This volume is dedicated to Shifra Chana, Mike, Chuck and Samuel.

Preface

This book is about two special topics in rheological fluid mechanics: the elasticity of liquids and asymptotic theories of constitutive models. The major emphasis of the book is on the mathematical and physical consequences of the elasticity of liquids; seventeen of twenty chapters are devoted to this. Constitutive models which are instantaneously elastic can lead to some hyperbolicity in the dynamics of flow, waves of vorticity into rest (known as shear waves), to shock waves of vorticity or velocity, to steady flows of transonic type or to short wave instabilities which lead to ill-posed problems. Other kinds of models, with small Newtonian viscosities, give rise to perturbed instantaneous elasticity, associated with smoothing of discontinuities as in gas dynamics.

There is no doubt that liquids will respond like elastic solids to impulses which are very rapid compared to the time it takes for the molecular order associated with short range forces in the liquid, to relax. After this, all liquids look viscous with signals propagating by diffusion rather than by waves. For small molecules this time of relaxation is estimated as 10^{-13} to 10^{-10} seconds depending on the fluids. Waves associated with such liquids move with speeds of 10^5 cm/s, or even faster. For engineering applications the instantaneous elasticity of these fluids is of little interest; the practical dynamics is governed by diffusion, say, by the Navier-Stokes equations. On the other hand, there are other liquids which are known to have much longer times of relaxation. Polymers mixed in Newtonian solvents and polymer melts, like high viscosity silicone oils or molten plastics, are examples. The longest times of relaxation for these liquids are of practical interest; times we can read on our clock, of the order of milliseconds to minutes, or longer. The study of hyperbolic dynamics is complicated by the presence of many relaxation times. The limiting wave speed is determined by the fastest rather than by the slowest relaxation so that the instantaneous elastic response has already begun before the slow relaxation has begun. The fast relaxation of small molecules gives rise to an effective viscosity which smooths slow waves. If the total viscosity is much greater than the effective viscosity we may consider the theory of perturbed elasticity, with relatively small effective rigidities associated with the long lasting relaxations. The effective wave speeds are slow, ranging roughly from 1 to 1000 cm/sec.

It follows from what has been said that the models which are instantaneously elastic and give rise to hyperbolicity and change of type are precise only for times too short for applications. For the applications, the effective theory appears to work well but not all issues have been resolved. One question is what type of theory may be developed when the effective viscosity is not just a small part of the total. A second question is to what extent we may expect robust values of the effective quantities which are not dependent on flow conditions. It is probable that the successful resolution of these issues will depend more on experiments than on theory.

The contents of the seventeen chapters on the elasticity of liquids is taken from relatively recent papers not before collected into one volume.

The three chapters on asymptotic theory treat some well-known things in a new way. In 15 I review theories of fading memory and show how different theories will lead to different types of constitutive equations. Various types of perturbation theories are considered in detail in 16. In 17 I deal with second order theory emphasizing features which I consider fundamental like the balance of inertia and normal stress effects, the persistence of normal stress, the correlation between extensional viscosity and the intensity of secondary motion, the importance of nonelastic contributions to extensional and secondary motions and the general rheometrical problem of determining values of the quadratic constants.

I have tried to avoid repeating things which are well expressed in other books listed in the references. Since only special topics are treated, this book cannot be used as a general reference, but for many of the special topics treated it is effectively the only reference. Some complementary results for wave propagation in viscoelastic materials and many results about existence and uniqueness of solutions for one-dimensional models can be found in the book by Renardy, Hrusa, and Nohel [1987].

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