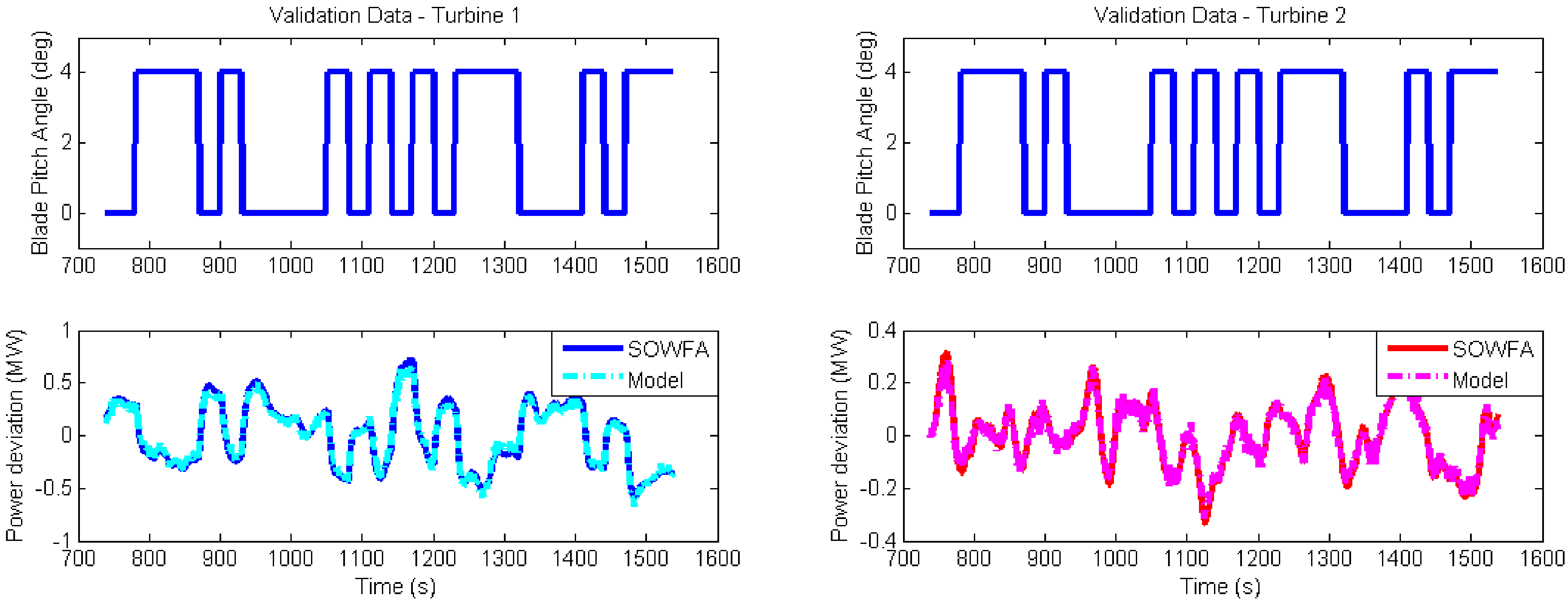


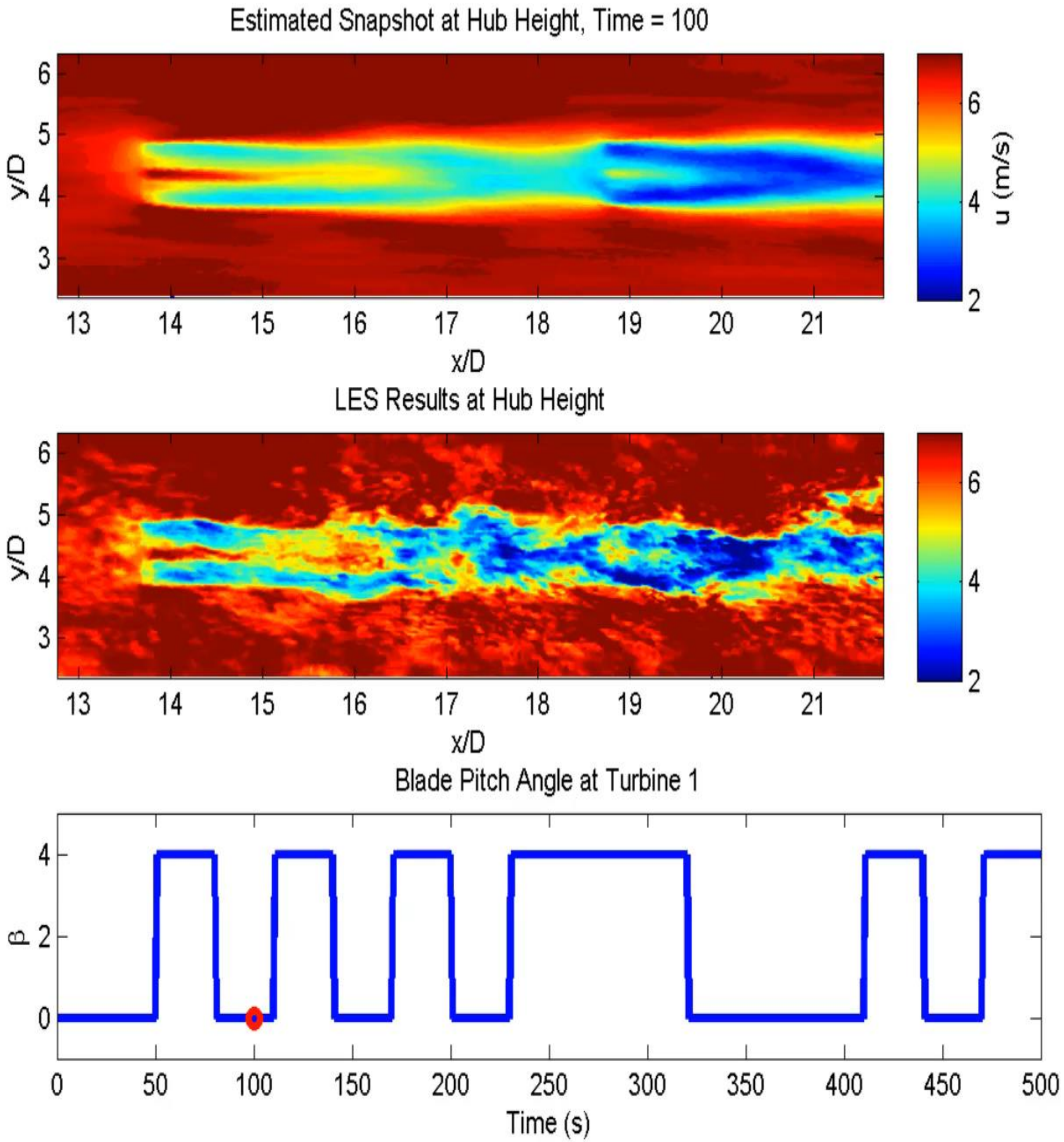


Blade pitch angle changes from 0^o to 4^o

Model applied to Validation Data

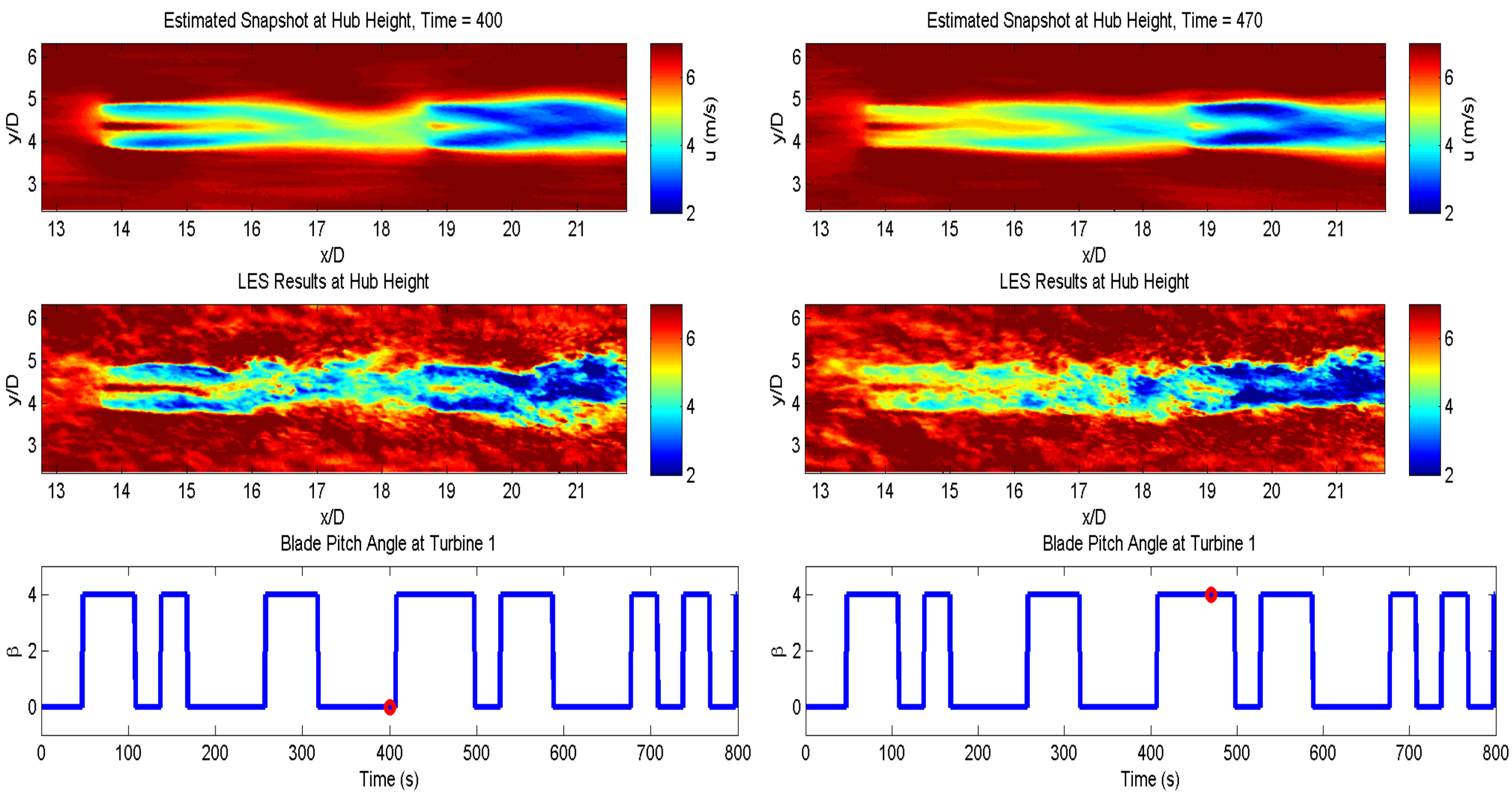
- Input-output behavior is retained on validation data





Estimated Snapshot at Hub Height, Time = 100
y/D
x/D
u (m/s)
LES Results at Hub Height
y/D
x/D
Blade Pitch Angle at Turbine 1
β
Time (s)

Compare Individual Snapshots



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- NASA
  - NRA NNX14AL36A: "Lightweight Adaptive Aeroelastic Wing for Enhanced Performance Across the Flight Envelope," Tech. Monitor: J. Bosworth.
  - NRA NNX12AM55A: "Analytical Validation Tools for Safety Critical Systems Under Loss-of-Control Conditions." Tech. Monitor: C. Belcastro.
  - SBIR contract #NNX12CA14C: "Adaptive Linear Parameter-Varying Control for Aeroservoelastic Suppression." Tech. Monitor. M. Brenner.
- Eolos Consortium and Saint Anthony Falls Laboratory
  - http://www.eolos.umn.edu/ & http://www.safl.umn.edu/

Conclusions

Main Contributions in LPV Theory:

- Robustness analysis tools
- Model reduction methods

Applications to:

- Flexible and unmanned aircraft
- Wind energy
- Hard disk drives

http://www.aem.umn.edu/~SeilerControl/