The uniform rectangular plate is shown in the figure above. The plate is hinged by a universal joint at point $O$. That is, it is free to rotate about all three axes about $O$ but it is constrained from translating in any of the directions. The orientation of the coordinate frame attached to the plate (defined by axes having unit vectors $\hat{b}_1$, $\hat{b}_2$ and $\hat{b}_3$) is related to an inertial or reference coordinate frame (defined by axes having unit vectors $\hat{a}_1$, $\hat{a}_2$ and $\hat{a}_3$) via a 321 Euler angle sequence. The Euler angles, denoted $\varphi$, $\theta$ and $\gamma$, respectively, are known functions of time. That is, $\varphi = \varphi(t)$, $\theta = \theta(t)$ and $\gamma = \gamma(t)$.

1. Derive the equations of motion for the small block using the Lagrangian approach. Make the assumption that the Euler angles are small. Clearly identify the generalized coordinates you have selected.

2. Derive the equations of motion for the small block using the Newtonian approach. Make the assumption that the Euler angles are small.

3. If your equations of motion derived by the two approaches above are not identical, explain why they are different.
4. Suppose we wanted to move the small block in a small circle of radius $R_0$ about the origin. What would the time histories of $\phi$, $\theta$ and $\gamma$ have to be for this occur? How would the time histories be different if the circle is centered at some position other than the origin?