

apparatus. The angle which diagonalizes  $\mathbf{T}$  is given by (18) as  $S'_{12} = 0$  or

$$\cos 2\theta = 0, \quad \theta = 45^\circ$$

(In the break-up of viscous drop experiments in plane shear flow done by G.T. Taylor [1934], the drops first extend at  $45^\circ$  from the direction of shearing.)

Then, using (19), in principal coordinates, we have

$$\begin{bmatrix} T_{11} + \pi & 0 & 0 \\ 0 & T_{22} + \pi & 0 \\ 0 & 0 & T_{33} + \pi \end{bmatrix} = \eta \frac{U}{L} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (30)$$

and

$$B_{11} = -p_c + \pi - \eta \frac{U}{L}, \quad (31)$$

$$B_{22} = -p_c + \pi + \eta \frac{U}{L}, \quad (32)$$

We may distinguish between the cavitation criteria (24) and (25) if

$$B_{22} - B_{11} = 2\eta \frac{U}{L} \quad (33)$$

is large. This difference is of the order of one atmosphere of pressure if

$$2\eta \frac{U}{L} = 10^6 \frac{\text{dynes}}{\text{cm}^2} \quad (34)$$

If  $\eta = 1000$  poise,  $U = 10$  cm/sec and  $L = 10^{-1}$  cm, we may achieve such a stress. It is possible to imagine such a shearing motion between concentric rotating cylinders filled with silicon oil, though the conditions are severe. If we could depressurize the system so that a threshold of pressure less than one atmosphere were required, we might see cavities appear in shear flow when  $B_{11} < 0$  and  $B_{22} > 0$ . I am not aware of reports of cavities forming in shear flows, but the conditions required are at the border of realistic experiments and may have escaped detection. Experiments of this kind ought to be tried.

## 6 Cavitation in extension

We have argued that cavities always appear in the extensional flows defined in principal axes coordinates even when the flow is pure shear. However,

the direct creation of a pulling flow without rotation (vorticity) may lead to a higher level of dynamic stresses than could be otherwise achieved. Let us suppose that a small diameter thread open to the atmosphere is anchored at a solid wall at  $x = 0$  and is being pulled out at a constant rapid rate  $\overset{\circ}{S}$  in the direction  $x$ .

$$u = \overset{\circ}{S}x, \quad \nu = -\frac{1}{2}\overset{\circ}{S}y, \quad \omega = -\frac{1}{2}\overset{\circ}{S}z. \quad (35)$$

The thread is in tension when  $\overset{\circ}{S}$  is large enough

$$T_{11} = -\pi + 2\eta\frac{\partial u}{\partial x} \approx -p_a + 2\eta\overset{\circ}{S} \quad (36)$$

where, for very thin threads  $\pi \approx -p_a$  where  $p_a$  is atmospheric pressure. According to the maximum tension criterion (23) cavities will form in the thread, and the thread may actually break, when

$$B_{11} \approx p_a - p_c - 2\eta\overset{\circ}{S} < 0 \quad (37)$$

The stretch rate  $\overset{\circ}{S}$  for breaking can be estimated assuming that the thread cannot sustain a tension, by  $p_c = 0$ ; then

$$\overset{\circ}{S} > 10^6/2\eta(\text{sec}^{-1})$$

For very viscous threads, say  $\eta = 500$  poise, the stretch rate for breaking

$$\overset{\circ}{S} > 10^5(\text{sec}^{-1})$$

is rather large.

An extensional flow something like (33) is generated by stripping of a drop suddenly exposed to a high speed airstream behind a shock which was considered by Joseph, Huang and Candler [1996]. First, the drop will flatten due to high pressures of potential flow at the front and back of the drop. This occurs in the first  $20\mu\text{sec}$ ; then threads of liquid are stripped from the outer edge of the flattened drop by wind shear. At these early times, say  $50\mu\text{sec}$ , the drop is accelerating, but its velocity is nearly zero and the threads stripped from the drop are extending very rapidly (see [http://www.aem.umn.edu/Aerodynamic\\_Breakup](http://www.aem.umn.edu/Aerodynamic_Breakup) to see a video animation of breakup on a high speed camera). These threads fragment into mist and possibly vapor but it remains to establish whether or not the fragmentation is related to cavitation.

## 7 Breaking tension of polymer strands

A less speculative example of breaking of viscous threads in tension has been documented in experiments by Wagner, Schulze, and Göttfert [1996] on the drawability of polymer melts.

In these experiments the tensile force needed to elongate an extruded polymer melt is measured as a function of the draw ratio  $V = \nu/\nu_0$  where  $\nu_0$  is the velocity of the spinline at the die and  $\nu$  is the velocity of the spinline at the takeup wheels. The tensile force  $F$  is measured at the wheel and the stress in the strand at the wheel is said to be given by

$$\sigma = FV/A_0$$

where  $A_0$  is the area of the crosssection of the die hole.  $V$  and  $F$  increase together and at a certain critical  $F_B$  (and  $V_B$ ) the strand breaks. The remarkable feature of this breaking is that the breaking stress  $\sigma_B$  is independent of the extrusion pressure (the wall shear stress) and temperature. Wagner et al. [1996] conclude that the breaking stress  $\sigma_B$  is a “pure material constant”.

The breaking stress in their LDPE sample A18 ( $\eta_0 = 10^4$  PaS) is

$$\sigma_B \approx 10^6 Pa.$$

The breaking stress in the HDPE sample H50 ( $\eta_0 > 4.8PaS$ ) is

$$\sigma_B \approx 1.1 \cdot 10^6 Pa.$$

Atmospheric pressure is roughly

$$Pa \cong 1.1 \cdot 10^5 Pa;$$

the pressure in the thread is somewhat larger than this because of surface tension. The radius of the die is 1mm; if the thread thins by 10 or more the surface tension addition to pressure in the thread will be sensible. It is nevertheless certain that the strand is in tension when it breaks.

## 8 Cavitation experiments at the nanoscopic level

Chen and Israelachvili [1991] and Kuhl, Ruths, Chen and Israelachvili [1994] have done important direct visualization studies of cavitation of ultrathin nanometer liquid films using the surface forces apparatus technique. They are able to visualize cavitation between mica surfaces in approach-separation

and shearing motions. They noticed that vapor cavities developed when two curved surfaces are moved away from each other faster than some critical velocity  $\nu_c$ . In the experiments described by Kuhl et al. [1994], the liquid between 1cm radius hemispheres of mica was a low molecular weight, Newtonian, 180 poise polybutadiene and the separating motions can be thought to give rise to extensional motions like those described in (33).

Chen & Israelachvili say that

We have found that cavitation bubbles can occur either totally within the liquid, that is, away from the surfaces, or at the solid-liquid interfaces. The adhesion of untreated (polar) mica surfaces to the PBD liquid is stronger than the cohesion between the liquid molecules themselves (“wetting” conditions); hence, the cavities form totally within the liquid. In contrast, for surfaces coated with a surfactant monolayer, the nonpolar solid-liquid adhesion is weaker ... and the cavities form at the interfaces.

A qualitative description of their observation for the case of strong adhesion is described in the caption for the cartoon in figure 2.

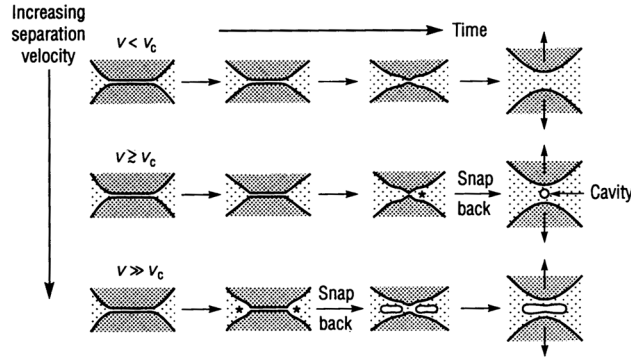


Figure 2: Schematic illustration of the separation of two observed mica surfaces at progressively increasing separation velocities as ascertained from the FEKO fringe pattern and direct optical microscope visualization. The most likely places where recoil and damage occurred are shown by the starred points (\*). **(Top)**  $\nu < \nu_c$ : smooth separation; no cavities. **(Middle)**  $\nu \geq \nu_c$ : abrupt separation; cavity and damage form at center. **(Bottom)**  $\nu \gg \nu_c$ : abrupt separation; cavities and damage form at rim (crater-like).

The experiments of Israelachvili and his associates show that cavities open in tension at a threshold value of the extensional stress and that the formation of cavities is analogous to the fracture solids, with the added caveat that the liquid can flow into the crack immediately after fracture. In the words of Kulh et al. [1994]

If the speed of separation is increased, the surfaces become increasingly more pointed just before they rapidly move apart. Then, above some critical speed  $\nu_c$  (here about 100  $\mu\text{m/s}$ ) a completely new separation mechanism takes over, as shown in Figure 4. Instead of separating smoothly, the liquid ‘fractures’ or ‘cracks’ open like a solid. It is known that when subjected to very high shear rates, liquids begin to behave mechanically like solids, for example, fracturing like a brittle solid. In our experiments, the point and time at which this ‘fracture’ occurred was just as the surfaces were about to separate from their most highly pointed configuration (Fig. 4C) - for had the separation velocity been any smaller than  $\nu_c$  they would have separated smoothly without fracturing. We consider that in the present case, the ‘fracturing’ or ‘cracking’ of the liquid between the surfaces must be considered synonymous with the “nucleation” or “inception” of a vapor cavity.

The stretch rate may be underestimated by  $\nu_c/l$  where  $2l$  is the shortest distance between the mica surfaces. To get cavitation it is necessary to cross a stress threshold which is consistent with the observation that “... The thicker the initial film thickness the higher the value of  $\nu_c$ ...”

Of course, the analysis of steady extension in section 6 does not apply to the highly unsteady cavitation being described here. An estimate of the stress level at cavitation can nevertheless be composed as

$$2\eta\dot{S}$$

with  $\dot{S}(t)$  the maximum value of the stress rate between  $t = 10.00$  sec when there is no cavity and  $t = 10.01$  sec when a cavity has definitely opened. It may be optimistic, but certainly possible, that the distance the bump on the top mica surface and the bottom surface changes by 1 nm in  $10^{-4}$  to  $10^{-5}$  sec. Then, with  $2\eta = 36PaS$  we get

$$3.6 \times 10^5 < 2\eta\dot{S} < 3.6 \times 10^6 \text{ Pa}$$

which is greater than atmospheric pressure. A tension of this magnitude could open up a vacuum cavity. According to Kuhl et al. [1994] "... When a cavity initially forms and grows explosively, it is essentially a vacuum cavity since dissolved solute molecules or gases have had time to enter into the rapidly growing cavity." The final collapse of the cavity is slower because the cavity fills with vapor.