

# **Instability of the equilibrium of a liquid below its vapor between horizontal heated plates**

*by*

Adam Huang & Daniel D. Joseph

Dept. of Aerospace Engineering and Mechanics

University of Minnesota

110 Union St. SE

Minneapolis MN 55455

September 1991

## ***Abstract***

We study the stability of a motionless liquid below its vapor between heated horizontal plates. The temperature of the bottom plate is held below the vaporization temperature and the top plate is hotter than the vaporization temperature. A water film is on the cold wall and a vapor film on the hot wall. We find a basic solution depending only on the variable  $y$  normal to the walls with steady distributions of temperature, a null velocity and no phase change. The linear stability of this basic state is studied in the frame of incompressible fluid dynamics, without convection, but allowing for phase change. An ambiguity in the choice of the conditions to be required of the temperature at the phase change boundary is identified and discussed. It is shown that the basic state of equilibrium is overstable under conditions of large temperature gradient, when the other parameters have suitable values. An analysis of the energy of the most dangerous disturbance shows that the source of the instability is associated with change of phase.

## 1. ***Introduction***

There have been many studies on the stability problem of laminar film condensation or evaporation (Nusselt's solution) on an inclined cool or hot plate (see Unsal and Thomas [1980] for a fairly thorough review of this literature) and of a falling film of liquid down an inclined plate, which was decisively analyzed by Yih [1954,1963] and Benjamin [1957]. Yih's problem has been generalized to a two-fluid channel flow problem for horizontal and vertical cases (Blennerhassett 1980, Renardy 1990). Problems of the Yih type have fully developed basic flows but no phase change, and of course a temperature equation is not needed. The laminar film condensation problem is a phase change problem, but it is posed in a semi-infinite region which excludes the existence of a fully developed steady flow; in the basic flow there is continuous change of phase at the interface, so that the stability analysis of this problem is difficult and is treated in a quasi-steady approximation. In many studies the vapor phase was inactive. The study of Busse & Schubert (1970) is probably closest to ours. They generalized the usual Rayleigh-Bernard problem by including a contribution of the latent heat from phase transformation in the energy balance. They restricted their consideration to the case when the viscosity and thermal conductivity of the two phases are the same, and they assumed that the variation of density is small compared with the mean density and the interfacial mass balance (our equation (2.4)) is neglected. Nevertheless their analysis shows that the fluid layer can be unstable even when the less dense phase lies above the dense phase. They assumed exchange of instability in the analysis of their problem. In our problem this assumption is not correct and the loss of stability occurs always as overstability.

In the course of our study we came to realize that the correct conditions on the temperature at a phase change boundary are not known. There are different possibilities. If we require that the temperature at a phase boundary is in thermodynamic equilibrium, then the saturation temperature is uniquely determined by the pressure through the Clapeyron relation. If the pressure on water side is different than on the vapor side as in our stability study, the temperature will be discontinuous. We can choose only two conditions for the temperature at a phase change boundary, so that thermodynamic equilibrium (the Clapeyron relation) excludes thermal equilibrium (continuity of temperature). This issue seems not to have been addressed in the film condensation literature, where it is conventional to require thermal equilibrium between water and its vapor together with thermodynamic equilibrium for the vapor, but not the water; that is, the temperature of the water is at the saturation value appropriate to pressure in the vapor (see Plesset and Zwick [1954], Gebhardt [1961], Ishii [1975]). In every study of this problem known to us the two phases are required to be in thermal equilibrium, the temperature must continuous across the phase change interface. But temperature discontinuities cannot be eliminated by rigorous application of first principles. Schrage [1953] says "... There is no reason why the temperature of the gas phase should necessarily be the same as that of the liquid or solid surface in all cases." Indeed, classic kinetic theory calculations (Pao [1971], Sone and Onishi [1978], Aoki and Cercignani [1983], Onishi [1984] and Cercignani, Fiszdon and Frezzotti [1985]) indicate that for monatomic vapor large temperature jumps exist at interfaces. Shankar and Deshpande [1990] have measured the temperature

distribution in the vapor between an evaporating liquid surface and a cooler condensing surface in water, Freon 113 and mercury. The temperature profiles obtained in mercury showed large jumps at the interface as large as almost 50% of the applied temperature difference.

In the present study we allow phase change, and the vapor phase is active. The second wall allows the system to attain a steady fully developed basic state. We are not in the frame of laminar film condensation because phase changes do not occur in the basic flow. We carried out calculations for three cases, when the water and vapor are at saturation values corresponding their own pressures (thermodynamic equilibrium) and when the vapor or the water is at saturation and the temperature is continuous (thermal equilibrium). The results of stability analysis do not depend strongly on the choice of the three aforementioned conditions for the temperature at the interface in the case of water and water vapor.

## 2. *Governing Equations and Basic Solution*

We assume that physical properties of the fluids, i.e. the viscosity, density and conductivity of the fluids are constants independent of pressure and temperature. This neglecting is usually done in the film condensation problems, and we follow this path. This means that the pressure will be a dynamical variable uncoupled from thermodynamics and that convective currents set up by the working of gravity on thermally induced variations of temperature will be ignored.

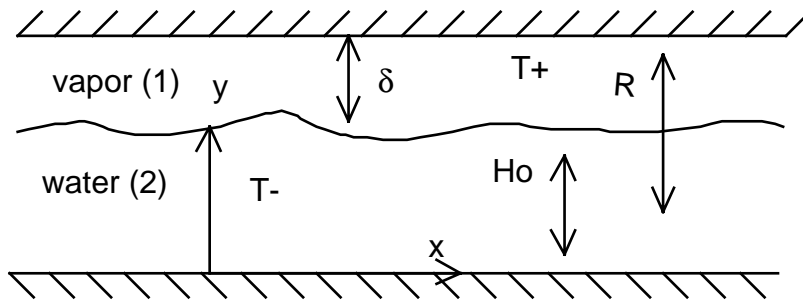


Figure 1. The physical configuration of this two-phase flow problem.

We consider two dimensional flow. The velocities of both phases satisfy the incompressible Navier-Stokes equations:

$$\nabla \cdot \mathbf{u} = 0, \tag{2.1}$$

$$\rho \frac{d\mathbf{u}}{dt} = -\nabla p + \rho \mathbf{g} + \mu \nabla^2 \mathbf{u}. \tag{2.2}$$

In the temperature equation we shall neglect the dissipation term:

$$\rho C_p \frac{dT}{dt} = k \nabla^2 T. \quad (2.3)$$

At the interface  $\mathbf{I}(x,t) = y - H(x,t) = 0$  we have mass, momentum and energy balances:

$$\text{Mass:} \quad -\dot{m}(\mathbf{x},t) = \rho_1 (\mathbf{u}_1 - \mathbf{u}_\Sigma) \cdot \mathbf{n}_{12} = \rho_2 (\mathbf{u}_2 - \mathbf{u}_\Sigma) \cdot \mathbf{n}_{12}. \quad (2.4)$$

$$\text{Momentum:} \quad \dot{m} \hat{u} \hat{\sigma} - \hat{u} \hat{p} \hat{\mathbf{n}}_{12} + 2 \hat{u} \mu \mathbf{D}[\mathbf{u}] \cdot \mathbf{n}_{12} \hat{\sigma} = 2H \sigma \mathbf{n}_{12}, \quad (2.5)$$

$$\text{Energy:} \quad -\hat{u} k \nabla T \hat{\sigma} \cdot \mathbf{n}_{12} = 2 \hat{u} \mu (\mathbf{u} - \mathbf{u}_\Sigma) \mathbf{D}[\mathbf{u}] \cdot \mathbf{n}_{12} \hat{\sigma} + \dot{m} \hat{u} h + \frac{1}{2} |\mathbf{u} - \mathbf{u}_\Sigma|^2 \hat{\sigma}, \quad (2.6)$$

with kinematic conditions:

$$\mathbf{u}_\Sigma \cdot \mathbf{n}_{12} = -\frac{\partial I / \partial t}{|\nabla I|}, \quad \hat{u} \hat{\sigma} \cdot \mathbf{t} = 0, \quad (2.7)$$

where  $\mathbf{n}_{12}$  is the normal of the interface directed from vapor to water, and  $\mathbf{t}$  is the tangent. We shall consider three different choices of the interfacial temperature conditions, referred as case I, II, III.

(I) Thermodynamic equilibrium of both phases:

$$T_{si} = T(P_i); \quad (2.8)$$

(II) Thermal equilibrium and thermodynamic equilibrium of vapor:

$$\hat{u} T \hat{\sigma} = 0, \quad T_1 = T(P_1). \quad (2.9)$$

(II) Thermal equilibrium and thermodynamic equilibrium of water:

$$\hat{u} T \hat{\sigma} = 0, \quad T_2 = T(P_2). \quad (2.10)$$

Boundary conditions are:

$$\text{at } y = 0, \quad u_2 = v_2 = 0, \quad T_2 = T_-, \quad (2.11)$$

$$\text{at } y = R, \quad u_1 = v_1 = 0, \quad T_1 = T_+. \quad (2.12)$$

There is a steady, developed, motionless solution

$$(u, v, p, T, H) = (0, 0, P, T, H_0)$$

where  $H_0$  is a constant :

$$H_0 = \frac{(T_s - T_-) k_2 R}{(T_+ - T_s) k_1 + (T_s - T_-) k_2}, \quad (2.13)$$

$$\left. \begin{aligned} T_1(y) &= T_+ - (T_+ - T_s) \frac{R-y}{R-H_0} \quad \forall y \in (H_0, R) \\ T_2(y) &= T_s - (T_s - T_-) \frac{y}{H_0} \quad \forall y \in (0, H_0) \end{aligned} \right\}, \quad (2.14)$$

$$\left. \begin{aligned} P_1(y) &= -\rho_1 g y + C_1 \quad \forall y \in (H_0, R) \\ P_2(y) &= -\rho_2 g y + C_2 \quad \forall y \in (0, H_0) \end{aligned} \right\}. \quad (2.15)$$

At the vapor-water interface  $y=H_0$ ,  $T_1=T_2=T_s$ , and the pressure is continuous, the basic flow is in thermodynamic equilibrium with

$$T_s = T(P(H_0)), \quad \dot{m} = 0. \quad (2.16)$$

So the different choices of interfacial temperature conditions give rise to the same basic state.

### 3. *Perturbation equations and normal modes*

To make our equations dimensionless we use following scales:

$$\begin{aligned} \text{length:} & \quad H_0, \\ \text{velocity:} & \quad V_0 = \frac{\mu_1 \zeta}{\rho_1 H_0 m}, \\ \text{time:} & \quad \frac{H_0}{V_0}, \\ \text{pressure:} & \quad \rho_i V_0^2, \\ \text{temperature:} & \quad T_s - T_- \end{aligned}$$

The dimensionless parameters listed below will appear in the dimensionless equations

$$\begin{aligned} \zeta &= \frac{\rho_2}{\rho_1}, & \text{Density ratio} \\ m &= \frac{\mu_2}{\mu_1}, & \text{Viscosity ratio} \\ \mathbb{R}_i &= \left(\frac{\zeta}{m}\right)^i, & \text{Reynold} \\ \text{Pr}_i &= \frac{\mu_i c_i}{k_i}, & \text{Prandtl number} \end{aligned}$$

$Pe_i$	$= \mathbb{R}_i Pr_i,$	Peclet number
$W$	$= \frac{\sigma \rho_1 H_0 m^2}{\mu_1^2 \zeta^2},$	Weber number
$\xi$	$= \frac{k_2}{k_1},$	Thermal conductivity ratio
$\Gamma$	$= \frac{k_1 T_s m (1 - \tau_2)}{\zeta h_{fg} \mu_1},$	Heat of vaporization number
$\Pi_i$	$= \frac{\zeta^2 \rho_i \mu_1}{h_{fg} H_0^2 m^2} \left( \frac{1}{\rho_1} - \frac{1}{\rho_2} \right) \frac{1}{(1 - \tau_2)},$	
$\tilde{G}$	$= \frac{g \rho_1^2 H_0^3 m^2}{\mu_1^2 \zeta^2},$	Gravity number
$\tau_1$	$= \frac{T_+}{T_s},$	Hot wall temperature ratio
$\tau_2$	$= \frac{T_-}{T_s},$	Cold wall temperature ratio
$r$	$= \frac{R}{H_0} = 1 + \frac{\tau_1 - 1}{\xi(1 - \tau_2)}.$	Vapor gap ratio

If we specify the fluids as water and water vapor and take all their material parameters at their table values at  $T_s = 100^\circ \text{C}$ ,

$\rho_1$	$= 0.59 \text{ g/m}^3,$
$\rho_2$	$= 0.97 \text{ g/cm}^3,$
$\mu_1$	$= 0.0121 \text{ cp},$
$\mu_2$	$= 0.28 \text{ cp},$
$k_1$	$= 2.5 \times 10^{-4} \text{ joule/cm sec } ^\circ\text{C},$
$k_2$	$= 6.8 \times 10^{-3} \text{ joule/cm sec } ^\circ\text{C},$
$C_{p1}$	$= 1.96 \text{ joule/g } ^\circ\text{C},$
$C_{p2}$	$= 4.18 \text{ joule/g } ^\circ\text{C},$
$h_{fg}$	$= 2.257 \times 10^3 \text{ joule/g},$
$\sigma$	$= 64.4 \text{ dynes/cm},$

we are left with three independent parameters  $H_0$ ,  $r$ ,  $\tau_2$ , and

$\zeta$	$= 1.69 \times 10^3,$
$m$	$= 22.47,$
$\xi$	$= 27.25,$
$Pr_1$	$= 0.9825,$

$$\begin{aligned} Pr_2 &= 1.7343, \\ \mathbb{R}_1 &= 73.44, \mathbb{R}_2 = 5393.43, \\ Pe_1 &= 72.15, Pe_2 = 9353.83, \end{aligned}$$

are constants and

$$\tilde{G} = 4.34 \times 10^6 H_0^3, W = 4.84 \times 10^4 H_0, \Gamma = 4.6 \times 10^{-3} (1 - \tau_2),$$

$$\Pi_1 = 1.0 \times 10^{-12} H_0^2 \frac{1}{(1 - \tau_2)}, \Pi_2 = 1.69 \times 10^{-9} H_0^2 \frac{1}{(1 - \tau_2)},$$

where  $H_0$  is measured in meters.

The basic temperature profile in dimensionless form is given by

$$T_1(y) = \frac{\tau_1}{(1 - \tau_2)} - \frac{(\tau_1 - 1)}{(1 - \tau_2)} \frac{r - y}{r - 1}, \quad (3.1)$$

$$T_2(y) = \frac{\tau_2}{(1 - \tau_2)} + y. \quad (3.2)$$

Let  $u, v, p, \theta, h$  be perturbations of basic state. Using the normal-mode decomposition of solutions

$$\begin{aligned} \mathbf{v}(x, y, t) &= \{u(y), iv(y)\} \exp(i\alpha(x - ct)), \\ p(x, y, t) &= p(y) \exp(i\alpha(x - ct)), \\ \theta(x, y, t) &= \theta(y) \exp(i\alpha(x - ct)), \\ h(x, t) &= h \exp(i\alpha(x - ct)), \end{aligned}$$

we get the following equations for the amplitudes:

$$\alpha u + v' = 0, \quad (3.3)$$

$$-\alpha cu = -\alpha p + \frac{i}{\mathbb{R}} (\alpha^2 u - u''), \quad (3.4)$$

$$-\alpha cv = p' + \frac{i}{\mathbb{R}} (\alpha^2 v - v''), \quad (3.5)$$

$$\alpha c \theta + v T' = -\frac{i}{Pe} (\alpha^2 \theta - \theta''), \quad (3.6)$$

together with boundary conditions:

$$u_1(r) = v_1(r) = \theta_1(r) = 0, \quad (3.7)$$

and

$$u_2(0) = v_2(0) = \theta_2(0) = 0, \quad (3.8)$$

and interface conditions at  $y=1$

$$v_1 - \zeta v_2 = \alpha c(\zeta - 1)h, \quad (3.9)$$

$$[u] = 0, \quad (3.10)$$

$$[m(-\alpha v + u')] = 0, \quad (3.11)$$

$$2\left[\frac{i\zeta v'}{\mathbb{R}}\right] - [\zeta p] - (W\alpha^2 + \tilde{G}(1-\zeta))h = 0, \quad (3.12)$$

$$\Gamma[\xi \theta'] - i(\alpha ch + v_1) = 0. \quad (3.13)$$

Temperature conditions will be chosen from following:

$$\theta_i = \Pi_i p_i + [-T_i' - \Pi_i \tilde{G}]h, \quad (3.14)$$

$$[\theta] + \left[\frac{dT}{dy}\right] h = 0. \quad (3.15)$$

We can eliminate  $u$  and  $p$  from (3.3) through (3.6), and get following equations:

$$v^{(4)} - 2\alpha^2 v'' + \alpha^4 v = i\alpha c \mathbb{R}(\alpha^2 v - v''), \quad (3.16)$$

$$\theta'' - \alpha^2 \theta - iPe v T' = -i\alpha c Pe \theta. \quad (3.17)$$

We introduce  $\hat{\alpha}$  and  $\hat{\gamma}$  by defining

$$\hat{\alpha}^2 = \alpha^2 - i\alpha c \mathbb{R} \quad \text{and} \quad \hat{\gamma}^2 = \alpha^2 - i\alpha c Pe.$$

The general solution of this system must have following form:

$$v(y) = Ae^{\alpha y} + Be^{-\alpha y} + Ce^{\hat{\alpha} y} + De^{-\hat{\alpha} y}, \quad (3.18)$$

$$\theta(y) = Ee^{\hat{\gamma} y} + Fe^{-\hat{\gamma} y} + \tilde{\theta}(y),$$

where  $\tilde{\theta}(y) = \frac{T'}{\alpha c} \left[ Ae^{\alpha y} + Be^{-\alpha y} + \frac{Pe}{Pe - \mathbb{R}} (Ce^{\hat{\alpha} y} + De^{-\hat{\alpha} y}) \right]$ .

Substituting these expressions into the boundary and interfacial equations forms an eigenvalue problem of a 13X13 matrix: the coefficients A,B,C,D,E,F for both vapor and water phase and the interface position variable  $h$ . We can not solve this matrix analytically. However for each set of parameters, we can get the eigenvalues of this matrix numerically.



#### 4. *Long waves*

For long wave disturbances ( $\alpha=0$ ), the system is slightly different. We are able to calculate the determinant of the 13X13 matrix and get an explicit equation for the zero-th order term of  $\alpha c$  in the long wave expansion, which agrees with the general solution (3.18)

Suppose  $v_0(y), \theta_0(y), h_0$  are the zero-th order terms in the long wave expansion, and  $\sigma_0$  is the leading order of  $\alpha c$  in the expansion. At this order the system becomes:

$$v_0^{(4)} = -i\sigma_0 \mathbb{R} v_0' \quad (4.1)$$

$$\theta_0'' - iPe v_0 \frac{dT}{dy} = -i\sigma_0 Pe \theta_0. \quad (4.2)$$

This system admits a solution in following form:

$$v_0(y) = Ae^{i\hat{\beta} y} + Be^{-i\hat{\beta} y} + Cy + D, \quad (4.3)$$

$$\theta_0(y) = Ee^{i\hat{\phi} y} + Fe^{-i\hat{\phi} y} + \tilde{\theta}_0(y), \quad (4.4)$$

where  $\hat{\beta}^2 = i\sigma_0 \mathbb{R}$ ,  $\hat{\phi}^2 = i\sigma_0 Pe$ . Bringing this expression into the boundary and interface conditions, after some manipulating we found that the eigenvalue  $\sigma_0$  satisfies the following equation:

$$\{(\zeta m)^{1/2}(e^{2i\hat{\beta} (1-r)} - 1)(e^{2i\hat{\beta} (2+r)} - 1) - (e^{2i\hat{\beta} (1-r)} + 1)(e^{2i\hat{\beta} (2-r)} - 1)\} \\ \{\lambda^{1/2}(e^{2i\hat{\phi} (1-r)} + 1)(e^{2i\hat{\phi} (2-r)} - 1) - (e^{2i\hat{\phi} (1-r)} - 1)(e^{2i\hat{\phi} (2+r)} + 1)\} = 0,$$

where  $\lambda = \zeta \xi \frac{C_{p1}}{C_{p2}}$  is a constant. Note that  $\sigma_0 = 0$  is a solution of above equation and only  $r$  appears as a parameter in this equation. For any given  $r$ , we can solve this equation and get all the other eigenvalues, and they are all negative. The results show that longwave disturbances are neutrally stable at zero-th order.

#### 5. *Energy Analysis*

The key to understanding the predictions of the linear theory of stability is the determination of the maximum growth rate in unstable cases. This gives rise to a distinguished length and wavespeed and when combined with an energy analysis of this fastest growing mode, it can even be used to forecast the flow type. The energy analysis in this problem is different than some other two fluid problems (see Hu & Joseph, 1989) because the temperature or thermal energy equation is coupled to mechanical energy

through the mechanism of phase change. In this case we may get two energy balances, one for mechanical energy designated with a subscript M and another for thermal energy designated with subscript T. Suppose  $(u,v,\theta)$  are the components of an eigenvector associated with the maximum growth rate, multiplying (3.4) and (3.5) by  $u^*$  and  $v^*$ , the complex conjugates of  $u$  and  $v$  respectively, and integrating the sum of them over both the liquid and vapor regions, we get

$$-\alpha c \langle (u^2+v^2) \rangle = \left\langle \frac{1}{\mathbb{R}}(\alpha^2(u^2+v^2) + \left(\frac{du}{dy}\right)^2 + \left(\frac{dv}{dy}\right)^2) \right\rangle - \left[ \frac{\zeta_i}{\mathbb{R}}(u'u^* + v'v^*) \right] + [\zeta p v^*], \quad (5.1)$$

where  $\langle \quad \rangle = \int_1 + \int_2$ ,  $u^2 = uu^*$ ,  $v^2 = vv^*$  and we used the boundary and interfacial conditions to

evaluate the integrands. The imaginary part of the right hand side of (5.1) represents the growth of the energy of the disturbance and the left side may be split into two parts

$$\dot{E}_M = B_M - D_M, \quad (5.2)$$

where  $\dot{E}_M = \alpha c_i \langle (u^2+v^2) \rangle$ ,

$$D_M = \left\langle \frac{1}{\mathbb{R}}(\alpha^2(u^2+v^2) + \left(\frac{du}{dy}\right)^2 + \left(\frac{dv}{dy}\right)^2) \right\rangle, \quad \text{viscous dissipation rate,}$$

$$B_M = \text{Im} \left\{ \left[ \frac{\zeta_i}{\mathbb{R}}(u'u^* + v'v^*) \right] - [\zeta p v^*] \right\}, \quad \text{energy production at the interface.}$$

We may transform the last term of  $B_M$  as follows

$$[\zeta p v^*] = [\zeta p]v_2^* + [\zeta v^*]p_1, \quad (5.3)$$

where  $[\zeta p]$  can be evaluated from the jump condition (3.12) as:

$$2 \left[ \frac{\zeta_i}{\mathbb{R}} v' \right] - (\alpha^2 W + \tilde{G}(1-\zeta))h.$$

Then  $B_M$  can be further decomposed into four parts:

$$B_M = B_1 + P + B_2 + G, \quad (5.4)$$

where

$$B_1 = \alpha^2 W \text{Im}(hv_2^*) \quad (5.5)$$

can be regarded as the energy supply rate due to the surface tension;

$$P = - \text{Im}([\zeta v^*]p_1) = \frac{(\zeta-1)\Gamma}{\zeta} \text{Im}([\xi \theta']p_1) \quad (5.6)$$

arises from phase change;

$$B_2 = \text{Im} \left\{ \left[ \frac{\zeta_i}{\mathbb{R}}(u'u^* + v'v^*) \right] - 2 \left[ \frac{\zeta_i}{\mathbb{R}} v' \right] v_2^* \right\} \quad (5.7)$$

represents interfacial friction due to the viscosity difference; and

$$G = \tilde{G} (1-\zeta) \text{Im}(hv_2^*) \quad (5.8)$$

is the gravity term. So (5.2) becomes:

$$\dot{E}_M = B_1 + P + B_2 + G - D_M. \quad (5.9)$$

Similarly we get the energy balance for the temperature disturbance as

$$-\alpha c \langle \theta^2 \rangle = \langle \frac{i}{Pe} (\alpha^2 \theta^2 + (\frac{d\theta}{dy})^2) \rangle - \langle v \theta^* \frac{dT}{dy} \rangle - [\frac{i}{Pe} \theta' \theta^*], \quad (5.10)$$

which can be written as

$$\dot{E}_T = I_T - D_T + B_T,$$

where

$$\begin{aligned} \dot{E}_T &= \alpha c_i \langle \theta^2 \rangle, \\ D_T &= \langle \frac{1}{Pe} (\alpha^2 \theta^2 + (\frac{d\theta}{dy})^2) \rangle, \\ I_T &= \text{Im} \langle v \theta^* \frac{dT}{dy} \rangle, \\ B_T &= R([\frac{1}{Pe} \theta' \theta^*]). \end{aligned}$$

## 6. Results and Discussions

We first compared results for three different choices for the interfacial temperature. We found that the difference for water and its vapor are rather small. Table 1 displays a comparison for some typical parameter values. *Case I* is the case when thermodynamic equilibrium is required for both phases, *case II* is when temperature continuity replaces water thermodynamic equilibrium in the water, and *case III* is when temperature continuity replaces thermodynamic equilibrium in the vapor. The fact that the differences between the three cases are small is understandable because the parameters  $\Pi_1$  and  $\Pi_2$  are small. This implies that the influence of saturation pressure on the saturation temperature is negligible.

Table1. Comparison of the results from three different choices of interfacial temperature conditions.

parameters	eigenvalue with the maximum growth rate		
	case I	case II	case III
$H_0 = 0.001\text{m}$ $\alpha = 0.001$ $r = 1.10$ $\tau_2 = 0.99$	$\pm 3.2593\text{e-}3, 3.9424\text{e-}4 i$	$\pm 3.2592\text{e-}3, 3.9430\text{e-}4 i$	$\pm 3.2592\text{e-}3, 3.9424\text{e-}4 i$
$H_0 = 0.0001\text{m}$ $\alpha = 0.0005$ $r = 1.10$ $\tau_2 = 0.99$	$\pm 1.8923\text{e-}3, -1.3569\text{e-}4 i$	$\pm 1.8916\text{e-}3, -1.3572\text{e-}4 i$	$\pm 1.8921\text{e-}3, -1.3569\text{e-}4 i$

In the following we shall present results only for case II, which is the case usually treated in the literature on film condensation. We have three independent parameters  $H_0$ ,  $r$ ,  $\tau_2$ . First we consider the case when  $H_0$  and  $r$  are fixed,  $\tau_2$  is the parameter. We get the neutral curve shown in figure 2. The energy analysis (table 3) shows that the instability is solely caused by the phase change term. Note that both parameters  $\Pi_1$ ,  $\Gamma$  change with  $\tau_2$ , however, for this parameter range  $\Pi_1$  is usually rather small. It follows that in the range of parameters under consideration  $\Gamma$  is the main stability parameter. This can also be seen from (4.6) which shows that the phase change term is "directly proportional to"  $\Gamma$ . The flow gets more unstable when  $\Gamma$  is larger.

We also considered the case when  $H_0$  and  $\tau_2$  are fixed and  $r$  varies. The result is shown as figure 3; when  $r$  gets larger, i.e. the vapor layer is thicker, the flow is more stable. This conclusion is more clearly seen in figure 4, which gives the neutral curve when  $H_0$  is fixed,  $r$  and  $\tau_2$  are control parameters. Each point of the curve is a critical point.

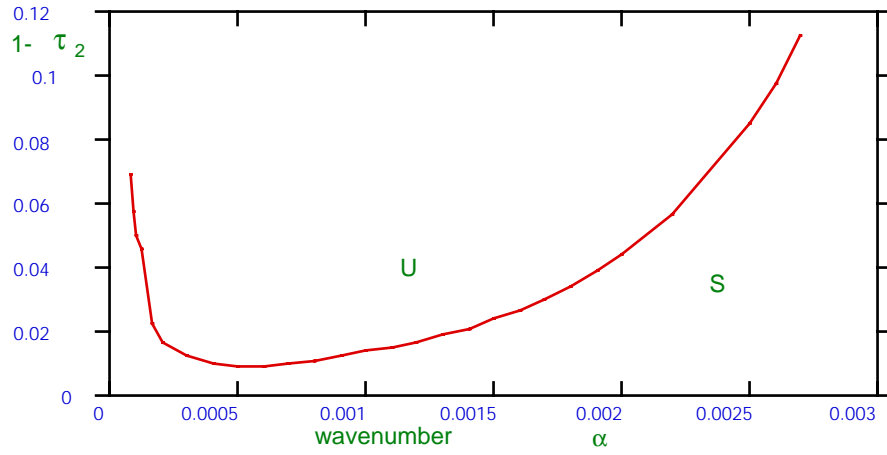


Figure 2. Neutral curve when  $H_0=0.001\text{m}$ ,  $r=1.3$ ,  $\tilde{G}=0.00434$ ,  $W=48.4$ ,  $\Gamma$ ,  $\Pi_1$  and  $\Pi_2$  change with  $(1-\tau_2)$ .

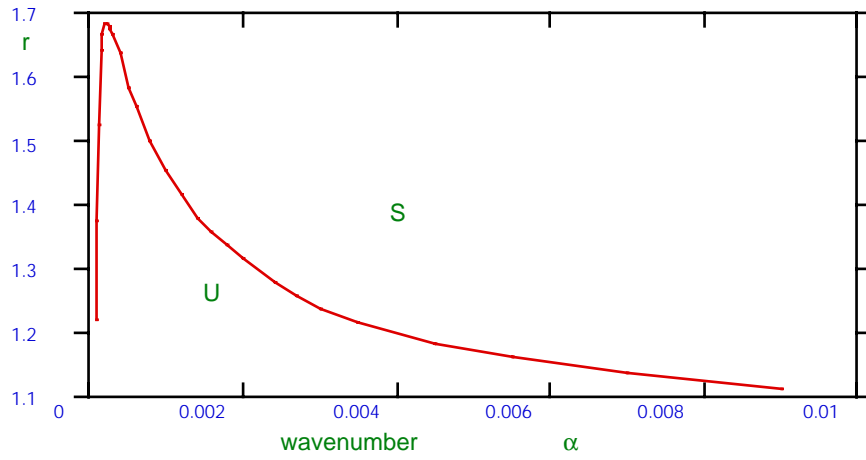


Figure3. Neutral curve when  $\tau_2$  (i.e. the temperature of the lower plane) is fixed (0.95):  $H_0=0.001\text{m}$ ,  $\tilde{G}=0.00434$ ,  $W=484$ ,  $\Gamma=0.231e^{-3}$ ,  $\Pi_1=0.2e^{-4}$ ,  $\Pi_2=0.0338$ .

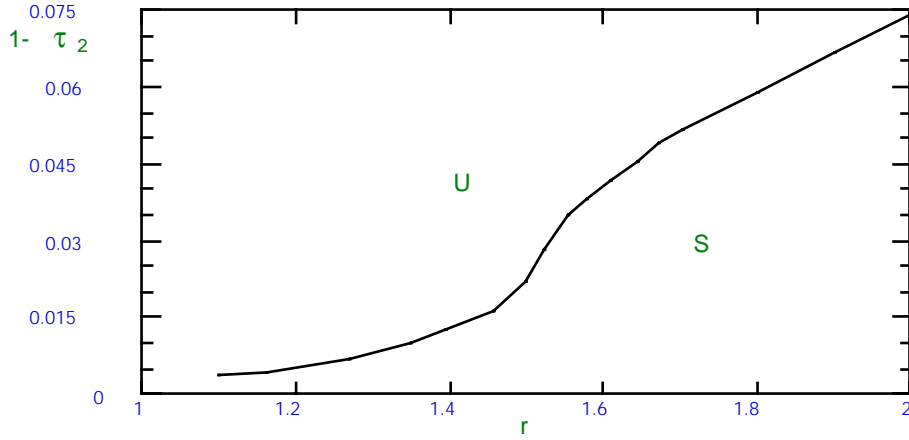


Figure 4. Neutral curve when  $H_0=0.001\text{m}$ ,  $\tilde{G} = 0.00434$ ,  $W = 48.4$ .

Table 2 and table 3 are results of the energy analysis of the mode with maximum growth rate. Note that the mechanical energy terms are normalized so that the viscous dissipation term  $D_M=1$ . The phase change is dominant and destabilizing. Surface tension and gravity are stabilizing but of negligible magnitude compared with the phase change term. The energy balance is essentially a balance between the phase change term and the viscous dissipation. This also helps to explain the fact that the stability does not change much with  $H_0$  (compare table 2 and table 3) when  $r$  and  $\tau_2$  are fixed because  $H_0$  appears through  $W$  and  $\tilde{G}$  which represent surface tension and gravity respectively, both of which are insignificant in the energy balance.

Table 2. Energy analysis for the mode of maximum growth rate in the case when  $H_0=0.01\text{m}$ ,  $\tilde{G} = 4.34$ ,  $W = 484$  and  $\tau_1 = 1.2$ .  $\tilde{\alpha}$  is the wave number for which the growth rate is maximum.

$1 - \tau_2$	$\tilde{\alpha}$	$\tilde{\alpha} c_i$	$\dot{E}_M$	$B_1$	$P$	$B_2$	$G$	$\dot{E}_T$	$B_T$	$I_T - D_T$
0.017	3.6e-5	1.3e-6	8.9e-4	-3.9e-16	1.00	1.8e-5	-4.5e-6	5.5e-6	1.0	-9.9e-1
0.030	1.1e-4	5.0e-4	1.7e-1	-3.3e-13	1.17	-1.5e-6	-4.2e-4	6.9e-4	1.0	-9.9e-1
0.040	1.9e-4	1.0e-3	2.3e-1	-1.2e-12	1.23	6.9e-6	-5.0e-4	8.2e-4	1.0	-9.9e-1
0.070	5.7e-4	4.0e-3	2.8e-1	-1.1e-11	1.34	-3.0e-6	-5.1e-4	1.0e-3	1.0	-9.9e-1

Table 3. Same as table2 except  $H_0=0.001\text{m}$ ,  $\tilde{G} =0.00434$ ,  $W = 48.4$  and  $\tau_1 =1.2$ .

$1 - \tau_2$	$\tilde{\alpha}$	$\tilde{\alpha} c_i$	$\dot{E}_M$	$B_1$	P	$B_2$	G	$\dot{E}_T$	$B_T$	$I_T - D_T$
0.017	3.6e-4	1.6e-6	1.2e-3	-4.6e-13	1.00	2.0e-5	-5.5e-7	7.1e-6	1.0	-9.99e-1
0.030	1.1e-3	5.0e-4	1.6e-1	-3.4e-10	1.16	-1.5e-6	-4.2e-5	6.9e-4	1.0	-9.99e-1
0.040	1.8e-3	1.1e-3	2.4e-1	-1.1e-9	1.24	4.6e-6	-5.2e-4	8.2e-4	1.0	-9.99e-1
0.070	5.2e-3	4.0e-3	3.8e-1	-1.0e-8	1.38	3.5e-6	-5.6e-4	1.0e-3	1.0	-9.99e-1

## 7. Conclusions

The influence on the flow stability of phase change has long been a major concern in the studies of film condensation or evaporation. However it seems that previous studies (Bankoff, S.G.; Unsal & Thomas; et al) were restricted to cases when the flow domain was semi-infinite. The basic flow in this case(usually the Nusselt solution) is not steady, the water film is continuously evaporating or condensing and it is found that condensation is stabilizing and evaporation is destabilizing (Unsal & Thomas). For the case when there is a steady temperature distribution without motion, Busse & Schubert proved when the less dense phase lies above, the denser phase can be unstable provided the temperature difference is large enough. We get the same conclusion from this work. Phase change is always destabilizing as can be clearly seen from our energy analysis. However there are some major differences between our problem and the problem treated by Busse and Schubert. They assumed that the variation of density was small compared with the mean density, which is inappropriate for water and water vapor, in our case the density difference between two phases is finite. They also assumed loss of stability occurs at real-valued eigenvalue at criticality. In our problem instability occurs with complex eigenvalues( see table 1), which means that either standing or traveling waves can be expected when the static state loses stability. The decision between standing and traveling wave awaits further nonlinear studies of bifurcation.

The issue of interfacial temperature conditions we believe is an important one, which has recently been considered for general phase problems by Truskinovsky [1990]. The problem arises whenever a pressure jump across the interface is allowed even when the phases are solid or liquid. Although in special cases treated here, water and its vapor, the stability results do not depend strongly on the choice of conditions on the temperature at the interface, we do not expect weak dependence in general especially when the dependence of the saturation temperature on the pressure is not small. The

choice of temperature conditions at a phase change boundary goes beyond continuum thermomechanics and appears to require some form of molecular theory.

### *Acknowledgments*

This work was supported under grants from the Department of Energy, the National Science Foundation, the US Army Research Office, AHPCRC and the Supercomputer Institute of the University of Minnesota.

### *References*

1. C.S.Yih, Stability of liquid flow down a inclined plane, *Physics Fluids* **6**,321, (1963).
2. Eckhaus, W. *Studies in nonlinear stability theory*. Springer, Berlin, (1965).
3. Bankoff, S.G. Stability of liquid flow down a heated inclined plane. *Int.J.Heat Mass Transfer* **14**, 377-385, (1970).
4. Lin, S.P. Profile and speed of finite amplitude wave in a falling liquids layer. *Physics Fluids* **14**, 263-268, (1971).
5. Lin, S.P. Stability of a liquid down a heated inclined plane. *Lett. Heat Mass Transfer* **2**, 361-370, (1975).
6. Unsal, M. & Thomas, W.C. Linear stability of analysis of film condensation. *J.Heat Transfer* **100**,629-634, (1978).
7. Unsal, M. & Thomas, W.C. Nonlinear stability of analysis of film condensation. *J.Heat Transfer* **102**, (1980).
8. Spindler, B. Linear stability of liquid film with interfacial phase change. *Int.J.Heat Mass Transfer* **25**, 161-173, (1982).
9. Hwang, C.C. & Weng, C.-I. Finite-amplitude stability analysis of liquid films down a vertical wall with and without interfacial phase change *Int.J.Multi phase Flow* **13** No.6 803-814, (1987).
10. Renardy, Y. Viscosity and density stratification in vertical Poiseuille flow. *Phys.Fluids* **30** (6) June (1987).
11. Gebhart, B. *Heat Transfer*, second edition, McGraw-Hill Book Company (1961, 1971)



12. Plesset, M.S. & Zwick, S.A. The growth of vapor bubbles in superheated liquids. *J. of Applied Physics*, **25** (4) April (1954).
13. Schrage, R. A Theoretical Study of Interphase Mass Transfer, Columbia University Press, 1953
14. Hu, H. & Joseph, D.D. Lubricated pipelining: stability of core-annular flow. Part 2. *J. Fluid Mech.* (1989), **205**.
15. Ishii, M. Thermo-fluid Dynamic theory of two-phase flow, Eyrolles, 1975
16. Hirschfelder, J.V., Curtis, C.F. and Bird, R.B. Molecular Theory of Gases and Liquids. John Wiley and Sons. Inc. NY (1954)
17. Blennerhassett, P.J. 1980 On the generation of waves by wind. *Proc. R. Soc. Lond. A* **298**, 451-494.
18. Busse, F.H. & Schubert, G. Convection in a fluid with two phases. *J. Fluid Mech.* (1971), vol. **46**, part **4**, 801-812.
19. Chandrasekhar, S. 1961 Hydrodynamic and Hydromagnetic Stability. Oxford: Clarendon.
20. Truskinovsky, Lev . (1990) Kinks versus shocks, to appear in: SHOCK INDUCED TRANSITIONS AND PHASE STRUCTURES IN GENERAL MEDIA, Springer-Verlag, 1991.
21. Shankar, P.N. and Deshpande, M.D. On the temperature distribution in liquid-vapor June 1990
22. Pao, Y.P. Application of Kinetic Theory to the Problem of Evaporation and Condensation. *Phys. Fluids* **14**, 306 (1971)
23. Sone, Y. and Onishi, Y. *J. Phys. Soc. Jpn.* **44**, 1981 (1978)
24. Aoki, K. and Cercignani, C. Evaporation and Condensation on Two Parallel Plates at Finite Reynolds Numbers. *Phys. Fluids* **26**, 1163 (1983)
25. Onishi, Y. 14th International Symposium on Rarefied Gas Dynamics, Tsukuba, Japan, 1984
26. Cercignani, C., Fiszdon, W. and Frezzotti, A. The Paradox of the Inverted Temperature Profiles between an Evaporating and a Condensing Surface. *Phys. Fluids* **28**, 3237 (1985).