A particle of mass $m$ may slide on the surface of a frictionless flat plate of mass $M$ and centroidal moment of inertia $I_G$ as shown. The particle is attached to the plate by a spring of constant $k$ and unstretched length $l_0$. The plate is attached to a fixed point $P$ by stiff wires and may swing about $P$ in the vertical plane. (a) Using the distance $x$ and the angle $\phi$ as generalized coordinates, derive the Lagrangian equations of motion for the system. (b) Assuming small oscillations, show that there is a possible motion of the system that satisfies $x = A \phi$, where $A$ is a constant, and determine $A$. (c) Express in terms of $A$ the frequency of the oscillation in part (b). (d) What is the sign of $A$ if $k = 0$ (i.e. the spring is removed)?