Figure 3: Surfaces separating at high speed, $\nu > \nu_c$, showing FECO fringes (top), schematic side-view (middle), and optical microscope view (bottom) of the surfaces. Homogeneous nucleation of a vapor cavity is shown in D (t = 10.01 sec), after which the cavity grows rapidly and then collapses (D to F). Note that in picture F (1 sec after inception) the cavity has still not totally disappeared (evaporated or collapsed).

16

9   What happens to the dissolved gas?

The cavitation of a liquid into a gas is a phase change. When the pressure is increased the vapor bubble collapses into a liquid. We may ask if a gas bubble (say air) which is not vapor can also condense into a liquid. The inward force at the boundary of a small spherical bubble due to surface tension is very strong, much too strong to be balanced by vapor pressure, and gas subjected to this pressure would quickly pass into solution in the liquid. Moreover, very small bubbles should collapse under the action of attractive Van der Waals forces even when surface tension vanishes.

A common postulate to explain cavitation is that pockets of gas and vapor are able to persist in equilibrium under normal conditions by being trapped in crevices in small hydrophobic solid particles such as dust particles which are thought to be usually present in liquids. Such impurities are sites for cavitation but not all or even most of the gas which disappears into a liquid can be trapped in these pockets. Moreover, the argument fails if there are no particles, or if the particles in suspension are mainly hydrophilic, with hydrophobic ones being removed by flotation.

The influence of nucleation sites, whatever they may be, can be greatly reduced by working with thoroughly cleaned and degassed liquids which are temporarily subjected to very high pressures. The cavitation thresholds in such prepared liquids are greatly increased because the gas there is thoroughly dissolved. Dissolved gas is thermodynamically part of the liquid and the maximum concentration of dissolved gas is determined by the temperature and pressure. It is important that dissolved gas at a certain temperature and pressure does not come out of, and is indistinguishable from, the liquid. I wonder what is meant by "dissolved gas": how is liquid with dissolved gas classified as a material or as a thermodynamic state? What are the intermolecular forces that hold the gas molecules in solution?

In dilute solutions, the gas concentration in the liquid neighboring a gas bubble is determined by the partial pressure of the gas according to Henry's law and mass will be transferred into or out of the bubble according to whether the concentration of dissolved gas is greater or less than the saturation concentration. This may explain why you can dissolve small bubbles into a degassed liquid at high pressure; the dissolved gas available is less than saturation. The transport of gas from gas bubbles is typically modeled as ordinary diffusion into a liquid, as in the case of miscible liquids (see Epstein & Plesset [1950], Birkhoff et al. [1958], Scriven [1959]).

We are interested here in exploring the utility of regarding the dissolution of gases into liquids as a phase change in which dissolved gas has condensed

into a liquid and the condensed liquid is *miscible*. At a minimum, I would like to know if a miscible liquid model can be used to model the truth, whatever that may be. In the cavitation literature one encounters phrases like "non-condensable dissolved gases" describing a condition of matter I do not understand.

Gases dissolved in liquids experience intermolecular forces between liquid and gas molecules. We could think of a gas molecule as a bridge between two liquid molecules. Of course, you can liquefy gases, but the thermodynamic conditions under which a substance is gas or liquid in isolation need not, indeed should not, apply when the substance is dissolved in another liquid. If our dissolved gas has condensed into a miscible liquid phase the solution properties, like the viscosity and density of the mixture, might resemble those of binary mixtures of miscible liquids. Water with dissolved air would have a different viscosity and density than pure water, but since the properties of condensed air are unknown, it is difficult to test our idea. The change in material properties due to mixing are difficult to detect when the solvent and solute have nearly the same properties.

To test the idea that dissolved gas can be modeled as condensed and miscible, we can use data on foamy oils, which are heavy oils containing dispersed gas bubbles. These oils give rise to obvious foaminess when the pressure is dropped rapidly at the well head. Foamy oil can be thought to be like a very viscous beer in which bubbles are released from gas in solution as the pressure is reduced. Foamy oils are good for our tests because the oil viscosity is so much larger than the viscosity of the dissolved gas.

Unlike miscible liquids, condensed gas cannot be mixed into the solvent in all proportions; there is a saturation concentration given as "solubility," a weight fraction, in the literature on foamy oil. Solubility is a thermodynamic quantity depending on temperature and pressure typically ranging from 0.1 to 10%.

Canadian researchers (see Peng et al. [1991] for a recent review) have done intensive studies of solubility of gases in bitumen from the Alberta tar sands. The solubility is the concentration of dissolved gas at saturation and it is a function of pressure and temperature. Viscosity and density of the saturated bitumen was measured for different gases. The results of these measurements are consistent with the idea that dissolved gases can be described as a condensed and miscible liquid.

Tumasyn et al. [1969] studied the effect of CO_2 on oil viscosity. Their experiments clearly showed that oil viscosity decreases with the amount of carbon dioxide dissolved. Furthermore, the drop in oil viscosity was higher for oils which has a higher viscosity when no CO_2 is present, as would be

true for miscible liquids.

Svrcek and Mehrotra [1982] studied gas solubility, viscosity and density for mixtures of Athabasca bitumens and dissolved CO_2, CH_4 (methane) and N_2 (Nitrogen) gases at saturation. They found that the viscosity and density of saturated oils depends on the gas, bitumen, temperature and pressure and is such that more gas is dissolved at higher pressures and lower temperatures. The viscosity of the solution drops precipitously with the amount of dissolved gas. The viscosity of bitumen without gas is more or less independent of pressure and drops more rapidly with temperature than does the saturated bitumen. The saturation concentration of CO_2, CH_4, and NO_2 in bitumen increases with pressure at each fixed temperature; the increase is linear for pressures below 5 MPa and is almost linear above. The solubility and viscosity are greater at lower temperatures; at room temperature the viscosity of dissolved CO_2 in bitumen drops nearly two orders of magnitude from the value of degassed bitumen, apparently because more condensed CO_2 is added as the pressure is increased. The solubility and reduction in viscosity is greatest for carbon dioxide and least for nitrogen.

The reduction of viscosity of bitumen with increasing concentrations of liquefied gas and the fact that different condensed gases lead to different reductions is consistent with the miscible liquid model. The parameters governing solubility are not so easily modeled.

The miscible liquid model may also be applied to density measurements; assuming that the solution is an ideal mixture the density of the solution ought to be equal to $f_g\phi + f_b(1 - \phi)$ where ρ_b and ρ_g are densities of the bitumen and the condensed gas and ϕ is the volume fraction of gas. The volume fraction of condensed gas can be obtained from the measured increase in the solution volume with concentration (the swelling factor in figure 5 of Peng et al. [1991]). The measurements of Svrcek & Mehrotra [1982] show that the density of carbon dioxide and nitrogen in bitumen is independent of pressure, hence concentration, whereas the methane in bitumen solution decreases linearly with pressure; that is, with concentration. The ideal mixture theory then implies that the density of carbon dioxide and nitrogen condensed in bitumen is nearly the same as the density of the bitumen, but the density of condensed methane is smaller.

The interfacial tension between partially miscible liquids is smaller than the tension before mixing and if the liquids are completely miscible the tension vanishes. Since gases are soluble in liquids, even in the dilute case to which Henry's law applies, a reduction in tension is expected and is observed. If the partial pressure of the gas is raised to a value in which all the gas is driven into solution, the tension between gas and liquid vanishes.

We may therefore expect the tension between gas and liquid to decrease as the gas pressure is increased, with an abrupt drop to zero as a bubble collapses to solution under the action of Van der Waals forces. A precise mathematical model for this reduction to zero tension has yet to be given.

An implication of this drop to zero surface tension is that the tension at cusp tip in a liquid-air cusping experiment of the type done by Joseph et al. [1991] would give us a true cusp and not to the molecularly small rounded tip required by finite tension according to the excellent analysis of Jeong and Moffat [1992]. The streaming of air bubbles from the cusp tip could then only start when the gas saturates in the neighborhood of the cusp tip.

Supercritical gases can dissolve into liquids, possibly without condensing. Supercritical gases exhibit liquid-like behaviors and are usually called supercritical fluids. A supercritical fluid is a highly compressed gas near or above its critical temperature and pressure points. At the critical temperature of a substance, the vapor and liquid phases have identical densities. A gas cannot be liquefied when it is above its critical temperature no matter how high the pressure. Above the critical temperature and pressure, the substance exists as a supercritical fluid. Critical temperatures are usually much higher than the substance's normal boiling or triple point (see White & Houk [1986]).

Carbon dioxide can exist as a liquid, solid, gas or supercritical fluid. The triple point pressure and temperature of carbon dioxide is 5.2 atm and -57°C; the critical pressure and temperature is 73 atm and 31°C. Carbon dioxide cannot be liquefied when the temperature is above 31°C.

Van Wassen et al. [1980] presented order of magnitude estimates for pure substances in the gaseous, supercritical fluid and liquid states:

Phase	Density g/cm^3	Diffusion cm^2/sec	Viscosity poise
Gas: 1 atm, 21C°	10^{-3}	10^{-1}	10^{-4}
Supercritical Fluid	0.3 - 0.8	$10^{-3} - 10^{-4}$	$10^{-4} - 10^{-3}$
Liquid	1	10^{-5}	10^{-2}

The properties of gases in solution are not known, but may be more like condensed liquid as is suggested by data on viscosity of saturated bitumen.

Supercritical fluids can be dissolved in liquids and used as solvent in extraction processes (Williams [1981]). Everyday experience of liquids provides the readily accepted proposition that liquid carbon dioxide could be used as a solvent. This would be understood in terms of the intermolecular

forces resulting from the close packing of solvent molecules about a solute molecule, and this could be related to the density of the liquid. It can therefore be intuitively accepted that supercritical carbon dioxide having a similar density could also act as a solvent.

Mackay and Paulaitis [1979] modeled the supercritical fluid as an expanded liquid instead of a highly compressed gas. Values of parameters of the mixtures they used were consistent with known results.

Studies of mixtures of gases in bitumen reveal that the distinction between supercritical fluids and condensed gases in solution cannot be supported. In the experiments of Svrcek & Mehrotra [1982] the mixture of carbon dioxide and bitumen which were created at high pressure and low temperatures of 23 and 24°C were well within the region in which carbon dioxide is liquid. No difference of kind can be observed in the mixtures of CO_2 and bitumen in the regions of high pressure and low temperature in which CO_2 itself is liquid or supercritical. In fact, the reduction in viscosity, apparent diffusivity etc. are greater at the high pressure and low temperatures for which the CO_2 alone is liquid. There is no indication in the data of a difference in the condition of the dissolved gas; there is nothing to suggest that the dissolved gas is not condensed.

10   The effect of polymeric additive on cavitation

Unfortunately, most of the studies of cavitation in non-Newtonian fluids are restricted to the same extremely dilute solutions of polymer in water which reduce drag. Reviews of these studies have been given by Arndt, et al. [1976], Hoyt & Taylor [1981], Arndt [1981] and Trevena [1987]; they show that the polymer reduces the cavitation index (suppresses cavitation) in some flows, whereas in others the results are ambiguous and even depend on the material of construction of the flow obstacle used to create low pressure. The suppression of cavitation with polymers might be expected to coincide with a reduction in the breaking strength of the solution; this does not appear to be the case. Sedgewick and Trevena [1978] studied the effects of polyacrilamide additives on the breaking tension of water using a static Berthelot tube method and a dynamic bullet piston method. In the static case, the presence of the polymer additive did not noticeably change the breaking tension, but in the dynamic tests the breaking tension was *reduced*; the greater the concentration of polymer the greater was the reduction. To understand these contradictory results it is necessary to better understand how polymer additives modify the state of stress in a flowing fluid.

Mathematical studies of the effects of polymer additives on cavitation have not been published. There are two difficulties: the choice of the constitutive equation and the calculation of the pressure; the "pressure" in an Oldroyd B fluid and in a second order fluid are not in the same relation to the mean normal stress. The composition of the stress into a pressure and a part related a-priori to deformation is not unique, but the state of stress a point given by different constitutive equations applied to the same problem can be compared.

Ellis and Ting [1970] constructed an argument based on the second order fluid noting that this fluid "... appears capable of explaining the observed cavitation pressure due to flow." Their idea is that cavitation suppression is due to an overpressure in the region of the obstacle where the pressure would ordinarily be low enough to permit cavitation in the case of water. Further, they attribute the difference to the presence of a correction term in the Bernoulli equation for a second order fluid.

Lumley agreed with the arguments and conclusions of Ellis and Ting but he objected to using a second order model "... since it includes only in a rudimentary way the effect of molecular extension..." which Lumley argued was responsible for drag reduction. He gets a similar result using a centerline Bernoulli equation for an Oldroyd B model.

Here, I am going to construct a mathematical analysis based mainly on the second order theory in which the idea of Ellis and Ting is put to test; we find there is a basis for the overpressure they mention, at the boundary of a rigid solid, and there is a Bernoulli equation in potential flow with an extra term. However, the extra term contributes to the promotion rather than to the suppression of cavitation. It is certain that the second order fluid is far from an ideal model for the fast flows usually studied in experiments on cavitation. On the other hand, the second order fluid is the only model of a viscoelastic fluid, other than the linear viscoelastic model, which admits a pressure function in potential flow (Joseph & Liao [1996]). Moreover, the second order fluid admits a pressure function of an entirely different origin, for perturbations of Stokes flow, which allows one to draw conclusions following from the no-slip or incomplete slip condition of real obstacles on which cavitation bubbles appear that cannot be studied in potential flows.

11 Potential flow cavitation of viscoelastic fluids

The constitutive equation for a second order fluid is usually given as

$$\mathbf{T} = -p\mathbf{1} + \eta\mathbf{A} + \alpha_1\mathbf{B} + \alpha_2\mathbf{A}^2, \tag{38}$$

$$\mathbf{B} = \frac{\partial \mathbf{A}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{A} + \mathbf{A}\mathbf{L} + \mathbf{L}^T\mathbf{A},$$

$$\mathbf{L} = \nabla\mathbf{u}\ \left(L_{ij} = \frac{\partial u_i}{\partial x_j}\right),$$

$$\mathbf{A} = \mathbf{L} + \mathbf{L}^T,$$

$$\alpha_1 = -\Psi_1/2,\ \alpha_2 = \Psi_1 + \Psi_2 \tag{39}$$

where $\Psi_1 > 0$ and $\Psi_2 < 0$ are the coefficients of the first and second normal stress differences. (A typical value for estimates is $\Psi_2 = -\Psi_1/10$)

The "pressure" p in (38) is not the mean normal stress; it is better for cavitation studies to write (38) as

$$\mathbf{T} = -\tilde{p}\mathbf{1} + \mathbf{S} \tag{40}$$

where, since $\mathrm{Tr}\mathbf{A} = \mathrm{div}\ \mathbf{u} = 0$ and $\mathrm{Tr}\mathbf{B} = \mathrm{Tr}\mathbf{A}^2$,

$$\tilde{p} = -\frac{1}{3}\,\mathrm{Tr}\mathbf{T} = p - \frac{1}{3}(\alpha_1 + \alpha_2)\,\mathrm{Tr}\mathbf{A}^2 \tag{41}$$

and

$$\mathbf{S} = \alpha_1\mathbf{B} + \alpha_2\mathbf{A}^2 - \frac{1}{3}(\alpha_1 + \alpha_2)\,\mathrm{Tr}\mathbf{A}^2, \mathrm{Tr}\mathbf{S} = 0 \tag{42}$$

The reader has already been alerted several times to the fact that the use of p in cavitation studies has no rational foundation.

Joseph [1992] has shown that equations of motion for a second order fluid admit all potential flow solutions; that is, velocity fields of the form

$$\mathbf{u} = \nabla\phi,\ \nabla^2\phi = 0 \tag{43}$$

give rise to solutions of

$$\rho\frac{d\mathbf{u}}{dt} = \mathrm{div}\mathbf{T},\ \mathrm{div}\mathbf{u} = 0 \tag{44}$$

where $\mathbf{T}$ is given by (38) and p has a Bernoulli equation with an extra term:

$$p + \rho\frac{|\mathbf{u}|^2}{2} - \frac{\hat{\beta}}{4}\mathrm{Tr}\mathbf{A}^2 + \rho\frac{\partial\phi}{\partial t} = 0 \tag{45}$$

23

The extra term

$$-\frac{\hat{\beta}}{4}\text{Trace }\mathbf{A}^2 = \hat{\beta}\nabla\mathbf{u}:\nabla\mathbf{u} < 0 \tag{46}$$

is positive whenever the climbing constant $\hat{\beta} = 3\alpha_1 + 2\alpha_2$ is positive and it acts in opposition to inertia $\rho|\mathbf{u}|^2/2$. Using the estimate $\Psi_2 = -\Psi_1/10$ we get

$$\hat{\beta} = 2\Psi_2 + \Psi_1/2 = \frac{3}{10}\Psi_1 > 0.$$

Since nearly all polymeric solutions climb rotating rods we may assume that $\hat{\beta} > 0$. Looking at p in (45) we might conclude that the compression due to the extra term in the Bernoulli equation is increased, suppressing cavitation as was supposed by Ellis and Ting [1970]. However, the mean normal stress $\tilde{p}$ is another (more relevant) pressure given by

$$\tilde{p} + \rho\frac{|\mathbf{u}|^2}{2} - \frac{1}{3}(5\alpha_1 + 2\alpha_2)\nabla\mathbf{u}:\nabla\mathbf{u} + \rho\frac{\partial\varphi}{\partial t} = 0 \tag{47}$$

gives rise to a different result. Since

$$5\alpha_1 + 2\alpha_2 = 2\Psi_2 - \Psi_1/2 < 0 \tag{48}$$

the extra term in the mean normal stress promotes rather than suppresses cavitation.

I have argued that cavitation at any point in a flowing liquid depends on the state of stress and breaking stress there. For potential flow

$$\mathbf{B} = (\frac{\partial}{\partial t} + \mathbf{u}\cdot\nabla)\mathbf{A} + \mathbf{A}^2 \tag{49}$$

and the deviatoric part $\mathbf{S}$ of the stress $\mathbf{T}$ is given by

$$\mathbf{S} = (\eta + \alpha_1\frac{d}{dt})\mathbf{A} + (\alpha_1 + \alpha_2)[\mathbf{A}^2 - \frac{1}{3}\mathbf{1}\text{Tr }\mathbf{A}^2] \tag{50}$$

and the mean normal stress $-\tilde{p}$ is given by (47) as

$$-\tilde{p} = +\rho\frac{\partial\varphi}{\partial t} + \rho\frac{|\mathbf{u}|^2}{2} - \frac{1}{12}(2\Psi_2 - \Psi_1/2)\text{Tr }\mathbf{A}^2 \tag{51}$$

since $-2\Psi_2 + \Psi_1/2 > 0$ the term with $\text{Tr}\mathbf{A}^2$ is positive, tensile promoting cavitation. Therefore the suppression of cavitation must arise from deviatoric part of the stress. However, the deviator has both positive and negative

eigenvalues (see Joseph [1992]) with the positive ones corresponding to tension, promoting cavitation.

The progressive reduction of the breaking strength with increasing concentration which was observed by Sedgewick and Trevena [1978] is consistent with the analysis just given, but the use of a second order fluid model to study cavitation in potential flows is not justified, too many effects are left out of the model.

12 Flow cavitation on solids

We can study flow cavitation for nearly steady flow of a viscoelastic fluid over bodies on which the no-slip condition applies as a viscoelastic perturbation of Stokes flow. The second order fluid arises universally as the asymptotic form taken by all the special models in common use. Though the motions to which such an analysis can rigorously apply are restricted, the rheology for those motions is completely general; every special model has η, α_1, α_2 as limiting values of model parameters in nearly steady slow motion.

The perturbation of Stokes flow with a second order fluid gives rise to a pressure function for motions in two-dimensions (Tanner [1966]) and for motions in three dimensions when $\alpha_1 = -\alpha_2$ (Giesekus [1963]). The implications for forces on solid particles of the two-dimensional reduction were studied by Joseph [1996] and Joseph & Feng [1966] who showed that the normal stress on each and every point on the boundary of a rigid solid is given by

$$T_{nn} = -p_s + \frac{\Psi_1}{2\eta}\left(\frac{\partial p_s}{\partial t} + \mathbf{u} \cdot \nabla p_s\right) - \frac{\Psi_1}{4}\overset{\circ}{\gamma}^2 \tag{52}$$

where n is a coordinate along the outward $\mathbf{n}$ on the boundary $\Psi_1 = -2\alpha_1, \overset{\circ}{\gamma}$ is the shear rate of the Stokes flow at that point and p_s is the Stokes flow pressure obtained from solving $\nabla p_s = \eta\nabla^2\mathbf{u}$ for no-slip boundary condition. For steady flows over stationary bodies like those used in cavitation studies we get

$$T_{nn} = -p_s - \frac{\Psi_1}{4}\overset{\circ}{\gamma}^2 \tag{53}$$

which is positive in tension; the viscoelastic contribution $-\Psi_1\overset{\circ}{\gamma}^2/4$ is a compression. In the three dimensional case with $\alpha_1 + \alpha_2 = 0$, the Stokes flow

problem can be solved and $p_s = p_\infty - \frac{3}{2}\eta\frac{U}{a}\cos\theta$ and $\overset{\circ}{\gamma} = \frac{5}{2}\frac{U}{a}\sin\theta$ where θ is the polar angle.

The shear rate $\overset{\circ}{\gamma}$ is greatest where the streamlines of the Stokes flow are most crowded; these are the "high" velocity points on the body. Since compressive stresses are generated at "high" velocity points, we get additional "pressure" due to viscoelasticity precisely at the "cavitating" points where the pressure is lowest in potential flow. The compressive contribution $-\Psi_1\overset{\circ}{\gamma}^2/4$ is largest, for example, near the equator of a sphere where cavitation bubbles would appear if no polymers were present.

Van der Meulen [1973] has shown that cavitation inception on a hemispherical-nosed stainless steel body in a water tunnel is greatly reduced by the presence of polyethylene oxide, while a teflon coating showed a much smaller effect. If gas is nucleated at the surface of the hydrophilic stainless steel body it will be replaced by water. The additional pressure due to (52) is effective in suppressing this nucleation of gas. The teflon coated sphere is hydrophobic. Since it likes gas, the liberation of cavitation bubbles is not so evident and not readily replaced by water. The addditional pressure should also work here, but its overall effect could be less evident due to the adhesion of nucleated gas on the teflon.

The formula (53) should not be expected to apply strictly to the high speed flows in which cavitation is suppressed in experiments. Moreover, Ψ_1 is not known in the dilute solutions used in these experiments. It is nevertheless of interest to use (53) for an estimate, to see if it gives rise to a pressure level large enough to effect cavitation.

To construct such an estimate we first use data from figure 3 of Arndt et al. [1976] for flow at $U = 18.3$m/s of a 20ppm solution in water of Polyox WSR-301 over a 2R=7.62cm diameter hemispherical nosed body. Estimating $\overset{\circ}{\gamma} = O(U/R)$ we get $\overset{\circ}{\gamma} \approx 500\text{sec}^{-1}$, $\overset{\circ}{\gamma}^2 \approx 2.5 \times 10^5\text{sec}^{-2}$. To get pressure increases of the order of the vapor pressure; i.e., one hundredth of an atmosphere we would need to satisfy

$$\frac{\Psi_1}{4}(2.5 \times 10^5\text{sec}^{-2}) = 10^4\frac{\text{dynes}}{\text{cm}^2} \tag{54}$$

or

$$10\Psi_1 = O(1)\frac{\text{gm}}{\text{cm}} \tag{55}$$

This is a value of Ψ_1 smaller than those which can be measured on rheometers but it is possibly larger than the value of Ψ_1 which could be expected from solutions so dilute as 20ppm.

26

A second estimate taken from data in Table 2 in the paper by Ellis, Waugh and Ting [1970] for flow of different fluids over a 1/4 in radius hemispherical nose body; typical shear rates are of the order of

$$\frac{U}{r} \approx \frac{500}{1/4} = 2000\text{sec}^{-1}$$

For fluids, like aqueous 50 ppm polyox, for which cavitation suppression was observed we need Ψ_1 larger than

$$100\Psi_1 = O(1)\frac{\text{gm}}{\text{cm}}. \tag{56}$$

Ting [1978] studied cavitation on flat top posts protruding from a disk rotating in aqueous polyox FRA solutions with concentrations of 100, 250 and 500 ppm. The 0.29cm diameter posts were 0.29cm high and cavitation suppression was observed for values of $Re = \rho U d/\eta > 10^5$ where, for water $\rho/\eta = 100^{-1}$ and $d = 0.29$. Hence

$$\overset{\circ}{\gamma} \cong \frac{U}{d/2} \cong 5000\text{sec}^{-1}$$

corresponding to estimates like (54) giving rise to

$$\Psi_1 \approx 1.6 \times 10^{-3}\frac{\text{gm}}{\text{cm}}. \tag{57}$$

Dilute polymer solutions might have normal stress coefficients larger than (57) but nothing is known. In section 12 we will argue that these viscoelastic effects, with very small $\Psi_1 = 2\eta\lambda_1$ where λ_1 is the relaxation time of the fluid, may be seen on small but not large bodies.

The estimates following from (53) are perhaps not so outrageous as to eliminate consideration of the mechanism of inhibition of cavitation inception by shear-induced normal stress. This mechanism has not much in common with mechanisms involving extensional effects due to the irrotational stretching of polymer coils which are popular in theory of drag reduction. Some of the problems encountered in estimating the viscoelastic effects in an irrotational strain field were discussed by Arndt et al. [1976] and they appear to support an alternate explanation presented first by Arakeri and Acosta [1973] which is discussed next.

Arakeri and Acosta [1973] have shown that polymer additives influence the point of laminar separation and inhibit cavitation inception. Van der Meulen [1976] did holographic studies in which he shows that when polymers are added, separation of the boundary layer does not occur and the

27

boundary layer exhibits a turbulent character. This may be compatible with the inhibition of cavitation inception by shear induced normal stresses. In a Newtonian fluid, the position of separation is controlled by the pressure variation on the solid surface. In a viscoelastic fluid the normal stress T_{nn} is not given by pressure alone and terms contributed by normal stresses due to shearing do not vanish; obviously the fluid at the boundary of a solid can sense T_{nn} only and various decompositions of the stress giving rise to different expressions for the pressure are not relevant. To their credit, Ellis et al. [1970] recognize this when they say (p. 463) that "... cavitation bubble inception is inhibited by an actual change of principal stress (or less precisely, pressure)..." The effects of viscoelastic contributions to the variation of T_{nn} on the boundary of a solid which determines the position of points of separation are not clear. Arndt [1981] notes that "... All that can be said at the moment is that available experimental and theoretical information do not provide any clear cut criteria for the determination of early transition in the flow of polymer solutions."

13   Size effects and the elasticity number

A very interesting and robust effect in cavitation inception inhibition is that the inhibition is large for small bodies and is negligible for large bodies. Arndt et al. [1976] studied cavitation in two recirculating water tunnels using four hemispherical nosed bodies ranging in size from 6.3mm to 203mm. They found that the desinent value of the cavitation index was reduced as much as 50% on the smaller bodies but the effect on the 203mm was negligible. The effect persisted even in well degraded polymer solution. Huang [1971] noted that the cavitation inception reduction was much smaller when a larger model was used in a water tunnel. A small propellor was observed by White [1971] to exhibit greatly reduced cavitation, in agreement with the diameter effect hypothesis.

A size effect scaling with the square of the radius is consistent with the equations governing the motion of viscoelastic fluids over bodies. For example, in the flow of a second order fluid the effects of inertia proportional to ρu^2 and viscoelastic normal stresses proportional to $\Psi \overset{\circ}{\gamma}^2$ are in opposition and estimating $\overset{\circ}{\gamma} \approx u/a$ where a is the particle size, we find that viscoelasticity dominates near the body when $\Psi_1/\rho a^2$ is large.

The same scaling follows from analysis of Oldroyd B fluids made dimensionless by $[a,\ U,\ a/U,\ \eta U/a]=$ [length, velocity, time, stress]. The

28

dimensionless equations for momentum and stress then take the form

$$Re\left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u}\right) = -\nabla p + \nabla \cdot \mathbf{T},$$
$$\mathbf{T} + De\overset{\triangledown}{\mathbf{T}} = \mathbf{A} + \frac{\lambda_2}{\lambda_1} De\overset{\triangledown}{\mathbf{A}}$$

where $\nabla \cdot \mathbf{u} = 0$ and

$$\overset{\triangledown}{\mathbf{T}} = \frac{\partial \mathbf{T}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{T} - \nabla \mathbf{u}, \mathbf{T} - \mathbf{T} \cdot \nabla \mathbf{u}^T.$$

The parameters are:

$$Re = U a \rho / \eta \quad \text{(Reynolds number)},$$
$$De = U \lambda_1 / a \quad \text{(Deborah number)},$$
$$\lambda_2 / \lambda_1 \quad \text{(retardation / relaxation time ratio)}.$$

The maximum elastic effects are for $\lambda_2 = 0$ (Maxwell model). The fluid is Newtonian when $\lambda_2 = \lambda_1$ and $0 \le \lambda_2/\lambda \le 1$.

Huang, Hu and Joseph [1997] used these equations to study the sedimentation of an ellipse with semi-major axis a in an Oldroyd B fluid. They found that the results correlated better with

$$E = De/Re = \lambda_1 \eta / \rho a^2 \quad \text{(elasticity number)}$$

and

$$M = \sqrt{ReDe} = U/c \quad \text{(viscoelastic Mach number)}$$

where $c = \sqrt{\eta/\lambda_1 \rho}$ is the speed of shear waves. They found that when the elasticity number is smaller than critical the fluid is essentially Newtonian with broadside-on falling at the centerline of the channel. For larger elasticity numbers the settling turns the long side of the particle along the stream in the channel center for all velocities below a critical one; identified with a critical Mach number of order one.

The elasticity number depends on the fluid and the particle size and not on the velocity. It says that small particles can experience elastic response in circumstances in which larger particles have essentially Newtonian response, as in experiments on cavitating flows over bodies. On the other hand, the Mach number does not depend on the particle size. The elastic response of small particles is thus rather firmly implied by direct and elementary study of the equations governing the motion of Oldroyd B fluids.

14   Discussion

It is perhaps useful to think of cavitation as a topic in a more general theory of breaking and mixing of incompressible materials in which cavitation and fracture of amorphous solids are different, but perhaps related phenomena. Such a theory might lead to better understandings of the dynamics of mixing and mixers which is an important but as yet undeveloped subject.

The breaking of an incompressible continuum is obviously controlled by breaking stresses and cavitation is not different. The fluid knows the state of stress at a point but it cannot decompose this stress into a pressure determined by the flow dynamics and a constitutively determined extra stress. It is however useful, and is always possible, to define the pressure as the mean normal stress, even when, as in many viscoelastic fluids, it is not originally so defined; in this case the extra stress is deviatoric, with plus and minus entries on its leading diagonal.

It may be assumed that in each liquid or solid there is a field of breaking strengths defined at each and every point of the continuum; this strength field could be discontinuous with singular values at nucleation sites. The breaking of a liquid requires that we compare some function of the principal stresses with the strength criterion; for instance it would be consistent with the literature on cavitation to look for the tensile strength of liquids. In this case we look at the magnitude of the maximum value in the diagonalized extra stress, which must be positive and compare it to the breaking strength (cavitation inception) of the liquid. Our comparison requires that we distinguish also the direction of the maximum tension in the moving continuum, which is determined by diagonalization of the stress tensor; we could look at equi-tension lines and seek the locus of maximum values. It is not completely clear that the maximum tension is the right criterion, though it is in some sense conventional and does appear to agree with experiments on the tensile strength of liquids. One interesting consequence is that we could test the criterion and the directionality of this criterion by setting the pressure level slightly above the breaking strength in a pure shear flow between parallel plates in which the maximum tension is 45° from the flow direction.

The state of stress in a flowing polymeric liquid could in principle have strong effects on cavitation. The evidence from experiments with dilute solutions used for drag reduction weighs strongly for cavitation inhibition, though neutral inhibition results for cavitation on large bodies and a reduction in the breaking strength of liquids with polymer concentration have been reported. It is argued that potential flow approximations are not appropriate for cavitation studies except possibly in the case of acoustic cavitation.

30

An analysis of cavitation inhibition of polymeric liquid based on the second order fluid model, in which the original pressure in the model is embedded in a more appropriate decomposition based on the mean normal stress, shows that inhibition will not arise in potential flows, increased cavitation is predicted. It was stressed that the definition of pressure in non-Newtonian fluids is not unique and differs from one constitutive equation to another, but the mean normal stress is uniquely defined and sets an appropriate level against which one can measure the differences between principal compressive and tensile stresses.

Another positive feature of the second order fluid model for studies of cavitation is that a pressure function can be derived a-priori for flows which perturb Stokes flow as well as for potential flows. In the Stokes flow perturbation the no-slip condition is applied so that the analysis is appropriate for discussions of cavitation which originate from nucleation on solid boundaries. An additional normal stress T_{nn} equal to $-\Psi_1 \overset{\circ}{\gamma}{}^2/4$ which is always compressive and proportional to the square of the shear rate at each point on the boundary of a solid body is equivalent to a viscoelastic pressure which is large at places where the flow is fast, the opposite of inertia. Though the speeds of cavitating flows are larger than those for which analysis based on second order models is valid, estimates of the size effects do give rise to additional pressures which could suppress nucleation at solids. An important additional effect is that the variation of "backpressure" which controls the position of points of separation would be changed by the presence and variation of this additional viscoelastic pressure.

All evidence from experiments indicates that cavitaton inhibition is much more pronounced on small than on large bodies. This observation is consistent with the analysis of response Oldroyd B fluids. The controling parameters of these fluids are the Reynolds number Ua/ν and the Deborah number $U\lambda/a$. The Deborah number shows a size dependence of the required kind, but the number depends on U which also depends on a. The elasticity number E which is the ratio $De/Re = \lambda\nu/a^2$ is independent of U and it indicates enhanced elasticity for small bodies in a given fluid and the size effect is proportional to a^2 rather than a. The predictions of viscoelastic response based on the elasticity number should be robust since it arises from direct analysis of dimensions but it has not yet been tested in cavitation studies.

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