Written Preliminary Examination
Control and Optimization Questions
Friday, April 11, 2005

(Closed-book and closed notes)

Prob. 1. Minimization with Equality Constraint

$$\min_{x_1,x_2,x_3} \quad J = -x_1x_2 + x_2x_3 + x_3x_1 \quad \text{subject to} \quad x_1 + x_2 - x_3 = -1$$

Prob. 2. Min Time Path Through a Region with Two Constant Velocities

$$\min_{\theta_1,\theta_2} \quad J = \frac{y_1}{V_1 \cos\theta_1} + \frac{(y_2 - y_1)}{V_2 \cos\theta_2} \quad \text{s. t.} \quad x_2 - y_1 \tan\theta_1 - (y_2 - y_1)\tan\theta_2 = 0$$

Find the relationship between (θ_1, θ_2) and (V_1, V_2). All variables except (θ_1, θ_2) are given.

Prob. 3. Minimum and Maximum Distance from A Circle to an Ellipse

Find the minimum as well as the maximum distance between the following circle and ellipse, and establish the sufficiency of these extreme distances.

$$x^2 + y^2 = 1, \qquad \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \qquad a > b > 1$$

Prob. 4. Optimal Control Problem Formulation

The 2-D point-mass conventional aircraft equations of motion in the vertical plane are

$$\begin{aligned} m\dot{V} &= T\cos\alpha - D - mg\sin\gamma \\ mV\dot{\gamma} &= T\sin\alpha + L - mg\cos\gamma \\ \dot{h} &= V\sin\gamma \\ \dot{x} &= V\cos\gamma \\ \dot{m} &= -f_e(V,h) \end{aligned}$$

where

$$L = \frac{1}{2}\rho V^2 S C_L, D = \frac{1}{2}\rho V^2 S C_D$$

and V is the airspeed, m is the mass, T is the thrust, α is the angle of attack, (L, D) are the (lift, drag), γ is the flight path angle, (x, h) are the (horizontal location, altitude), f_e is the fuel burn rate, ρ is the air density, S is the aircraft reference area, and (C_L, C_D) are (lift, drag) coefficient. It is assumed that C_D is a function of C_L. The state variables for this set of equations are $[V, \gamma, h, x, m]$, and the control variables are $[T, C_L]$.

Formulate optimal control problems to reflect the following performance needs: (1) Minimum-time climb from a given initial state to a specified final altitude of h_f. (2) Minimum-fuel climb from a given initial state to the specified final condition of $h(t_f) = h_{fs}$, $V(t_f) = V_{fs}$, and $t_f = t_{fs}$. In each formulation, please specify the performance index, initial conditions, terminal constraints, and **all applicable and independent** path constraints.