INDEPENDENT CONFIRMATION THAT DELAYED DIE SWELL IS A HYPERBOLIC TRANSITION

by

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Abstract

We measured shear wave speeds in the same aqueous Xanthan solutions used to study delayed die swell by Allain, Cloitre, Perrot and Quemada [1993]. They reported delayed die swell for solutions of 500, 1000, 2000 and 4000 ppm Xanthan in water when the shear rate, 8U/d, was above a critical value $8U_C/d$, where U is the mean velocity through a round pipe of diameter d=0.6 mm and U_C is a critical value. Their photographs of delayed die swell are exactly like those reported by Joseph, Matta and Chen [1987] for 17 different polymer solutions, by Joseph [1990] for 18 and by Hu et al. for M1. Allain et al. sent us samples of the four solutions, prepared by them, and we measured wave speeds on our meter. As in all the earlier experiments, the delay occurs when the centerline velocity in the pipe $\tilde{U} = 2U$ is greater than the wave speed C, and after a distance L the swell is complete and the mean velocity is less than C. This adds four more cases, 23 in all, to those for which the delay can be correlated with the wave speed measured on our meter. So far 23, and only 23, cases have been studied and they are all consistent with calculations from theory giving rise to hyperbolicity and change of type.

1. Introduction

Delayed die swell is graphically described in the cartoon and caption of Figure 1.

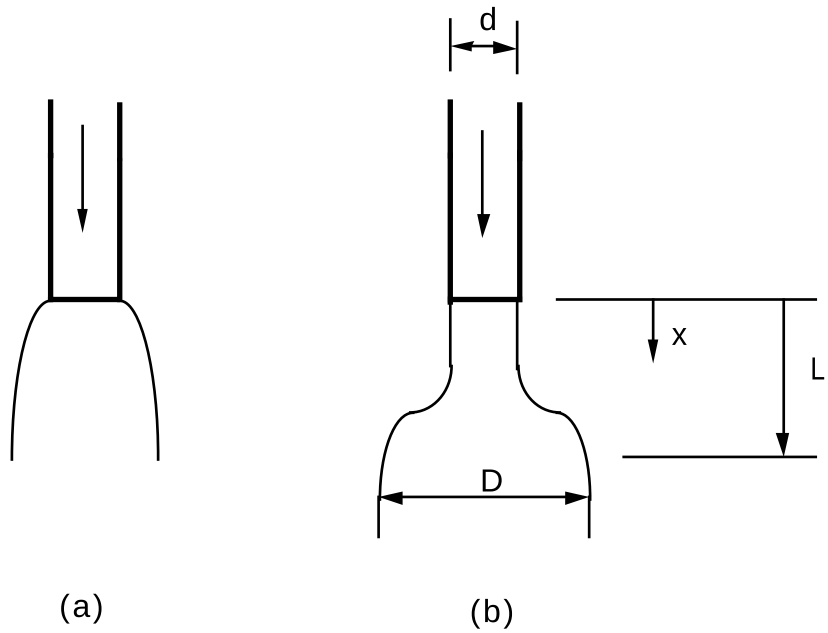

Figure 1. Cartoon of delayed die swell with a not-too short time of relaxation: (a) Subcritical, (b) Supercritical. The speed at the centerline of the pipe is greater than the wave speed before the delayed swell and less than the wave speed at x=L after the swell.

There have been many photographs and some discussions of delayed die swell over the years, but the first systematic study is by Joseph, Matta and Chen [JMC, 1987]. In their summary they wrote that

The experiments reported here establish that there is a general critical condition associated with die swell which we call delayed die swell. This condition is defined by a critical speed which is the area-averaged velocity, the extrusion velocity, at the exit of the pipe when the swell is first delayed. The delayed swell ratio and delay distance first increase for larger, post-critical values of the extrusion velocity; then the increases are terminated either by instabilities or by smoothing. The maximum post-critical velocity at the pipe exit was always greater than the shear wave speed measured on the shear-wave-speed meter. The post critical area-averaged velocity at the position of maximum swell before termination was always less than the shear wave speed. There were always points in the region of swelling where the ratio of the local velocity to the shear wave speed, the viscoelastic Mach number, was unity. The swelling of the jet is a nonlinear phenomenon which we suggest is finally terminated either by instability or when the variations of the velocity, vorticity and stress field are reduced to zero by the inward propagation of shear waves from the free surface of the jet. This propagation is generated by discontinuous "initial" data along x in which the prescribed values of velocity at the boundary change from non-slip in the pipe to no-shear in the jet. The measurements raise the possibility that the delay may be associated with a change of type from supercritical to sub critical flow.

A convenient and detailed discussion of the history of research on delayed die swell can be found in the recent book on the fluid dynamics of viscoelastic liquids by Joseph [J, 1990]. Further results which show that the celebrated M1 fluid are like the other 18 are given by Hu, Riccius, Chen, Arney and Joseph [1990]. In this paper we show that the most recent results of Allain, Cloitre, Perrot and Quemada [ACPQ, 1993] on delayed die swell in Xanthan solutions correlate perfectly, like all the other 19 cases so far treated, with wave speeds measured on our wave-speed meter (US patent 4,602,502). These results were summarized already in the abstract of this paper and are the focus of this paper.

The theoretical idea that we are promoting is that the delay is a critical hyperbolic phenomenon (resembling a hydraulic jump) associated with a change of the type of the steady vorticity equation analogous to transonic flow. Yoo and Joseph [1985] showed that the vorticity equation for an upper convected Maxwell model changes type from elliptic to hyperbolic, in a region in the center of a channel, when the centerline velocity of the Poiseuille flow exceeds the speed $C=\sqrt{\eta/\rho\lambda}$ of vorticity waves into regions at rest. Ahrens, Yoo and Joseph [1987] extended this result to round pipes. Chen [1993] has derived similar change of type criteria for two viscoelastic liquids which are extruded from a pipe.

Delvaux and Crochet [1989] have done a numerical study of delayed die swell in a plane jet using an Oldroyd B model, the upper convected model with a small retardation time. Their results confirm the ideas outlined above and add some new understanding. The main new result can be described as "the breakout of the region of hyperbolic vorticity". At small supercritical values for which the centerline velocity in the channel is somewhat larger than the wavespeed, the hyperbolic region in the channel center extends slightly downstream into the jet at $x>0$, but does not touch the free surface at the boundary of the jet. As the velocity increases, more and more of the jet is consumed by the hyperbolic region. At a certain velocity, the hyperbolic region first touches the jet boundary, then consumes more and more of the jet boundary. Evidently the change in the curvature of the free surface of the jet is associated with the breakout of the hyperbolic region. This explains why the delay is not observed at small supercritical values of the velocity (M>1) but only at larger postcritical breakout values. Other features of the experiments have yet to be elucidated by theory and simulation.

The results given in this paper are direct measurements and are independent of constitutive hypotheses.

2. List of symbols

The following list are symbols which were introduced by ACPQ [1993] and others already defined in J [1990].

C speed of shear (vorticity) waves $C=\sqrt{G/\rho}=\sqrt{\eta/\lambda\rho}$

η viscosity of Maxwell model

d (=0.6 mm) inside diameter of the pipe used in the experiment of ACPQ. Their d_i is our d.

D Final diameter of the swelled jet, see figure1.

G Shear modulus or rigidity. For a Maxwell model $G=\eta/\lambda$. G_C is the effective value given by $G_C=\rho C^2$.

$\dot{\gamma}_a$ (=8U/d) apparent shear rate

L distance from the pipe exit x=0 to the position of final swell, see figure 1.

λ relaxation time. We compute $\lambda=\eta/\rho C^2$ from measured values of η and C.

$\tilde{\mu}$ zero shear rate viscosity $\tilde{\mu}=\mu+\eta$ where μ is the Newtonian viscosity which other rheologists think should be the solvent's viscosity. In table 1 and other calculations we put $\mu=0$, $\tilde{\mu}=\eta$

M(0)   (=U/C) viscoelastic Mach number on the centerline at the exit.

M(x)   (=u/C) viscoelastic Mach number at any point

M(L)   (=ũ(L) /C) viscoelastic Mach number at x=L

u(x)   velocity

ũ(x)   area averaged velocity

U   (=ũ(0))mean extrusion velocity (=4Q/πd^2)

ũ(L)   (=U$(D/d)^2$) Flow speed at the position x=L of the final swell. The velocity is assumed to be uniform there.

3. Measurements of ACPQ [1993] of delayed die swell in Xanthan solutions

ACPQ [1993] measured properties of delayed die swell in solutions of Xanthan in deionized water in concentrations of 500, 1000, 2000, and 4000 ppm. The critical velocities and swell ratios which we need are given in their Fig.5. In Figure 2 we have reproduced one panel from their figure to instruct our readers on the method we used to compute the entries in Table 1.

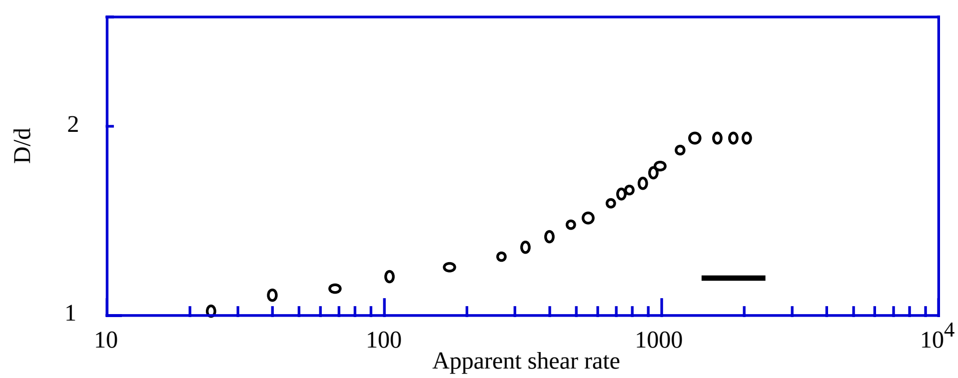

Figure 2. Sketch of a panel from figure 5 of ACPQ [1993] showing the swell ratio versus apparent shear rate for 2000 ppm Xanthan solution in deionized water. The thick bar give

the interval of uncertainty of $\dot{\gamma}_a$ where the delay is first observed. The vertical axis gives the swell ratio.

Concentration	D/d	$\dot{\gamma}_a$	$\tilde{U} = 2\overline{u}(0)$	$\overline{u}(L)$
500 ppm	1.44	202	3.03 cm/s	0.734 cm/s
	1.81	1120	16.8	2.55
1000	1.44	404	6.06	1.46
	1.75	1608	24.1	3.94
2000	1.84	1122	16.8	2.48
	1.95	2175	32.6	4.29
4000	2.01	2113	31.7	3.92
	2.10	3447	51.7	5.83

Table 1. Critical parameters for delayed die swell of Xanthan solutions after ACPQ [1993] (see caption of figure 2 for an explanation of the interval of uncerainty)

<u>4. Measurement of wave speeds</u>

The shear wave speed for Xanthan solutions was measured with the wave-speed meter in our laboratory. The meter allows us to determine transit times for an impulsively generated shear wave in a fluid at rest. The wave traverses the gap between two concentric cylinders. Transit time measurements are made for different gap sizes. A shear wave speed is defined if we measure one and the same speed over a range of small gaps.

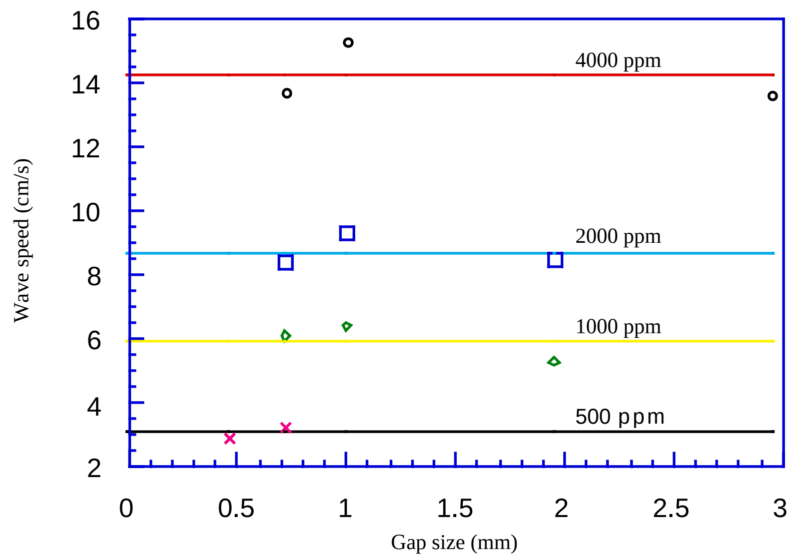

Figure 3. Wave speeds for Xanthan in deionized water

ppm	$\tilde{\mu}$ (Pa s)	$\bar{C}$ (cm/s)	G_c (Pa)	λ_c (10^{-3}s)	Gap (10^{-3} m)	C (cm/s)	T (^oC)	ρ (Kg/m^3)
500	0.028	3.1	0.96	29.2	0.467 0.724	2.9 3.2	24	998
1000	0.21	5.9	3.46	60.7	0.724 1.003 1.956	6.1 6.4 5.3	24	999
2000	5.66	8.7	7.57	748	0.724 1.003 1.956	8.4 9.3 8.5	24	1000
4000	129.0	14.2	20.2	6386	0.724 1.003 2.959	13.7 15.3 13.6	24	1003

Table 2. Properties of Xanthan solutions in deionized water

The theory of the wave speed meter was given by Joseph, Narain and Riccius [1986]. The apparatus and the measuring technique are described by Joseph, Riccius and Arney [1986] and in more detail by Riccius [1989]. All this has been summarized in the book by Joseph [1990]. We have recently modified the meter to facilitate the handling of samples and made other small improvements. Our new instrument is the same as the old one in essential details and it gives rise to basically the same speeds.

The results of our measurements are summarized in Figure 3 and Table 2. The speeds are computed as the ratio of the measured transit time to the gap size. The transit times are read off oscilloscope traces of the motion of reflected laser light focused on photodiodes from mirrors on the inner and outer cylinders. We take the wave speed as the average value over about ten trials. In figure 4 we have exhibited typical raw data for the 4000 ppm solution in a gap of 2.959 mm. In figure 5 we give raw data for the 2000 ppm solution in a gap of 1.003 mm and in figure 6 raw data for the 1000 ppm solution in a gap of 1.956 mm. All raw data is available upon request.

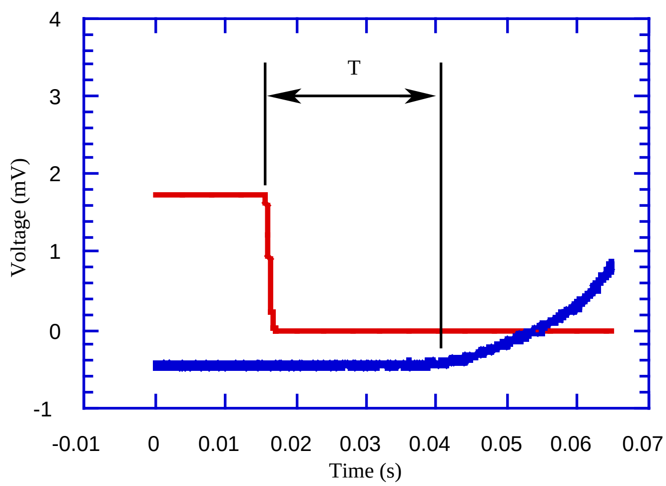

Figure 4. Transit time T of a shear signal to traverse a 2.959 mm gap in Xanthan solution at 24^{o} C. The speed is T/2.959. the average speed over different realizations [13.5, 13.9, 12.2, 15.1, 12.9, 14.1, 13.5, 13.6, 13.4, 13.7] is 13.6 cm/s

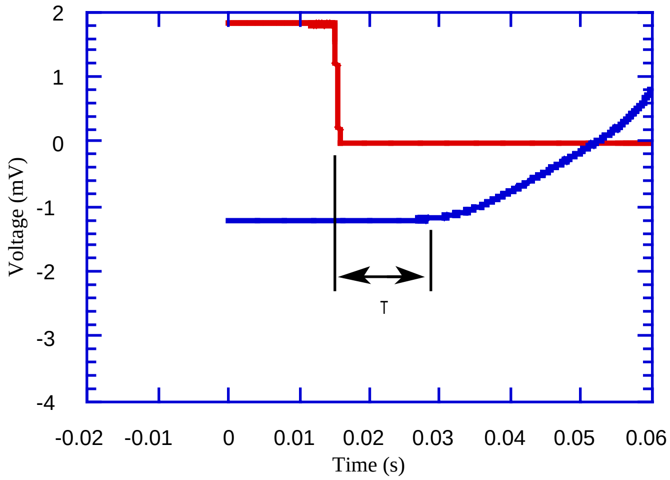

Figure 5. Transit time T of a shear signal to traverse a 1.003 mm gap in a 2000 ppm Xanthan solution at 24^{o} C. The speed is T/1.003. the average speed over different realizations [9.3, 9.4, 9.4, 9.2, 9.3, 9.5, 9.3] is 9.3 cm/s

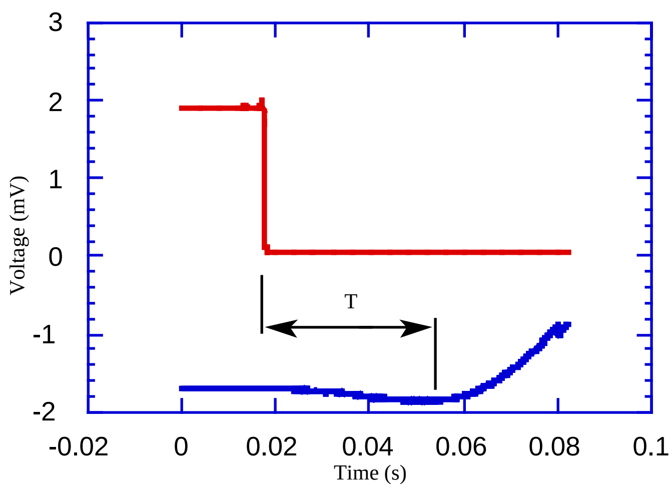

Figure 6. Transit time T of a shear signal to traverse a 1.956 mm gap in a 1000 ppm Xanthan solution at 24^{o} C. The speed is T/1.956. the average speed over different realizations [5.3, 5.2, 5.4, 5.4] is 5.3 cm/s

The wave speeds we measured in Xanthan solutions are relatively robust without too much scatter. Other fluids are more sensitive to the spectrum of the kick signal. We are, after all, dealing with dispersive waves so that the frequencies excited in a bang will disperse with different speeds and their dispersion depends on the fluid.

There are much more sophisticated instruments which are used to take dynamic measurements of propagation speeds of monochromatic waves of fixed but different frequencies. We have the idea that these sophisticated instruments cannot be used directly to measure how fast bangs will propagate and bangs are what you get in real flows. It would not be the first time that a coarse and robust instrument works better than a delicate and sensitive one.

5. Comparison of the critical velocities for delayed die swell measured by ACPQ [1993] with wave speeds measured on the wave-speed meter

The comparison is given as Table 3. The reader should recall that ACPQ [1993] give an interval of uncertainty for the critical velocity and swell ratio at delay. The wave speeds used here are taken from figure 3. The viscoelastic Mach number M(0) is greater than (or nearly) one at the lower limit of uncertainty and certainly greater than one at the higher limit. The Mach number M(L) after the swell is definitely less than one.

Concentration ppm	Wave speed C (cm/s)	Centerline velocity $\tilde{U}$(cm/s)	M(0)=$\tilde{U}$ /C	Final Velocity $\tilde{u}$(L) (cm/s)	M(L)=$\tilde{u}$(L) /C
500	3.1	3.03	0.98	0.734	0.24
		16.8	5.42	2.55	0.82
1000	5.9	6.1	1.03	1.46	0.25
		24.1	4.1	3.94	0.67
2000	8.7	16.8	1.93	2.48	0.29
		32.6	3.75	4.29	0.49
4000	14.2	31.7	2.23	3.92	0.28
		51.7	3.64	5.83	0.41

Table 3. Wave speed, critical velocity and Mach number before and after swell.

6.Discussion

Steady flow past bodies is another class of problems which appear to exhibit properties of hyperbolicity and change of type. This problem has been treated in the works of Ultman and Denn [1970], Joseph [1985], Delvaux and Crochet [1990], Hu and Joseph [1990] and Fraenkel [1988, 1991]. These and related matters are discussed in the book of Joseph [1990]. The nonlinear studies were based on the upper-convected Maxwell model, but the linearized studies were basically model-independent. They show that the flow goes supercritical when the viscoelastic Mach number $M=U/C$ passes through one; the vorticity equation of the steady flow changes type from elliptic when $M<1$ to hyperbolic when $M>1$. In the supercritical case there is a Mach cone of vorticity. In front of the cone there is a "region of silence", actually potential flow, with vortical flow behind the cone. The supercritical transitions do seem to correspond to the flow transitions observed in the experiments on the flow over wires reported by James [1967], James and Acosta [1970] and Ambari, Deslouis and Tribollet [1984], as well as to the flow features observed in the experiments of flow over flat plates of Hermes and Fredrickson [1967].

The experiments on anomalous transfer of heat and mass in the flow over small wires were carried out in dilute drag reducing solutions of polymers in water, say 10 to 100 ppm of polyethylene oxide in water. The critical issue posed by these experiments is the comparison of the critical values of the velocity above which anomalous behavior occurs with measured values of the wave speed. Some few values for wave speeds for very dilute solutions are presented in the tables at the end of the book by Joseph [1990], but they are not reliable. We need to find the techniques to do reproducible measurements of wave speeds in dilute solutions. Gas dynamics would look good on paper, but only there, if we could not measure the speed of sound.

Drag reduction is another field in which wave speeds are needed. Goldshtik, Zametalin and Shtern [1981] wrote a remarkable paper on near wall turbulence and drag reduction which also hangs crucially on the unsolved problem of the experimental determination of wave speeds in dilute solutions. Their analysis makes a small number of debatable assumptions but is entirely deductive, without empirical inputs. It is astonishing that calculations from this purely deductive theory in the Newtonian case give the values of the two constants that Nikuradse measured for the logarithmic law of the wall. When applied to the upper convected Maxwell model, their theory gives rise to a criterion for the inception of drag reduction: the friction velocity must be larger than the shear wave speed $C=\sqrt{\eta/\rho\lambda}$. Drag reduction starts when the viscoelastic Mach number based on the friction velocity passes through unity. In this case the slope of the mean velocity profile in the buffer layer increases. Goldshtik et al also considered near wall turbulence for an Oldroyd B fluid with different times of retardation. Again, there appears to be a criterion for onset which depends on the viscoelastic Mach number based on the friction velocity, but the critical value is larger than one and it increases with the retardation time. The effect of the retardation is to increase the thickness of the buffer layer. Elasticity then increases the slope of the velocity in the buffer layer and viscosity increases its thickness.

In recent works, Liu and Joseph [1992] found that long bodies which fall under gravity in viscoelastic solutions of moderate dilution undergo a tilt transition which can be interpreted as a change of type. This is another example of flow around bodies. When the particle is light and falls slowly it falls nose down with its long side parallel to gravity, but heavier particles put their long side perpendicular to gravity, just the opposite. Particles of intermediate weight fall with their axis at a fixed angle of tilt. Fortunately reliable measurements of the wave speeds of solutions in which tilt transitions occur can be made. In every one of many cases the transition commences when the fall velocity exceeds the wave speed, in all cases without exception.

Goldshtik et al [1981] gave an interesting interpretation for the criterion for the onset of drag reduction in a Maxwell fluid. They said that there are two lengths to consider, a viscous length ν/u^* where ν is the kinematic viscosity and u^* is the friction velocity, and an elastic length λu^*. Onset occurs when the elastic length exceeds the viscous one.

$$\lambda u^* > \nu/u^*$$

We can apply the same idea to the tilt transition by claiming onset when

$$\lambda U > \nu / U$$

The question is what λ should be used. Our answer is the $\lambda = \nu / C^2$ which we get by measuring ν, and C on our meter. Then, $M^2 = U^2/C^2 > 1$, correlating unrelated dynamic data from many different experiments.

Acknowledgments

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