

Figure 1

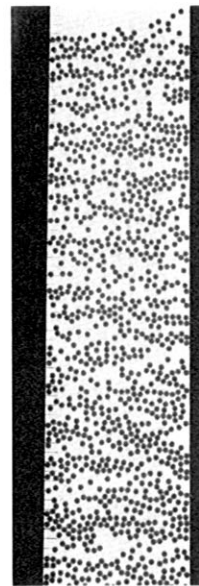


Figure 2

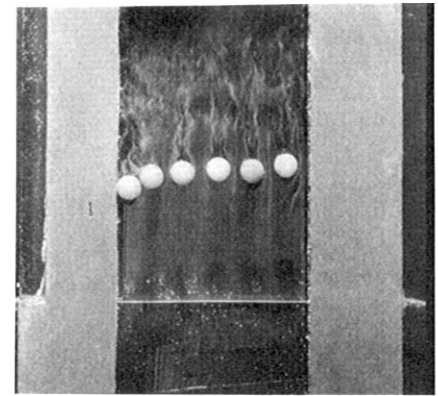


Figure 3

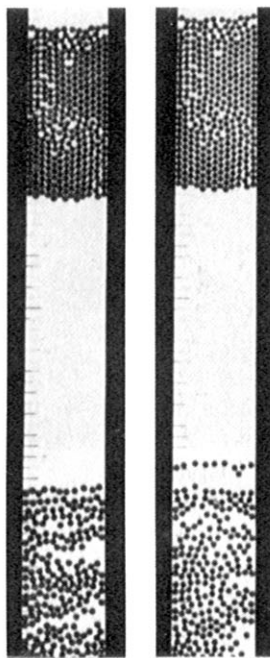


Figure 4

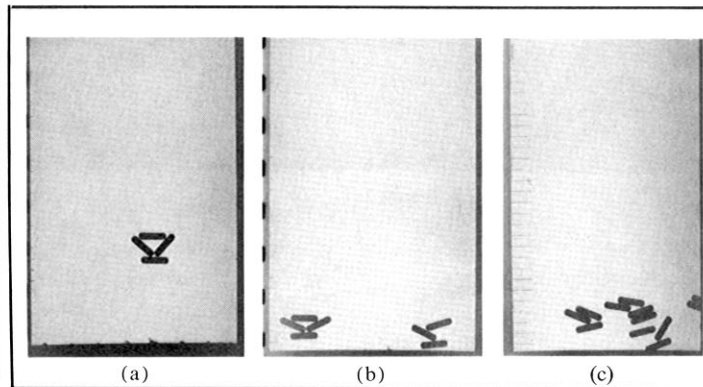


Figure 5

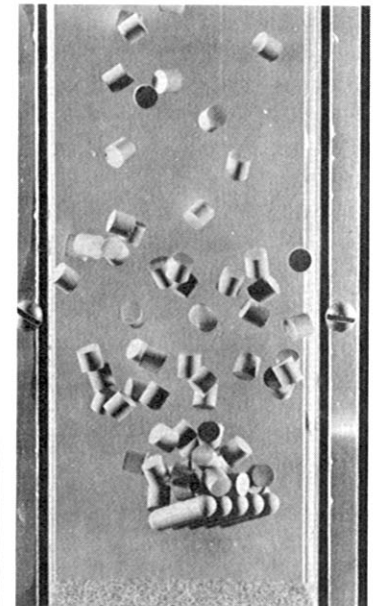


Figure 6

## NONLINEAR MECHANICS OF FLUIDIZATION OF SPHERES, CYLINDERS, AND DISKS IN WATER

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In all the figures, particles fluidized by water are held in place by a balance of weight and drag. In all the figures except Figs. 1 and 6, the particles are constrained by closely spaced side walls to motion in two dimensions.

The motion of particles is dominated by wakes and turning couples. Wakes are important in the first part of the scenario we call drafting, kissing, and tumbling (Fig. 1). The second sphere is drafted into the wake of the first (a), they kiss (b), and then tumble (c). The falling motion of kissing spheres is unstable to couples of the type that turn long bodies broadside on. The kissing spheres tumble because they are a long composite body. Neighboring spheres with centers aligned cross stream appear to have the greatest stability. Figure 2 shows a fully expanded bed of plastic beads at  $Re = 300$  and voidage = 0.88. Most spheres line up in horizontal arrays. Vertical pairs are drafting. The stability of

single lines of spheres constrained to move in two dimensions is amazing. Figure 3 shows that these lines can be stationary even though there is considerable unsteady activity in the wakes. Figure 4 shows fluidization of plastic beads in a 3 in. wide two-dimensional bed inclined  $23^\circ$  from the vertical at  $Re = 290$ . The spheres at the top of the bed are stable and close packed forming a shock wave of voidage. The bottom configuration with stable cross-stream rows is in an expanded state. Figure 5 shows cylinders fluidized at  $Re = 650$ . Long cylinders are stable broadside on. Wake interactions can produce the stable stationary architectures shown in (a) and (b) or the "log stacking" broadside on configuration shown in (c). In Fig. 6 a "raft" of glued cylinders turns broadside on and short cylinders are sucked into its wake like debris behind a truck.

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