THE EFFECTIVE DENSITY AND VISCOSITY OF A SUSPENSION

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Abstract

This paper presents results of a series of experiments on the settling velocity of spheres in two-component solid-liquid suspensions. Particular emphasis has been given to the effective values of density and viscosity of the mixture which allows us to describe the settling of the spheres in the mixture using appropriate modifications of the equations valid for the settling of spheres in pure fluids.

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Introduction

In this paper we give the results of modeling a monodisperse suspension of spheres as an effective, pure fluid with properties computed from the suspension. Particular emphasis has been given to the effective values of density and viscosity of the mixture which allows us to describe the settling of single spheres in the mixture using appropriate modifications of the equations valid for the settling of spheres in pure fluids. The idea explored here is motivated by the work of Di Felice, Foscolo, Gibilaro and Rapagna [1991]. They consider a binary-solid suspension idealized by two particle diameters with the fluid plus small particles modeled as an effective or pseudo-fluid. The density and combined flux of the fluid plus smaller particles were modeled as simple mixtures, using well-known linear weights for the constituents, and the viscosity of the pseudo-fluid was modeled with the cell model of Happel [1957]. They then tested the pseudo-fluid concept against four different experiments. The first experiments were on the settling velocity of a single particle in a fluidized bed in which the fluidized suspension was represented as the upward motion of the pure pseudo-fluid and the terminal velocity of the falling sphere was computed from the Dallavalle correlation with velocities measured relative to the pseudo-fluid velocity. The second type of experiment was on the fluidization of two distinct sizes of particles of the same density with predictions for the upward velocity of the large particle modeling the small particles plus fluid as a pseudo-fluid using the monocomponent correlation of Rapagna, et al. [1989] with the terminal settling velocity of a single large particle in the pseudo-fluid. A third set of experiments focused on the circulation of a bidisperse suspension in a circulating fluidized bed, and the fourth set focused on the sedimentation of binary-particle mixtures, modeled as sedimentation of the large particles through the pseudo-fluid. In all cases, the effective equations for the pseudo-fluid yielded predictions which are in remarkably good agreement with experiments.

Kothari and Turian [1983] considered the settling of particles of different sizes with the same density. They used the Richardson-Zaki correlation for the slip-velocity of the particles relative to the fluid and proposed that the Stokes settling velocity for a particle of species *i* be modified by replacing the fluid density ρ_w in buoyancy force with the average density of a suspension consisting of the fluid and the particles smaller than that of species *i*. They found very good agreement between their model and experiments.

Sengun and Probstein [1989] modeled a polydisperse coal slurry as a bimodal dispersion in which the fluid and colloidal particles are regarded as a single composite, non-Newtonian fluid, and the large non-colloidal particles as a monodispirse (polydisperse), non-colloidal dispersion. Since the colloidal particles tend to be in suspension by Brownian motion, the composite fluid is something like a mixture of miscible liquids or more like a polymeric surfactant solution. They found excellent agreements between the predictions of their theory and experiments on coal slurries. They did not introduce the notion of an effective density probably because the motion of slurries is not driven by gravity, but the fluid plus colloidal particles do form an effective fluid with a composite density.

The present work differs from others in several ways. We are able to back out an effective viscosity μ_e and density ρ_e of a fluidized suspension by linear regression of measured values of the fall velocity of test spheres and rise velocity of test bubble through the suspension, fitting the experimental data to an empirical formula of Francis [1933] for the sedimentation of a single particle in a channel filled with pure fluid with the fluid viscosity μ_f and density ρ_f replaced μ_e and ρ_e. When the test particle is large relative to the suspended particle, the measured values of μ_e and ρ_e are just the ones which are expected from the empirical formula of Thomas [1965] for viscosity and the composite density of the mixture is linear in the volume fraction. Moreover, using the same effective values of the viscosity and density, we are able to modify the correlation between the drag coefficient and Reynolds number given by Barnea and Migrahi [1973], which applies in fluidization and sedimentation to describe the settling of particles in a fluid-solid suspension. Again, the agreements are good when the test spheres are rather larger than the suspended spheres, but the discrepancies increase as the ratio of diameters of test to suspended spheres decreases.

A proper theory of the effective density of a mixture ought to identify the borders of the applicability of the theory and the ratio of the sizes of test to suspended particles is one such border. The concept of effective density is an interpretation of Archimedes' principle and the different interpretation of it are controversial. Suppose we have two fluids and they are arranged in layers as in Figure 3. Never mind that the heavy fluid will sink. It may sink very slowly relative to the time scale of the thought experiment described in the caption to Figure 3.

The layered configuration is not completely appropriate to fluidized beds and suspensions. For these we may think of small particles giving rise to a relatively smooth pressure gradient close to the mean pressure gradient, as in Figure 4.

In a fluidized bed, one wants a formula for the drag on a particle. If the particle is fluidized in the compositive fluid under steady conditions, then the drag D on a particle is equal to the effective weight We,

$$D = \mathrm{We} = \left(\rho_P - \tilde{\rho}\right) g V_P \tag{1}$$

where ρ_P is the particle density, V_P is the particle volume and $\tilde{\rho}$ is the effective density of the composite fluid.

There is a controversy about the effective density in the case when the fluidized solid particles have the same volume V_P and density ρ_P as the test particle with drag D given by (1). Foscolo, Gibilaro and Waldram [1983] claim that

$$\tilde{\rho} = \overline{\rho} = \rho_f \varepsilon + (1-\varepsilon)\rho_P \tag{2}$$

where ρ_f is the fluid density, ε is the fluid fraction or voidage and $\overline{\rho}$ is the mass average density. Since $\rho_P - \overline{\rho} = \varepsilon\left(\rho_P - \rho_f\right)$, we have

$$\left(\rho_P - \overline{\rho}\right) g V_P = D_G = \varepsilon\left(\rho_P - \rho_f\right) g V_P = \varepsilon D_c \tag{3}$$

where D_c is the correct drag according to Clift, et al. [1987], who claim that $\tilde{\rho} = \rho_f$.

The point of controversy appears at first glance not to have substance. It depends on a decision about the decomposition of the total weight of a particle into buoyancy and drag

$$\rho_P V_P g = \tilde{\rho} V_P g + \left(\rho_P - \tilde{\rho}\right) V_P g \tag{4}$$

where $\tilde{\rho} = \overline{\rho}$ for Gibilaro and $\tilde{\rho} = \rho_f$ for Clift. Since $D_G = \varepsilon D_c$, both choices are correct within the context of their own definitions. The choice $\tilde{\rho} = \overline{\rho}$ made in the decomposition becomes important in deriving approximate theories in which the drag formula depends on the choice. Foscolo and Gibilaro [1984, 1987] generalized the Richardson-Zaki correlation into an unsteady drag law $D_{PG} = D(u_P, \varepsilon)$

depending on the particle velocity u_P and fluid fraction ε. They used this drag to get an expression for bubbling in a fluidized bed. Clift et al. [1987], choosing $\tilde{\rho} = \rho_f$, find a drag $D_C = D_{PG}/\varepsilon$ and an expression for bubbling which also differs by a factor of ε.

(Figure 3 from "Studies of Two-Phase Flows of Solids and Liquids [Joseph])

Figure 1. The dark layers are of heavy fluid, the light areas are of light fluid. A large sphere will be buoyed up by the effective density of the composite fluid. There is a mean pressure gradient made of steps. If the small sphere moves relative to the media, it will sometimes be buoyed up by light fluid and sometimes by heavy fluid. The small sphere in the picture is buoyed up by the light fluid.

(Figure 4 from "Studies of Two-Phase Flows of Solids and Liquids [Joseph])

Figure 2. The particles are heavier than the fluid. They fall unless fluidized by a stream moving up against gravity. A large sphere will be buoyed up by the effective density of the composite fluid. What will happen to a small sphere? Will it be buoyed up as an effect of the mean pressure gradient?

There is therefore a genuine issue in the controversy about effective density. We are dealing with approximate theories in which the 3-D dynamic is replaced by an equivalent 1-D theory, based entirely on an expression for the drag on a particle in a fluidized suspension. The question is: What is the best equivalent? Here the choice makes a difference and there is a "best" choice, where best need not be all that good. There will be an error due to approximation, and the smallest error could be large in some problems.

Experimental

Low Reynolds numbers

An effective fluid was created by suspending glass beads in a highly-viscous liquid. The liquid (Dow Corning 200 oil) had a viscosity of 12.2 Pa s and a density of 975 kg m^{-3}. The diameter of the suspended particles was approximately 800 μm and each one weighted 2,140 kg m^{-3}. These suspended particles settled only a few centimeters in one hour, in sharp contrast to the settling velocity of centimeters per second achieved by the test particles. The vertical sedimentation channel used in these experiments was of Plexiglas, 37-cm high and of rectangular cross-section, 8.5-cm wide and 5.5-cm deep. Each concentration of glass beads in Silicon oil was thoroughly mixed in a separate container and poured into the sedimentation channel just prior to each experiment. Tests were started some minutes later after allowing entrained air to rise.

The solid fluid mixtures used had a certain degree of opacity and therefore strong back lighting was provided to track the settling particles and measure their falling velocity. A certain care had to be given to the duration of the lighting time. Long exposure to light caused overheating of the system and the reduction of the fluid viscosity, in spite of the small thermal sensitivity of the viscosity of the Dow Corning 200 fluids to temperature.

The test particles were made of different materials with a high degree of sphericity and a narrow diameter tolerance. Other experiments were carried out using bubbles of fluid as test particles. Bubbles of watered colored with blue ink were injected in the channel with calibrated pipet. The precision of the injected volume was ±0.05 ml. The weight was measured in advance for each dropped particle. Settling velocities were evaluated either by measuring the time for the particle to pass by a fixed reference position with a stopwatch or by following the particle from the image taken with a high-speed video recording system.

High Reynolds number

Sedimentation of particles at higher Reynolds numbers was carried out in water-fluidized beds. The column used was 1-m high and 5.17-cm ID, made of transparent plastic material. The distributor was made with a 2-cm high fixed bed of 3-mm lead shots held down by a steel net and a fabric cloth. The water outlet was on one side of the column, 10-cm below the top, which was open to allow the dropping of test particles. Glass beads of 6.35-mm diameter and 2,453 kg m^{-3} density and glass beads of 800μm and 2,410 kg m^{-3} density were fluidized. The same kind of test particles and the same kind of measurement techniques were used in the fluidized bed and sedimentation bed. The expansion curves of the two beds are given in Figure 3. According to Richardson and Zaki [1954]

$$\frac{U}{u_o} = \varepsilon^n \tag{5}$$

where u_o is the terminal velocity of one sphere when the solid fraction is zero and $\varepsilon = 1 - \phi$ is the fluid fraction. From a regression of our experimental data we obtained values of $n = 2.26$ and $u_o = 0.375$ m s^{-1} for the 0.635 cm beads and $n = 3.09$ and $u_o = 0.120$ m s^{-1} for the 800 μm beads. Solid test particles and measurement techniques similar to those used in the sedimenting bed were used in the fluidized bed.

Theoretical equations

Low Reynolds numbers

The terminal velocity, u_p, of a single spherical particle at low Reynolds numbers in an infinite fluid is given by the Stokes equation

$$u_p = \frac{g(\rho_p - \rho_f)d_p^2}{\gamma \mu_f} \tag{6}$$

where g is the acceleration due to gravity, ρ_p is the particle density, ρ_f is the fluid density, d_p is the particle diameter, μ_f is the fluid viscosity and γ is a coefficient which depends on the fluid and particle viscosity: $\gamma = 12\left[\left(\mu_f + \tfrac{3}{2}\mu_p\right)/\left(\mu_f + \mu_p\right)\right]$. γ=18 in case of solid particles ($\mu_p = \infty$) and γ=12 in the case $\mu_p << \mu_f$, as in the case of the water bubble that we injected in silicon oil and silicon oil-glass mixtures. This means that in our experiments γ was always known and independent of the medium and the particle viscosity. It is well known that when the particle is confined in a small channel, the fall velocity of the particle is smaller due to the displacement of the fluid in the direction opposite to the particle motion. A modification of (6) proposed by Francis [1933] for small containers can be used to account for the back flow on the particle fall velocity

$$u_p = \frac{g(\rho_p - \rho_f)d_p^2}{\gamma \mu_f}\left(1 - \frac{d_p}{D_c}\right)^{2.25} \tag{7}$$

where D_c is the channel diameter. Equation (3) can be used to measure the viscosity of a fluid of known density. If both density and viscosity are unknown, more than one experiment is required. To facilitate the interpretation of experiments, it is convenient to rearrange (7) in the following way:

$$\rho_p = \rho_f + \mu_f \frac{\gamma u_p}{g d_p^2}\left(1 - \frac{d_p}{D_c}\right)^{-2.25} \tag{8}$$

We can plot the data from experiments using different particles in a diagram were the abscissa is given by $\gamma u_p / g d_p^2 \left(1 - d_p / D_c\right)^{2.25}$ and the ordinate is the particle density. Equation (8) shows that data should fall on a line whose slope is the fluid viscosity and the intercept is the fluid density. Equation (8) can be also used to correlate the sedimentation of particles in solid-fluid mixtures. In this case, however, values of ρ_f and μ_f should to be replaced with effective values ρ_e, the effective density of the mixture, and μ_e, the effective viscosity of the mixture.

Many authors (i.e. Richardson and Meikle 1961, Barnea and Mizrahi 1973, Di Felice et al. 1991) have suggested and shown that with some fitting of experimental results, the effective density of a suspension is given by the mean density of the mixture

$$\rho_m = \rho_f (1 - \phi) + \rho_s \phi \tag{9}$$

where ρ_s is the density of the solid and ϕ is the solids fraction. Brenner [1958] has shown that in creeping motion $\rho_e = \rho_m$ only when particles are uniformly distributed in the channel cross section.

Several equations have been proposed for the mixture viscosity μ_m. An extensive review of the subject is given by Barnea and Mizrahi [1973]. The Einstein equation for dilute suspensions of spherical particles is well known, but its validity cannot be extended to solid concentrations of order 0.1 or higher. For $\phi < 0.25$, Thomas [1965] showed that the mixture viscosity could be calculated with the equation

$$\mu_m = \mu_f \left(1 + 2.5\phi + 10.05\phi^2\right) \tag{10}$$

with an accuracy of over 97.5% of the measured value of the viscosity of the mixture.

All Reynolds numbers

Barnea and Mizrahi [1973] proposed a model to describe the expansion of the fluidized bed. In that model, the Reynolds number-drag coefficient relationship for a single solid particle $C_{Do} = C_{Do}(\mathrm{Re}_o)$ is extended to a particle falling in a suspension of other similar particles. Corrected expressions for the Reynolds number Re_ϕ and the drag coefficient $C_{D\phi}$ are used to account for the different fluid dynamic conditions to which the particles are subjected. The expression for the Reynolds number is

$$\mathrm{Re}_\phi = \frac{u_\phi\, d_s\, \rho_f}{\mu_m} = \mathrm{Re}_o \left(\frac{u_\phi}{U_o}\right) \left\{\frac{1}{\exp\left[5\,\phi / 3(1-\phi)\right]}\right\} \tag{11}$$

where u_ϕ is the relative average interstitial velocity between the suspended particles and the fluid and d_s is the diameter of the suspended particles. The term in round brackets in (7) is the correction term for the actual fluid velocity, while the term in curly brackets accounts for the modified fluid viscosity. Barnea and Mizrahi [1973] obtained the numerical coefficients for the viscosity term from a regression of experimental data on the expansion of fluidized beds and sedimenting velocities of settling beds in creeping flow. The drag coefficient for a particle subject to gravity is given by

$$C_{D\phi} = \left[\frac{4 d_s\,(\rho_s - \rho_m) g}{3 \rho_m\, u_\phi^2}\right] \frac{1}{\left\{1 + \phi^{1/3}\right\}} = C_{Do} \left(\frac{u_\phi}{U_o}\right)^{-2} \left[\frac{(\rho_s - \rho_m)}{(\rho_s - \rho_f)}\right] \frac{1}{\left\{1 + \phi^{1/3}\right\}} \tag{12}$$

where the term in round brackets has the same meaning as before, the term in square brackets account for the effective density of the mixture and the term in curly brackets accounts for the wall effect generated by the surrounding particles. Equations (11) and (12) were tested by Barnea and Mizrahi [1973] on fluidization and sedimentation data found in literature and the tests showed good agreement between measured values and in a range of Re_ϕ between 10^{-4} to 10^3 using the empiral drag law

$$C_{D\phi} = \left(0.63 + \frac{4.8}{\sqrt{\mathrm{Re}_\phi}}\right)^2 \tag{13}$$

Equations (11) and (12) can be modified to describe the settling of solid particles in two-component suspensions by replacing d_s and ρ_s in (11) and (12) with the test particle diameter d_p and density ρ_p and by replacing u_ϕ in (11) and (12) _________. After making these replacements, we find that

$$\mathrm{Re}_p = \frac{(u_p - u_f)d_p \rho_f}{\mu_m} = \mathrm{Re}_o \left(\frac{u_p - u_f}{U_o}\right)\left\{\frac{1}{\exp[5\phi/3(1-\phi)]}\right\} \tag{14}$$

$$C_{Dp} = \left[\frac{4d_p(\rho_p - \rho_e)g}{3\rho_m(u_p - u_f)^2}\right]\frac{1}{\{1+\phi^{1/3}\}} = C_{Do}\left(\frac{u_p - u_f}{U_o}\right)^{-2}\left[\frac{(\rho_p - \rho_e)}{(\rho_p - \rho_f)}\right]\frac{1}{\{1+\phi^{1/3}\}} \tag{15}$$

where $u_f = U/\varepsilon$. Equation (15) is coupled with an empirical drag law of the form

$$C_{Dp} = \left(0.63 + \frac{4.8}{\sqrt{\mathrm{Re}_p}}\right)^2 \tag{16}$$

Equations (14) and (15) with $\rho_e = \rho_m$ is one estimate of the settling velocity of solid particles in effective fluid models of solid-fluid mixtures at any Reynolds number. In creeping flow, the main difference between (8) and (14) through (17) is the wall correction. In (8) the velocity is corrected for wall effects by the coefficient $\{1 - d_p/D_c\}^{-2.25}$ while in (14) through (16) it is corrected for similar effects from other particles by the coefficient $\{1+\phi^{1/3}\}$. If we replace the last factor in (15) with a wall correction factor, we find that

$$C_{Dp} = C_{Do}\left(\frac{u_p - u_f}{U_o}\right)^{-2}\left[\frac{(\rho_p - \rho_e)}{(\rho_p - \rho_f)}\right]\left\{1 - \frac{d_p}{D_c}\right\}^{2.25} \tag{17}$$

Equations (14), (16) and (17) with $\rho_e = \rho_m$ is a possible variant of our theory (14), (15) and (16) for settling velocity.

Experimental Results

Equation (8) was tested against a preliminary set of experiments in pure silicon oil. The results are given in Figure 4. Near the intercept, data points for experiments with water bubbles overlap with data points for experiments for plastic particles. A linear regression of the data gives values of fluid density $\rho_f = 969$ kg m^{-3} and of fluid viscosity $\mu_f = 12.1$ Pa s, which are very near to the actual values.

The linear correlation coefficient $r = 999$ shows that the linear model describes fairly well the experimental data in the conditions tested.

Two mixtures of glass beads and silicon oil were tested, one at $\phi = 0.1$ and the other at $\phi = 0.2$. The results are given in Tables 1, 2 and 3 and Figures 5 and 6. Hollow circles in Figure 4 represent experiments carried out with water bubbles (Table 3). Water bubbles rise and solid fall in our mixtures. For $\phi = 0.1$ (Figure 5, Table 1) values of fluid properties from the linear regression are $\rho_e = 1.11 \cdot 10^3$ kg m^{-3} and $\mu_e = 14.4$ Pa s. These values compare with $\rho_m = 1.10 \cdot 10^3$ kg m^{-3} and $\mu_m = 16.5$ Pa s evaluated from (9) and (10). For $\phi = 0.2$ (Figure 6, Tables 2, 3), regression values of fluid properties are $\rho_e = 1.27 \cdot 10^3$ kg m^{-3} and $\mu_e = 20.2$ Pa s. These compare with $\rho_m = 1.21 \cdot 10^3$ kg m^{-3} and $\mu_m = 23.2$ Pa s. These compare with $\rho_m = 1.21 \cdot 10^3$ kg m^{-3} and $\mu_m = 23.2$ Pa s from (9) and (10).

For all the data with $\phi = 0.1$ and $\phi = 0.2$, Re_p and C_{Dp} were evaluated first using (14), (15) and (16), and then using (14,) (16) and (17). A comparison of these two methods of evaluation is shown in Figure 9. The agreement seems satisfactory. It appears that in the conditions tested (15) and (17) produce similar results, and we differentiate between the two on the basis of our experimental results. Values of Re_p and C_{Dp} evaluated according to (14) and (15) are reported in Tables 1 and 2 as well.

Data from experiments carried out in the fluidized bed are given in Tables 4 and 5. Evaluation of this data using (14) and (15) are compared with (16) in Figures 8 and 9 and in Tables 4 and 5. Agreement between theory and experiment for fluidized beds of spheres is better for the 800 μm glass beads (Figure 8) than for the 0.635 cm glass beads (Figure 9). A possible interpretation of this difference in behavior may be found in the different in the diameter d_p of test particles and the diameter d_s of suspended particles. When $d_p / d_s \cong$ 10-20 and when d_s = 6.35 mm, $d_p / d_s \cong$ 1-2 with d_s = 800 μm. This suggests that the earlier model may be valid only in the limiting case $d_p / d_s >> 1$. Inspection of (15) indicates that the big discrepancy between the model and the data in the bed of 6.35 mm particles cannot be simply overcome with the use of an effective density $\rho_e \neq \rho_m$, which varies in the range $[\rho_f, \rho_m]$. This implies that in the conditions tested, there is a failure of the whole model.

In Figures 8 and 9, experimental results interpreted by (14) and (17) are reported as well. The difference between (15) and (17) is small, although results obtained with (17) appear to be slightly more scattered.

Discussion and Future Developments

Experiments on the settling of particles in fluidized suspensions correlate fairly well with (8) and (17) when ρ_f and μ_f are replaced with effective value ρ_m and μ_m.

Results of experiments at low Reynolds number tend to sustain the hypothesis that the effective density experienced by the falling particles is the average density of the suspension expressed by (9 nevertheless scatter of data is considerable and likely to affect values of the effective density extrapolated with (8). Experiments carried out with rising water bubbles help in the evaluation of the intercept by placing points near to the intercept of (8).

A possible effect of the diameter ratio of sedimenting to suspended particles has been highlighted by sedimentation experiments in water fluidized beds.

Work will proceed completing experiments with water bubbles in silicon oil mixtures and trying to better assess the effect of the d_p/d_s ratio by using different and as big as possible particles to make the silicon oil-solid mixtures.

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Particle	u_p (10^{-3} m s^{-1})	$\frac{18u_p}{gd_p^2}\left(1-\frac{d_p}{D_c}\right)^{2.2}$ (10^2 s m^{-2})	Re_p (10^{-3})	C_{Dp} (10^3)
teflon	1.14	0.651	0.480	46.9
d_p =6.35 mm	1.06	0.606	0.447	54.5
ρ_p =2150 kg m^{-3}	1.18	0.627	0.496	44.2
	1.54	0.877	0.649	26.0
	1.19	0.681	0.502	43.6
aluminum	1.80	1.03	0.758	30.4
d_p =6.35 mm	1.97	1.12	0.830	25.6
ρ_p =2790 kg m^{-3}	2.09	1.19	0.880	22.7
	2.02	1.15	0.852	24.3
	2.01	1.15	0.848	24.5
aluminum	7.52	1.01	7.98	4.16
d_p =16.0 mm	8.80	1.18	9.34	3.05
ρ_p =2700 kg m^{-3}	9.07	1.22	9.62	2.85
	8.46	1.13	8.98	3.30
	8.85	1.19	9.39	3.01
ceramic	7.46	1.68	5.51	4.99
d_p =11.1 mm	8.50	1.91	6.28	3.85
ρ_p =3820 kg m^{-3}	9.14	2.06	6.75	3.33
	8.09	1.82	5.97	4.23
	8.91	2.01	6.58	3.50
steel	7.32	4.18	3.09	7.13
d_p =6.35 mm	7.82	4.47	3.30	6.24
ρ_p =7660 kg m^{-3}	8.25	4.71	3.48	5.60
	7.66	4.38	3.23	6.52
	8.30	4.74	3.50	5.56

Table 1. Sedimentation data in glass-silicon oil mixtures, ϕ = 0.1.

Particle	u_p (10^{-3} m s^{-1})	$\frac{18u_p}{g d_p^2}\left(1-\frac{d_p}{D_c}\right)^{2.2}$ (10^1 s m^{-2})	Re_p (10^{-3})	C_{Dp} (10^3)
teflon	0.996	5.69	0.333	50.9
d_p =6.35 mm	0.983	5.62	0.329	52.3
ρ_p =2150 kg m^{-3}	1.00	5.74	0.336	49.7
aluminum	1.36	7.77	0.455	45.8
d_p =6.35 mm	1.46	8.33	0.488	39.9
ρ_p =2790 kg m^{-3}	1.15	6.58	0.385	63.9
	1.20	6.86	0.402	59.1
	1.14	6.54	0.383	64.8
	0.970	5.54	0.324	90.2
	1.01	5.77	0.338	83.7
aluminum	5.03	6.76	4.23	7.99
d_p =16.0 mm	5.77	7.74	4.86	6.09
ρ_p =2700 kg m^{-3}	5.78	7.77	4.86	6.05
	5.93	7.97	4.98	5.75
	5.39	7.24	4.53	7.02
	5.35	7.20	4.50	7.12
aluminum	6.98	7.59	7.00	5.29
d_p =19.1 mm	7.32	7.96	7.34	4.81
ρ_p =2800 kg m^{-3}	7.45	8.11	7.48	4.63
	7.09	7.71	7.11	5.12
ceramic	5.85	13.2	3.42	7.17
d_p =11.1 mm	4.83	10.9	2.83	10.5
ρ_p =3820 kg m^{-3}	5.79	13.0	3.39	7.31
	6.35	14.3	3.72	6.09
	5.58	12.5	3.27	7.88
	5.65	12.7	3.31	7.68
	5.98	13.4	3.50	6.87
steel	3.56	20.4	1.19	27.3
d_p =6.35 mm	4.58	26.2	1.53	16.5
ρ_p =7660 kg m^{-3}	4.55	26.0	1.52	16.8
	5.96	34.1	1.99	9.76
	4.89	27.9	1.64	14.4
	5.51	31.5	1.84	11.4
steel	17.0	38.2	9.94	2.15
d_p =11.1 mm	16.7	37.5	9.77	2.21
ρ_p =7790 kg m^{-3}	14.6	32.7	8.52	2.91
	12.7	28.5	7.43	3.83
	14.8	33.2	8.65	2.82
	15.6	35.0	9.07	2.59

Table 2. Sedimentation data in glass-silicon oil mixtures, ϕ = 0.2.

Volume (ml)	u_p (10^{-3} m s^{-1})	$\frac{12u_p}{gd_p^2}\left(1-\frac{d_p}{D_c}\right)^{2.25}$ (10^1 s m^{-2})
0.310	-0.486	-1.14
0.560	-0.692	-1.18
0.560	-0.731	-1.25
0.810	-0.922	-1.30
1.06	-0.951	-1.17
1.06	-1.01	-1.24
1.31	-1.19	-1.32
1.56	-1.25	-1.28
1.56	-1.03	-1.06
1.81	-1.45	-1.39
2.06	-1.48	-1.35
2.31	-1.61	-1.40

Table 3. Sedimentation data for bubbles of water, ρ_p=990, in glass-silicon oil mixtures, ϕ = 0.2. Negative velocities are directed upwards.

Particle	ϕ (-)	u_f (10^{-2} m s^{-1})	u_p (10^{-1} m s^{-1})	Re_p (10^3)	C_{Dp} (10^{-1})
teflon d_p=6.35 mm ρ_p =2150 kg m^{-3}	0.66	3.31	1.90	0.646	4.73
aluminum d_p=6.35 mm ρ_p =2700 kg m^{-3}	0.66	3.31	2.84	0.899	5.28
	0.79	5.77	3.26	1.62	4.68
aluminum d_p=16.0 mm ρ_p =2700 kg m^{-3}	0.66	3.31	5.31	3.93	4.10
	0.66	3.31	4.54	3.41	5.45
	0.79	5.77	5.15	6.01	5.10
	0.79	5.77	5.60	6.47	4.40
aluminum d_p=19.1 mm ρ_p =2800 kg m^{-3}	0.66	3.31	6.56	5.69	3.59
	0.66	3.31	5.53	4.86	4.93
ceramic d_p=11.1 mm ρ_p =3820 kg m^{-3}	0.66	3.31	6.15	3.13	4.31
	0.66	3.31	5.58	2.86	5.16
	0.79	5.77	6.51	5.15	4.26
	0.79	5.77	6.94	5.46	3.80
steel d_p=6.35 mm ρ_p =7660 kg m^{-3}	0.66	3.31	6.73	1.94	5.67
	0.79	5.77	7.37	3.29	5.00
	0.79	5.77	7.56	3.37	4.77
steel	0.66	3.31	11.0	5.41	3.99

d_p =11.1 mm	0.66	3.31	11.4	5.58	3.75
ρ_p =7660 kg m^{-3}	0.79	5.77	9.38	7.20	5.70

Table 4. Sedimentation data in a bed of 800 μm glass beads fluidized by water.

Particle	ϕ (-)	u_f (m s^{-1})	u_p (m s^{-1})	Re_p (10^3)	C_{Dp} (10^{-1})
teflon d_p =6.35 mm ρ_p =2150 kg m^{-3}	0.75	0.20	0.0086	0.986	9.99
aluminum	0.80	0.23	0.305	6.18	5.38
d_p =16.0 mm	0.80	0.23	0.222	5.38	7.30
ρ_p =2700 kg m^{-3}	0.77	0.23	0.267	5.17	6.15
	0.77	0.23	0.276	5.25	5.97
	0.77	0.23	0.274	5.24	6.00
	0.70	0.17	0.234	3.71	7.10
	0.70	0.17	0.219	3.60	7.56
	0.70	0.17	0.254	3.87	6.51
	0.58	0.11	0.111	1.43	14.6
	0.58	0.11	0.123	1.49	13.4
	0.58	0.11	0.112	1.43	14.4
aluminum	0.80	0.23	0.412	8.71	4.91
d_p =19.1 mm	0.80	0.33	0.353	8.07	5.88
ρ_p =2800 kg m^{-3}	0.77	0.21	0.354	7.16	5.83
	0.70	0.17	0.284	4.89	7.46
	0.58	0.11	0.134	1.83	16.3
ceramic	0.80	0.23	0.325	4.44	6.26
d_p =11.1 mm	0.80	0.23	0.320	4.41	6.37
ρ_p =3820 kg m^{-3}	0.80	0.23	0.296	4.23	6.90
	0.80	0.23	0.300	4.31	6.80
	0.80	0.23	0.286	4.21	7.15
	0.77	0.21	0.292	3.76	7.09
	0.77	0.21	0.346	4.13	5.90
	0.77	0.21	0.282	3.69	7.36
	0.70	0.17	0.244	2.64	8.88
	0.70	0.17	0.233	2.58	9.29
	0.70	0.17	0.246	2.65	8.79
	0.58	0.11	0.160	1.16	15.1
	0.58	0.11	0.149	1.12	16.1
	0.58	0.11	0.156	1.14	15.4
steel	0.74	0.19	0.200	1.60	15.3
d_p =6.35 mm	0.74	0.19	0.200	1.65	15.2
ρ_p =7660 kg m^{-3}	0.77	0.21	0.235	1.93	12.8
	0.77	0.21	0.253	2.00	12.0
d_p =7.14 mm	0.77	0.19	0.273	2.17	12.9

Table 5. Sedimentation data in a bed of 6.35 mm glass beads fluidized by water.