Theoretical aspects of High-Speed Supercavitation Vehicle Control

Bálint Vanek\textsuperscript{1} József Bokor\textsuperscript{1,2} Gary J. Balas\textsuperscript{1}

\textsuperscript{1}Department of Aerospace Engineering and Mechanics
University of Minnesota

\textsuperscript{2}on sabbatical leave from Hungarian Academy of Sciences
This work was supported by ONR, Dr. Kam Ng program officer

American Control Conference, Minneapolis 2006
Theoretical aspects of HSSV Control

Outline

1 Motivation
   Problem Description
   Vehicle Configuration

2 Control Design
   Control Approach
   Controllability Analysis
   Simulation

3 Summary
   References

(Image: PSU/ARL)
Supercavitation

- Pressure of fluid drops due to high speed, leading to vaporization
- Reduced skin friction drag
- Planing force can be used to sustain the tail
- Transition to supercavitation needs effort
- Control surfaces immersion changes
- Large, nonlinear planing forces
Longitudinal Vehicle States

- The switching hyperplane depends on the delayed state variable \( x(t - \tau) \),
- First mode the system dynamics is linear, second mode is nonlinear input affine
- Switching condition does not depend on the control inputs
System Equations

\[ \dot{x}(t) = Ax(t) + Bu(t) + F_g + F_p(t, x, \delta) \]  

Bimodal, Switched System:

\[ F_p(x, \delta) = P(1 - \frac{R'}{h'(x, \delta) + R'})^2(\frac{1 + h'(x, \delta)}{1 + 2h'(x, \delta)})\alpha(x, \delta), \]  

\[ h'(x, \delta) = \begin{cases} R^{-1}c(\delta)x(t) & \text{if } c(\delta)x(t) \leq 0, \\ 0 & \text{if } c(\delta)x(t) \geq 0, \end{cases} \]  

\[ \alpha(x, \delta) = \begin{cases} c_\alpha(\delta)x(t) - V^{-1}\dot{R}_c & \text{if } c(\delta)x(t) \leq 0, \\ 0 & \text{if } c(\delta)x(t) \geq 0, \end{cases} \]
Control Approach

- Feedback laws for each mode transforms system to LTI
- Control design synthesized in a new multivariable canonic coordinate frame
- Extending controllability results on bimodal switched LTI systems to time delayed, switched, with the analysis of time delayed zero dynamics
- Tracking problem using multivariable pole placement
Control Architecture

\[ v \rightarrow Act._{\text{fin}} \rightarrow B_c \rightarrow \int \rightarrow x_0 \rightarrow C_s \rightarrow y_s \]

- \( F_{\text{grav}} \)
- \( cavitator \)
- \( F_{\text{plane}} \) (Bimodal)
- Feedback Linearizing Controller (switching)

Vehicle Configuration

Theoretical aspects of HSSV Control

Motivation

Problem Description

Vehicle Configuration

Control Design

Control Approach

Controllability Analysis

Simulation

Summary

References
Feedback Linearization

Input variables enter linearly in both modes. Coordinate transformation for canonical coordinates:

\[
T_c = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & -V & 1 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\] (5)

Linear dynamics in new coordinates:

\[
A_c = \begin{bmatrix}
0 & 1 & 0 & 0 \\
-\alpha_{110} & -\alpha_{111} & -\alpha_{120} & -\alpha_{121} \\
0 & 0 & 0 & 1 \\
-\alpha_{210} & -\alpha_{211} & -\alpha_{220} & -\alpha_{221}
\end{bmatrix}
\]

\[
B_c = \begin{bmatrix}
0 \\
c_1 AB \\
0 \\
c_2 AB
\end{bmatrix}
\] (6)

Two outputs: \(y_1 = x_1\) and \(y_2 = x_3\), vector relative degree in both modes are identically (2,2).
State Feedback

Applying a switched nonlinear feedback:

\[
\begin{align*}
    u_{flc} &= \begin{cases} 
    (CAB)^{-1}(\dot{y}_{13}(t) - F_{x}x(t) - \bar{F}_{g} + v_{I}(t)) \\
    (CAB)^{-1}(\dot{y}_{13}(t) - F_{x}x(t) - \bar{F}_{g} - \bar{F}_{p}(x, \delta) + v_{II}(t)) 
    \end{cases}
\end{align*}
\]

(7)

the switching condition is given by the sign of \( y_s = c(\delta)x_c \)

- Nilpotent system with identical linear dynamics in both modes
- The system is continuous on the switching hypersurface
- The relative degrees are equal in both modes
System Decomposition

Selecting a direction $p \in \text{Im}\{B\}$ such that the system is left and right invertible:

$$\dot{x} = Ax + pu_p + \bar{B}\bar{u}, \quad y_s =Cx.$$  \hfill (8)

the largest $(A, p)$ - invariant subspace in $\ker\{C\}$ and the smallest $(C, A)$ invariant subspace over $\text{Im}\{p\}$ induce the following decomposition:

$$\dot{\xi} = A_{11}\xi + \gamma v$$  \hfill (9)

$$u_p = \frac{1}{\gamma}(-A_{12}\eta - \bar{B}_{21}\bar{u} + v)$$  \hfill (10)

$$\dot{\eta} = P\eta + \bar{Q}\bar{u} + Ry_s,$$  \hfill (11)

Last equation denoting the zero dynamics on the switching hyperplane assuming $Q$ is monic.
Controllability Conditions

If the pair \((P, Q)\) is controllable, then \(\eta\) is controllable without using \(y_s\).

If the pair \((P, Q)\) is not controllable, then using unconstrained \(\bar{u}\) and nonnegative \(y_s\)

1. The pair \((P, [Q R])\) has to be controllable.
2. Consider the decomposition induced by the reachability subspace \(R(P, Q)\),

\[
\begin{align*}
\dot{\eta}_1 &= P_{11}\eta_1 + P_{12}\eta_2 + Q\bar{u} + R_1 y_s \\
\dot{\eta}_2 &= P_{22}\eta_2 + R_2 y_s,
\end{align*}
\]

(12) (13)

where \(R_2 \neq 0\). Then the imaginary part of the eigenvalues of \(P_{22}\) cannot be zero.
Application for the HSSV

HSSV has time delay dependency. Discretize the system using backward difference scheme:

\[ C_d = [1, 0, L, 0, -1] \]  \hspace{1cm} (14)

The relative degrees are identically \( r = 2 \) for both modes. Zero dynamics obtained:

\[
T_{cd} = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 \\
0 & \beta_{41} T & 0 & -\beta_{21} T & 0
\end{bmatrix}
\]  \hspace{1cm} (15)
Switched Zero Dynamics

Decomposing the system to: \[
[\xi^T(t), \eta^T(t)]^T = T_{cd}x(t)
\]

\[
\xi(t + 1) = \begin{bmatrix} 0 & a_{12} \\ 0 & a_{22} \end{bmatrix} \xi(t) + \begin{bmatrix} 0 & 0 & 0 \\ 0 & e_{22} & e_{23} \end{bmatrix} \eta(t) + \\
+ \begin{bmatrix} 0 \\ b_{21} \end{bmatrix} v_1(t) + \begin{bmatrix} 0 \\ f_{22} \end{bmatrix} v_2(t)
\]

\[
y_s = \begin{bmatrix} 1 & 0 \end{bmatrix} \xi(t) \quad \text{switching condition}
\]

\[
\eta(t + 1) = P\eta(t) + R\xi(t) + Qv_2(t),
\]

where

\[
P = \begin{bmatrix} p_{11} & p_{12} & p_{13} \\ 0 & p_{22} & p_{23} \\ 0 & 0 & p_{33} \end{bmatrix}, \quad R = \begin{bmatrix} 0 & r_{12} \\ 0 & r_{22} \end{bmatrix}, \quad Q = \begin{bmatrix} 0 \\ 0 & q_{31} \end{bmatrix}.
\]

The zero dynamics are described by the last equation. The HSSV is controllable.
Theoretical aspects of HSSV Control

Motivation
Problem Description
Vehicle Configuration

Control
Design
Control Approach
Controllability
Analysis
Simulation

Summary
References

Tracking: Multivariable Pole Placement

\[
\begin{bmatrix}
  u_1(t) \\
  u_2(t)
\end{bmatrix} = (CAB)^{-1}\left( \begin{bmatrix}
  \dot{x}_1(t) \\
  \dot{x}_2(t)
\end{bmatrix}_{\text{ref}} - [\alpha_u] \begin{bmatrix}
  x_1(t) \\
  x_2(t)
\end{bmatrix} - \\
  - [\alpha_l] \begin{bmatrix}
  x_3(t) \\
  x_4(t)
\end{bmatrix} - [G_c] - [P_c(t, \tau)] - \begin{bmatrix}
  v_1(t) \\
  v_2(t)
\end{bmatrix}\right) \tag{20}
\]
Reference Tracking Controller

\[
\begin{bmatrix}
    v_1(t) \\
    v_2(t)
\end{bmatrix}
= [\bar{\alpha}_u]
\begin{bmatrix}
    x_1(t) - x_{1,\text{ref}}(t) \\
    x_2(t) - x_{2,\text{ref}}(t)
\end{bmatrix}
+ [\bar{\alpha}_l]
\begin{bmatrix}
    x_3(t) - x_{3,\text{ref}}(t) \\
    x_4(t) - x_{4,\text{ref}}(t)
\end{bmatrix}
\]  

(21)

- The system behaves the same regardless of the interior switching state
- One linear outer loop controller can guarantee stability and appropriate tracking
- Position and angle command dynamics are decoupled
Vehicle Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value and Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>$g$</td>
<td>Gravitational acceleration</td>
<td>9.81 m/s²</td>
</tr>
<tr>
<td>$m$</td>
<td>Density ratio, $\frac{\rho m}{\rho}$</td>
<td>2</td>
</tr>
<tr>
<td>$R_n$</td>
<td>Cavitator radius</td>
<td>0.01905 m</td>
</tr>
<tr>
<td>$R$</td>
<td>Vehicle radius</td>
<td>0.0508 m</td>
</tr>
<tr>
<td>$R_c$</td>
<td>Cavity radius at tail</td>
<td>0.0647 m</td>
</tr>
<tr>
<td>$L$</td>
<td>Length</td>
<td>1.8 m</td>
</tr>
<tr>
<td>$V$</td>
<td>Velocity</td>
<td>75 m/s</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Cavitation number</td>
<td>0.035</td>
</tr>
<tr>
<td>$C_1$</td>
<td>Cav. lift coefficient</td>
<td>2685 N/rad</td>
</tr>
<tr>
<td>$C_2$</td>
<td>Fin lift coefficient</td>
<td>1343 N/rad</td>
</tr>
</tbody>
</table>
Simulation Setup

- Obstacle avoidance maneuver, $75 \text{m/s}$, $17.5 \text{m}$ translation within $4 \text{s}$
- Pressure, temperature, viscosity etc. are constant
- $10\%$ disturbance on cavity wall
- Model mismatch due to actuators ($G_{\text{act}} = \frac{1}{200s+1}$)
- Trajectory reference commands and reduce limit cycle oscillations.
17.5 m Maneuver

- High planing depth induces high actuator deflections
- Good reference tracking
- High pitch rate oscillations and accelerations due to the cavity disturbance
Role of Disturbance Model

- Higher noise $\Rightarrow$ higher pitch rate and accelerations on tail but same immersion depth
- Control scheme is sensitive to bandwidth of disturbance model
Different Amplitude Maneuvers

- Planing depth is influenced by maneuver amplitude
- Fin deflection is not changed
- Cavitator deflection is more sensitive
Summary

- Supercavitation is a promising way to increase the speed of underwater vehicles
- Control design challenges including delayed state dependency, nonlinearities and switching with noisy switching surface were analyzed
- The controllability problems related with the system were analyzed
- An inversion based control methodology was developed
- A successful implementation of a two-loop control strategy was demonstrated
- Further development requires increased collaboration between fluid and control researchers

Future goals: three dimensional trajectory tracking (asymmetric fin immersion and non-vertical planing forces), robust constraint fulfillment, control the vehicle using one surface.
References

G.J. Balas, J. Bokor, B. Vanek and R.E.A. Arndt
*Control of Uncertain Systems: Modelling, Approximation and Design.*
Control of High-Speed Underwater Vehicles
Springer-Verlag, 2006.

DARPA Advanced Technology Office
Underwater Express
*BAA06-13 Proposer Information Pamphlet (PIP),* 2005.

G.J. Balas, Z. Szabó and J. Bokor
*Controllability of bimodal LTI systems*