Longitudinal Motion Control for High Speed Supercavitation Vehicles
Comparison of different approaches

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1 Problem Description
   Motivations for Supercavitation
   Vehicle configuration

2 Control design
   Control with fins and cavitator
   Control with cavitator
   Control with fins

3 Simulation Results

4 Summary
Benefits using Supercavitation

- The pressure of the fluid drops due to its high speed leading to vaporization
- Reduced skin friction drag
- Planing force can be used to sustain the tail
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Drawbacks of Cavitation

- Transition to supercavitation needs effort
- Cavity bubble can be destabilized with actuator
- Control surfaces immersion to liquid change
- High, nonlinear planing forces
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Vehicle states in the longitudinal plane

Note the switching, delay dependency and the different fin immersions.
Possible control configurations

- Cavitation
- Fin
- Thrust vectoring
- Side skids
Equations of Motion

\[
\begin{bmatrix}
\dot{z} \\
\dot{\theta} \\
\dot{w} \\
\dot{q}
\end{bmatrix} = A \begin{bmatrix} z \\ \theta \\ w \\ q \end{bmatrix} + B \begin{bmatrix} \delta_e \\ \delta_c \end{bmatrix} + G F_{grav} + P F_p
\]  \hspace{1cm} (1)

Empirical description for the force generated by planing:

\[
F_p = -\rho R^2 \pi V^2 \left(1 - \frac{R'}{h' - R'}\right)^2 \left(\frac{1 + h'}{1 + 2h'}\right) \alpha_p
\]

\(h'\): normalized immersion of the body;
\(\alpha_p\): planing contact angle
Both variables are switched delay dependent.
## Vehicle Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value and Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>$g$</td>
<td>Gravitational acceleration</td>
<td>$9.81 \frac{m}{s^2}$</td>
</tr>
<tr>
<td>$m$</td>
<td>Density ratio, $\frac{\rho m}{\rho}$</td>
<td>2</td>
</tr>
<tr>
<td>$R_n$</td>
<td>Cavitator radius</td>
<td>0.01905 m</td>
</tr>
<tr>
<td>$R$</td>
<td>Vehicle radius</td>
<td>0.0508 m</td>
</tr>
<tr>
<td>$R_c$</td>
<td>Cavity radius at tail</td>
<td>0.0966 m</td>
</tr>
<tr>
<td>$L$</td>
<td>Length</td>
<td>1.714 m</td>
</tr>
<tr>
<td>$V$</td>
<td>Velocity</td>
<td>$77 \frac{m}{s}$</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Cavitation number</td>
<td>0.029</td>
</tr>
<tr>
<td>$C_1$</td>
<td>Cav. lift coefficient</td>
<td>2685 N/rad</td>
</tr>
<tr>
<td>$C_2$</td>
<td>Fin lift coefficient</td>
<td>4015 N/rad</td>
</tr>
</tbody>
</table>
Simplifications

- Small angle approximations around level flight
- Constant horizontal speed, fin immersion
- Constant environmental parameters (density, pressure etc.)
- Planing only on the tail
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Dynamic Inversion Control

- With suitable feedback the system can be transformed to linear
- The position and pitch dynamics can be decoupled
- The resulting control scheme is switched (three different modes)
Receding Horizon / Model Predictive outer loop control

- Position and Angle tracking
- Planing can be avoided using predictions
- Limits on actuator deflections cannot be incorporated
Equations of Motion with Cavitator

After Coordinate transformation:

\[ A = \begin{bmatrix}
0 & 1 & 0 & 0 \\
0 & 4.0091 & 617.4 & -80.21 \\
0 & 0 & 0 & 1 \\
0 & -17.504 & -2695.6 & 1.3987 \\
\end{bmatrix}, \quad B_c = \begin{bmatrix}
0 \\
308.7 \\
0 \\
-781.01 \\
\end{bmatrix} \]  \tag{2}

\[ C = \begin{bmatrix}
1 & 0 & -1.04 & 0 \\
0 & 0 & 1 & 0 \\
0 & 1 & 0 & -0.8 \\
0 & 0 & 0 & 1 \\
0 & 4.01 & 617.4 & -80.21 \\
\end{bmatrix}, \quad P = \begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & -9265.5 \\
\end{bmatrix}, \quad G = \begin{bmatrix}
0 & 0 \\
9.81 & 0 \\
0 & 0 \\
\end{bmatrix} \]  \tag{3}
Dynamic Inversion Control

Applying a feedback:

\[
\delta_c = -\frac{P(4)}{B_c(4)} \left(1 - \frac{R'}{h' - R'}\right)^2 \left(\frac{1 + h'}{1 + 2h'}\right) \alpha_p
\]  

(4)
- Position reference tracking
- Rejection of artificial disturbance on second state
• Position reference tracking
• Rejection of artificial disturbance on second state
Limitations imposed by the vehicle dynamics

\[ TF_{\delta_{\text{cav}}} \rightarrow z = \frac{934.1(s + 61.87)(s + 8.074)}{s^2(s - 1.583 \pm 38.3i)} \] (5)
Limitations imposed by the vehicle dynamics:

\[
TF_{\delta_{\text{fin}}} \rightarrow z = \frac{605.2(s + 103.6)(s - 7.442)}{s^2(s - 1.583 \pm 38.3i)}
\]  \hspace{1cm} (6)

Right Half Plane Zero at 7.442 Poles at 1.583 \pm 38.3i
\[\Rightarrow\] Bandwidth should be \( \approx 4.5 \text{rad/s} \)
Roll stabilization is possible \( \Rightarrow \) vehicle can be trimmed before launch. Planing free flight during straight level path.
Dynamic Inversion Control

The special structure of $B_f$ and $P$

$$B_f = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -568.8 \end{bmatrix} \quad P = \begin{bmatrix} 0 & 0 \\ 0 & -9265.5 \end{bmatrix}$$

(7)

allow to apply a feedback:

$$\delta_f = -\frac{P(4)}{B_f(4)} \left(1 - \frac{R'}{h' - R'}\right)^2 \left(\frac{1 + h'}{1 + 2h'}\right) \alpha_p$$

(8)

leading to pure linear closed loop dynamics.
Bandwidth with Fin control,
Good disturbance attenuation (for gravity compensation),
Inverse response is significant.
Simulation Setup

• **Actuator model**: 30 Hz bandwidth with 0.2 rad max. deflection.

• Benchmark maneuver: obstacle avoidance with different amplitudes and time spans.

• Simulations with three fin sizes 75%, 100, and 125%.

• Fin and Cavitator drag approximated with fourth order polynomials, identified from full nonlinear simulation.

• Skin friction drag is calculated based on Paryshev’s theory.

• Noise on the cavity wall with max. 10% of the cavity gap magnitude, up to 200 Hz frequency.
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Benchmark Maneuver: Cavitation, Fin and Cav.+Fin control

Work by Actuators: Cav.: 367J, Fin: 192J, Cav.+Fin: 0.44J;
Drag: Cavitator, Fin and Cav.+Fin control

Fin Drag $\approx 110N$  Cav. Drag $\approx 2863N$.  
Overall work of drag:  
Cav.:16420J,Fin:$-0.12\%$,Cav.+Fin:$-0.3\%$
Stability and Performance map

Better tracking with Cav.+Fin in shorter maneuvers
Cav.+Fin different amplitude maneuvers

- Work of drag does not change, remains smaller than Cav.
- Work by fin actuators: $-4.5\% ; 0.22\% ; +23\%$ respectively
- Max. tracking error: $0.145 ; 0.18 ; 0.224 \text{ m} (0.53 \text{ m with cav.})$
Drag remains slightly lower than cav. control
Work by actuators increase (+100%) as time span decreases to 4.5 sec, remains same for 5 and 5.5 sec
Cav.+Fin different fin lengths

- Tracking error does not change
- Work by cav. actuators:
  - $-9\%(n = 0.75)$; 0.22 J; $+32\%(n = 1.25)$
- Work by fin actuators:
  - $+60\%(n = 0.75)$; 0.22 J; $-11\%(n = 1.25)$ respectively
- Work by Drag:
  - $-0.9\%(n = 0.75)$, 16370J; $+0.9\%(n = 1.25)$ (Cav. : $+0.3\%$)
Cav. Control with different path amplitudes

Planing lasts longer with sharper maneuvers
Overall contribution on drag is low
Work of cav. actuator: −19% (4m), 375J, +8% (6m)
Tracking error scales linearly
Cav. Control with different time spans

- Longer maneuver slightly better tracking
- Actuator work has little change
- Work by cav. actuators also change slightly
- Work by Drag remains higher than Cav.+Fin (0.2% change)
Cav. Control with different fins

- No effect on tracking error
- Cav. defl. is smaller with larger fins
- Planing is longer with small fins
- Overall drag $-0.5\%(n = 0.75)$ 16370 J $+1.1\%(n = 1.25)$
- Actuator energy $+57\%(n = 0.75)$ 375J $-12\%(n = 1.25)$
Summary

- With only fin control the vehicle has poor tracking performance
- Cavitator control is a good alternative to Fin+Cav. control especially with larger radius maneuvers, although planing causes high accelerations on tail, roll control is not possible
- Cav.+Fin control provides better tracking, with reduced drag and lower control energy invest at the expense of additional system complexity.
Outlook

Problems for the future:

- Design of active or passive skids for better damping
- Parameter Varying (Gain Scheduled) model depending on fin immersion
- Water tunnel hardware in the loop simulations with feedback control
- Lateral directional equations of motion and stabilizing tracking controller for them

Thank You!