## PhD Written Preliminary Question, Fluid Mechanics, 2007

(1) Show the Navier Stokes equation for an incompressible fluid with a constant viscosity

$$\rho\left[\frac{\partial \mathbf{u}}{\partial t}+\frac{1}{2}\nabla|\mathbf{u}|^2-\mathbf{u}\wedge\boldsymbol{\omega}\right]=-\nabla(p+\psi)+\mu\nabla^2\mathbf{u}\,,$$

where $\boldsymbol{\omega}=\mathrm{curl}\mathbf{u}$ and $\psi$ is a body force potential, is satisfied by a potential flow $\mathbf{u}=\nabla\phi$, $\nabla^2\phi=0$.

(2) Derive a Bernoulli equation for this potential flow.

(3) Derive a formula for the viscous stress $2\mu\mathbf{D}[\mathbf{u}]$ where $\mathbf{D}[\mathbf{u}]$ is the rate of strain evaluated on potential flow.

(4) Derive a formula (in terms of the potential) for the viscous dissipation $\mathcal{D}=2\mu\int_V \mathbf{D}:\mathbf{D}\mathrm{d}V$ in potential flow.

Consider a spherical gas bubble of radius $a$ rising with a velocity $U\mathbf{e}_x$ in a potential flow for which $\phi=-\frac{1}{2}Ua^3\frac{\cos\theta}{r^2}$.

(5) Show that the dissipation in the liquid outside the gas sphere is given by $\mathcal{D}=2\mu\int_V \mathbf{D}:\mathbf{D}\mathrm{d}V=12\pi\mu aU^2$. Equate the dissipation to the rate at which the drag force does work and compute the drag.

(6) Now consider a real flow associated with a bubble rising through a fluid. Give a qualitative description of the pressure and vorticity fields in the flow. In your description, make sure to state any relevant assumptions.