Time Benefits of Free-Flight for a Commercial Aircraft

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Introduction

The nationwide increase in air traffic has severely burdened the National Airspace System (NAS). The volume of air traffic is expected to double over the next 20 years, prompting the development of new innovations in air traffic control such as free flight. One of the primary concepts behind free-flight is allowing pilots to change routes in real time without consulting with Air Traffic Control (ATC). Consequently, this allows the airlines to save time and fuel by allowing them the freedom to choose flight routes that take advantage of atmospheric conditions (such as wind and temperature).

The theory of optimal flight routing was investigated as early as 1981 during a six-month study called Operation Free Flight[1]. The study included voluntary participation from Eastern, United, and Pan Am Airlines along flight routes serving 27 different cities. The Operation Free Flight study projected fuel savings for the airlines to be around $40 million annually while at the same time having insignificant adverse affect on the air traffic control system. A similar study was conducted by American Airlines from 1992 to 1995[2]. Over the period of this study American Airlines flew preferred "wind routes", whenever possible, which demonstrated fuel savings of $2.2 million annually. A NASA study estimated that user preferred routing could result in an annual cost savings for the Airline Industry of over $1 billion[3].

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The main purpose of this note is to show dynamic optimization algorithms can be used to find flight trajectories that take advantage of atmospheric conditions, resulting in flight times shorter than those of direct routes.

Aircraft Model

This note uses a point mass model of a commercial passenger jet. The calculations performed represent flight in a horizontal plane with variations in altitude restricted. The only control variable is the heading angle of the aircraft. The point mass equations of motion are given by the following:

$$\dot{x} = V \sin\psi + w_x \tag{1}$$

$$\dot{y} = V \cos\psi + w_y \tag{2}$$

Where V is the true airspeed of the aircraft, w_x and w_y are the wind velocities in the east-west and north-south directions respectively, and ψ is the heading angle of the aircraft referenced from north.

The true airspeed of the aircraft is dependent on the Mach number and temperature. In order to simulate the flight conditions for a commercial aircraft a Mach number of 0.8 and atmospheric temperature of –70.9 degrees Fahrenheit was used. All heading changes are instantaneous.

Atmospheric Modeling

Calculation of minimum-time trajectories using a dynamic optimization algorithm depends on wind gradients along the flight route, therefore, it is desirable to use an interpolation method that provides the wind speed and direction at any point along the flight route, gives reasonable wind gradients from one region of the country to next, allows real winds aloft data to be used at interpolation nodes, and minimizes processor time and memory storage. The wind model for this note is a steady state model; therefore wind gradients are due solely to position. It should be noted that the purpose of this interpolation scheme was to provide "realism" to the wind profile and not necessarily accuracy.

The main premise of this wind model is to use winds aloft data from certain cities around the flight path. As the aircraft moves along its trajectory the winds at the aircraft's location depends on the aircraft's proximity to each city.

Winds aloft data is given in the form of a wind magnitude and direction. This data was converted to wind velocities in the x and y directions, (i.e. w_x and w_y). These velocities are then applied to an interpolating set of equations. For the sake of simplicity Figure 1 and equations (3) and (4) use only four general cities to display the concept behind the wind model. The actual calculations used seven nodes for the first set of results and fourteen nodes for the second.

$$\Pi = \sqrt{(r_2 r_3 r_4)^2 + (r_1 r_3 r_4)^2 + (r_1 r_2 r_4)^2 + (r_1 r_2 r_3)^2} \quad (3)$$

$$w_{xA} = \frac{w_{x1}(r_2 r_3 r_4) + w_{x2}(r_1 r_3 r_4) + w_{x3}(r_1 r_2 r_4) + w_{x4}(r_1 r_2 r_3)}{\Pi} \quad (4)$$

Where w_{xA} is the wind magnitude at the aircraft's location in the east-west direction and the other w_x's are the wind magnitudes at the interpolation nodes. The wind in the north-south direction, w_{yA} is calculated in the same manner.

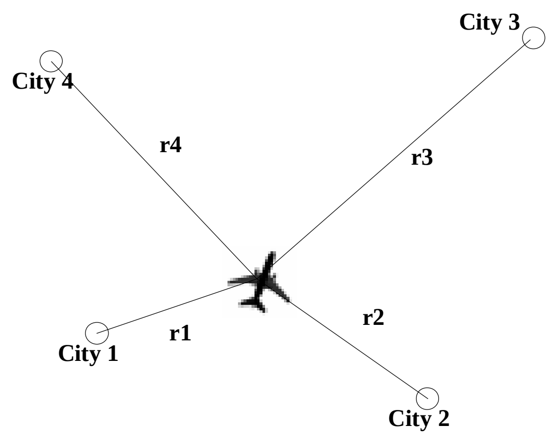

Fig. 1 Simplified city to aircraft distances used for weighted interpolation.

Problem Formulation

The problem investigated in this note is to find the minimum time of flight from departure city to destination city subject to equations (1) and (2). Minimum time flights that took advantage of the winds aloft were found using dynamic optimization algorithms written in C.

Dynamic optimization is the process of determining the control histories of a dynamic system over a finite time in order to minimize a performance index. The Combined Function and Parameter Optimization Algorithm implemented for the optimal trajectories in this note solves Bolza type dynamic optimization problems with terminal constraints and open final time[4]. The algorithm requires that the initial states be specified along with initial guesses of the control history and unknown parameters (in this case final time). In addition, controller, parameter, and terminal improving step sizes must be selected. The control improve step size was 10^{-3} rads, while the parameter improve step size and terminal improvement factor were 10^{-2}. Terminal, optimality, and parameter accuracy were of the order 10^{-8}. The initial guess of constant control and final time depend on the direction of flight and the user's experience. Convergence of the algorithm does not guarantee a 'global' optimum; therefore several initial guesses are investigated.

Forward and backward integration is performed using Hamming's modified predictor-corrector method, along with a special Runge-Kutta starter. Definite integrals are calculated using Simpson's rule.

Nominal Calculation

For the purpose of comparison, nominal calculations were also performed for the same Mach number and atmospheric conditions as the optimal calculations. The nominal calculation consisted of a fourth order Runge-Kutta integration of equations 1 and 2 along a direct path between the departure and arrival cities. In order for the velocity vector of the nominal flight to always point towards the Destination City the heading angle needed to be adjusted using equations (5)-(9).

$$\Delta = w_{xA} - w_{yA}\tan\theta \tag{5}$$

$$A = V^2\left(\tan^2\theta + 1\right) \tag{6}$$

$$B = -2\Delta V \tan\theta \tag{7}$$

$$X = \Delta^2 - V^2 \tag{8}$$

$$\psi_c = \cos^{-1}\left(\frac{\pm B + \sqrt{B^2 - 4AX}}{2A}\right) \tag{9}$$

Where ψ_c is the wind corrected heading angle. The integration for the nominal calculations is performed until the aircraft position is within a certain tolerance of the Destination City. This is achieved through numerical capture methods.

Results

The first set of results in this note represents flights between Minneapolis, Minnesota and San Francisco, California. The atmospheric model used seven interpolation nodes and flights were calculated to and from each city over a ten-day period. The winds aloft data for each day was taken from the Aviation Weather Center Website[5]. The time saved on flights from Minneapolis to San Francisco averaged 2.7 minutes per flight. The average from San Francisco to Minneapolis was slightly less at 2.65 minutes per flight. Generally, flights in the eastern direction have the advantage of a tail wind and therefore flight times will be lower. Obviously, the shorter the flight-time, the lower the opportunity to save time. The time saved for each flight over this ten day period is shown in Figure 2. The flight trajectory, ground speeds, and control histories for both the nominal and optimal flight from San Francisco to Minneapolis on Day 10 is shown in Figure 3. This optimal flight path resulted in a time saving of 7.2 minutes.

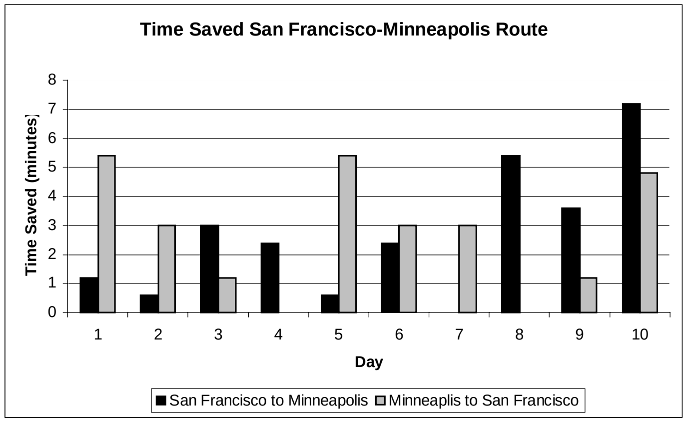

Fig. 2 Time saved on San Francisco-Minneapolis Route.

The second set of calculations modeled flights from Boston, Massachusetts to Seattle, Washington and flights again from Boston to Orlando, Florida. The wind model used fourteen interpolation nodes. Again, calculations were performed for a ten day period however, for these calculations only a few of the nodes had their data changed from day to day in order to get an idea of how sensitive an optimal flight would be to wind changes only in a small region around the flight route. The average savings from Boston to Seattle were 2.4 minutes and 1.56 minutes for the Boston to Orlando route. The results for all calculations are shown in Figure 4.

A study performed by Delta Air Lines utilized computer programs that optimized flight routes for time and fuel subject to winds, weather, and air traffic[6]. The results of the Delta study found that implementation of free flight may result in an average reduction in block times of two minutes.

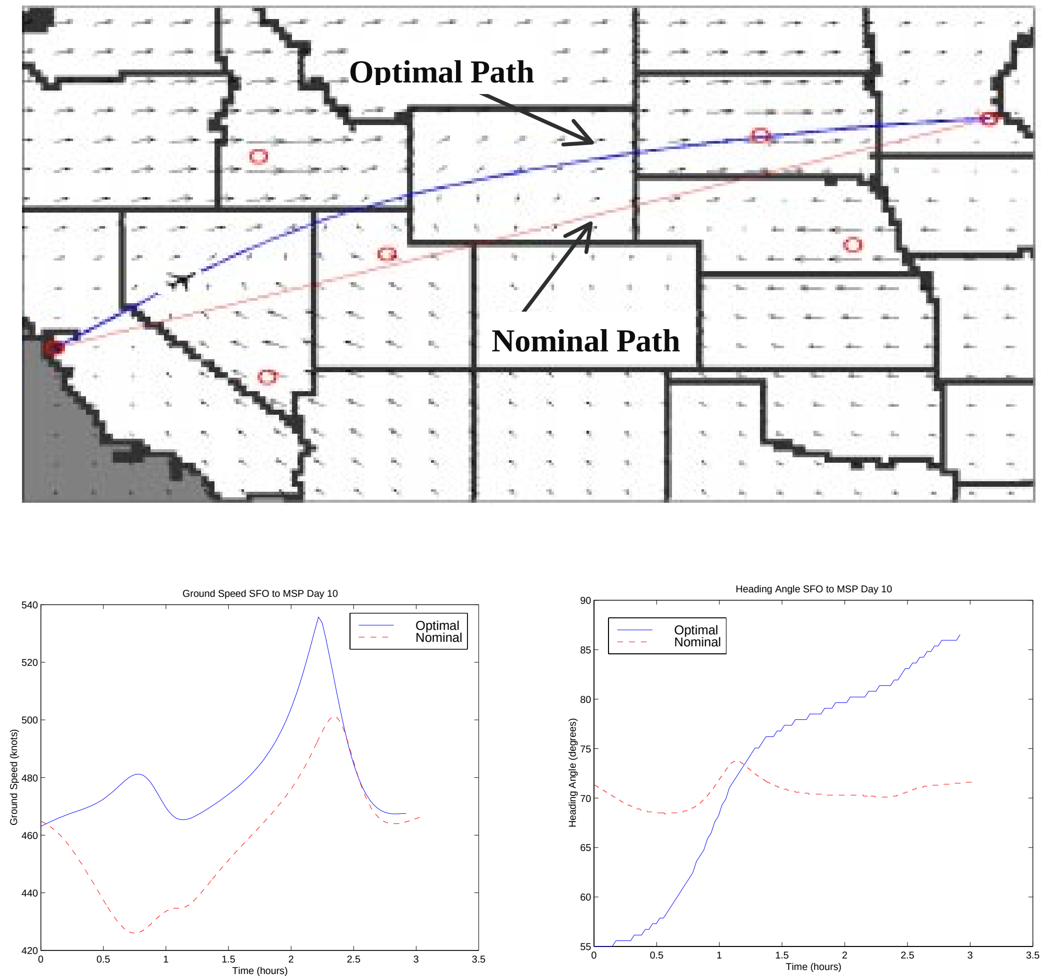

Fig. 3 Flight path, ground speed, and control history for San Francisco to Minneapolis flight on Day 10.

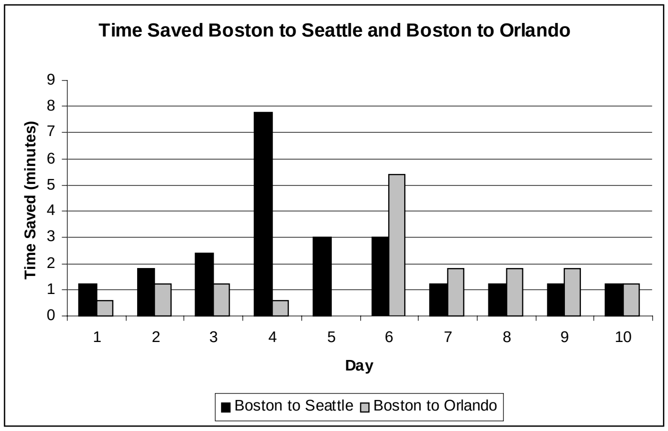

Fig 4 Time saved for Boston to Seattle and Boston to Orlando Routes.

It should be noted that the day to day changes in the wind were relatively small. However, even slight variations in wind can result in significant changes in the time saved from one day to the next.

Conclusion

This note investigates the timesaving an aircraft can achieve by taking advantage of the atmospheric conditions. Heading angle was used as the variable control. By minimizing a performance index a control history for the heading angle could be found that minimized the time of flight between a departure and destination city. A weighted interpolation scheme was used to model the atmosphere. Optimal calculations were performed for ten day periods and compared with a nominal flight time. Longer flights such as those between Minneapolis and San Francisco as well as Boston and Seattle saved on average over two minutes while the slightly shorter route between Boston and Orlando saved about a minute and a half. One of the findings from this note is that on certain days there will be no improvement over the nominal time. Ideal wind conditions that will allow for large improvements over the nominal time will occur when there are large wind gradients on either side of the straight flight path.

Acknowledgement

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