WAKE ARCHITECTURES IN TWO-DIMENSIONAL FLUIDIZATION OF SPHERES

Experiments and phenomenogical description

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Summary

*The structure of a shear flow past freely suspended spheres at low Reynolds numbers is visualized employing sheets of hydrogen bubbles illuminated by laser light as well as tungsten halogen lamps. We describe the wake interactions of the spheres as the fundamental contributions to the observed stability of the arrangements. Stable planar clusters of two, three and four particles were obtained when 22<**Re**<43.*

Introduction

The study of viscous flow past a single sphere or past a freely suspended assemblage of spheres is the centerpiece of an extensive literature dedicated to fluidized systems. The motions that occur at the particle scale determine the dynamic behavior of the suspension, but the complexities of the fluid-particle interactions pose severe difficulties for a full analytical and/or numerical treatment of the problem. Thus we turn to an experimental study of the dynamics of particle interactions in a fluidized system, focusing our attention on the hydrodynamics of planar motions around suspended spheres in a flow confined between two vertical walls.

We report here the occurrence of massively stable wake architectures in the planar motion of spheres suspended in a Poiseuille flow. Equilibrium configurations of up to 4 particles, each rotating stably in the boundary of the wake from the particle below were easily reproducible at a Reynolds numbers range that is well above the limit applicable to creeping-flow problems. There is clear evidence from the flow visualizations that the shear layer originated in the wake of one particle defines a region for the downstream particle where it clings to a strongly stable equilibrium position. In this position, the particle rotates steadily about an axis perpendicular to the plane of motion, which is a necessary stability requirement imposed by the crosswise velocity gradient of the wake. Of course, the lead particle does not rotate about the same axis.

To our knowledge, the works of Jayaweera, Mason and Slack (1964), Hocking (1964) and the numerical simulation for three-dimensional Stokes flows of Ganatos, Pfeffer and Weinbaum (1977) are the only ones studying the behavior of clusters of spheres freely interacting in a relatively slow viscous flow. In none of these studies, however, the authors have observed or analyzed the formation of stable wake structures. Happel and Pfeffer (1960) also study the interaction of three freely falling spheres at Reynolds numbers up to 0.7, and they report an unstable dynamical scenario even if inertial effects are negligible. The dynamical behavior of a single sphere in a shear flow just beyond the creeping flow regime has been studied [see, for example, chapter 7 of Happel and Brenner's book, pp. 316-ff (1991)], mostly as perturbations of Stokes's law. In the experimental investigation of Bagnold (1974), the problem of the shear flow past a rotating sphere is analyzed, not in the framework of a search for the best Stokes's law correction. His conclusions about lift forces related to the local fluid spin about a transverse axis can be used here as an argument for the observed stability of the particle arrangements. We advance in the present work the same basic rationale of Segré and Silberberg (1962), viz., that the cause for stability in this wake dominated flow must be sought either in the inertia terms of the Navier-Stokes equation, or in the presence of the walls, or in both.

We understand the formation of the wake architectures found by us as a fundamental stability mechanism of shear flows, not only an incidental property of a specific flow geometry.

Our study with few particles in a shear flow disclose one more fundamental fluid-particle interaction mechanism in fluidized systems within the framework of a deterministic approach. Since the total number of spheres are not enough to span the channel width, the particles do not form horizontal arrays. Instead they rotate and arrange themselves in a wake architecture. The rotation of the spheres plays a crucial part in the stability of these architectures, and there is no vortex shedding. Beyond the Reynolds number range reported here, vortex shedding takes place, breaking down the stability of the arrangement.

Experimental Apparatus

We built a return flow apparatus driven by a 3HP, 82l/m gear pump which allows us to stably fluidize a few particles with different aqueous solutions of glycerin. The physical properties of the aqueous glycerin and of the different fluidized spheres is given in Table 1. The tests with each kind of spheres were performed with previously selected spheres of the same free falling velocity in a cylindrical stagnant column of aqueous glycerol.

Table 1. Particle data. U_t= terminal falling velocity.

Sphere	**ρ,** g/ml	**d,** mm	$\mathbf{U_t}$, m/s
Teflon®	2.12	3.175	0.068
Teflon®	2.12	6.35	0.174
Glass	2.61	5.969	0.221
Delrin®	1.425	11.1125	0.075

The vertical working section consisted of a rectangular channel 25∞12.7∞1270mm made of Plexiglass plates 15.875mm thick. The equalizing section upstream of the working section was composed of a convergent plenum chamber followed by two closely assembled stainless steel screens, of meshes 270 and 325 [Agarwal (1979)]. The flow rate was measured with an Omega FP-5300 turbine flowmeter coupled to a Racal-Dana 9000A Timer/Counter.

The flow visualizations were made with the hydrogen bubble technique in conjunction with a continuous Lexel 4W Argon-ion laser. A laser sheet was obtained using a cylindrical lens 6.35mm diameter to give a sufficiently wide divergence angle to the incident laser beam.Tungsten halogen lamps illumination was also used independently to obtain additional, evenly illuminated, pictures. Two 40μm diameter stainless steel wires were stretched over the entire channel width. One wire was placed at the center of the channel gap, while the other was placed at approximately 3.8mm from the front wall, both at about 250mm downstream of the equalizing section. In this way, two different hydrogen bubble sheets could be independently generated, in order to locate the particles properly between the two vertical walls, and to disclose the flow structure along these two different planes.

Flow visualizations and analysis

Figure 1 shows the visualization of the flow past one stably locked pair of Teflon spheres 6.35mm diameter. The Reynolds number defined by $Re = \frac{u\,d}{\nu}$, where *u* is the superficial velocity, *d* is the particle diameter and ν is the kinematic viscosity of the aqueous glycerol, is 22.48. The plane of the laser sheet is passing right through one secant plane of the sphere, closer to the front wall. This enables us to see the particular deformation of the velocity profiles of the main flow due to the presence of the particle.

Figure 1. Visualization of the velocity profiles past a stably locked pair of Teflon spheres 6.35mm diameter.

The upstream sphere rotates very slowly about an axis *parallel* to the wall, which comes from the applied torque originating in the transverse velocity gradient of the Poiseuille flow. The wake structure acts as a composite body balanced by the drag and its weight. Each particle individually is dynamically in equilibrium under the action of distributed forces that can be decomposed into a lift force towards the undisturbed flow and normal to the direction of motion of the fluid, a 'Bernoulli' force towards the wake and parallel to the lift force, the drag, and the weight. The lift force is a consequence of the right-and-left asymmetry of the pressure distribution around the sphere. The 'Bernoulli' force is due to the velocity gradient across the shear layer.

The spin of the particle can be approximated by $\omega = \frac{a-b}{d}$ where *a* and *b* are the components of the fluid velocity at the vertical planes tangent to the particle.

The slight displacement of the sphere from the center of the channel is believed to be of the same sort as the one reported by Segré and Silberberg (1962), insofar as it seems plausible to believe that the constriction of the boundary flow may create a local excess of dynamic pressure to keep the particle in equilibrium [Bagnold (1974)]. The rotation of the lead sphere mentioned earlier is associated with this Segré-Silberberg effect on the composite structure as a whole.

The most important observation, though, is that the downstream sphere places itself in the shear layer created by the upstream sphere, where it stays steadily rotating about an axis *normal* to the walls. This is shown in Figure 2.

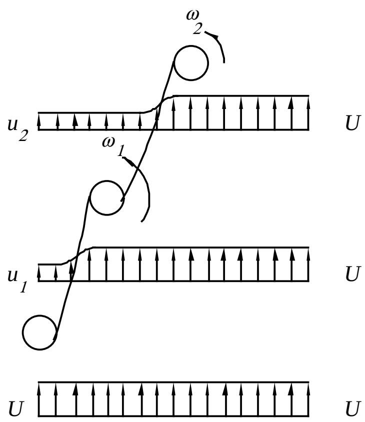

Figure 2. The wake architecture with three particles.The rotations ω_1 and ω_2 due to the velocity gradient of the wake are such that $\omega_1<\omega_2$.

It is clear that the stronger crosswise velocity gradient of the wake of the first particle is alone the main cause for the observed stability. The same pattern is reproducible up to 4 particles, as will be shown in the sequel. The elliptic nature of the problem and of the wake dominated flow directs the mathematical description to a treatment of the full Navier-Stokes equations. As shown in Table 2, the angles of the line of centers with the vertical decrease downstream, an indication that the build up of the structure rests on an increasing crosswise velocity gradient. If we were to approximate the particle spin to the half the difference between the velocities at planes tangent to the sphere on the inside and outside of the wake, we should write

$$\omega_i \approx \frac{U-u_i}{d}, \qquad i = 1,2, \tag{1}$$

according to the nomenclature of Figure 2.

Table 2. Flow visualization experiments with fluidized spheres. The fluid is a solution of glycerol and water, 80% by weight of glycerol, ρ=1.21 g/ml @ 22˚C.

Spheres	Re	Angles, □
Glass, 4 spheres	43.05	a1 =11.0; a2=10.5; a3=8.0
Teflon, 4 spheres	28.40	a1=24.3; a2=21.3; a3=15.3
Teflon, 2 spheres	22.48	a1 =20.9
Teflon, 3 spheres	22.00	a1=23.6; a2=15.9
Teflon, 3 spheres	4.95	—
Glass, 1 sphere	36.80	—
Delrin, 3 spheres	8.50	—

Figures 3 and 4 show four and three Teflon spheres 6.35mm diameter in stable suspension, respectively. The Reynolds numbers are, respectively, 28.4 and 22.0. Three Teflon spheres 3.173mm diameter, at the Reynolds number of 4.95 just before the carry-over, do not stabilize themselves in any architecture whatsoever. The same was observed for the Delrin spheres 11.113mm diameter, for which the interaction dynamics of drafting, kissing and tumbling takes place at a Reynolds number of 8.5. They are then free to drift horizontally and to end up being caught in the wake of another sphere. Since they do not rotate noticeably, they are not under the action of a lift force, and thus can not find a stable position in the shear layer. At much higher Reynolds numbers, this dynamic scenario has already been reported elsewhere [Fortes, Joseph and Lundgren (1987)].

Figure 3. Four Teflon spheres locked together in the wake structure. *Re* = 28.4.

Figure 4. Three Teflon spheres locked together in the wake structure. *Re* = 22.0.

The fluidization of four spheres of glass 5.969mm average diameter, at a Reynolds number of 43, was the limiting condition for the stability of spheres in staggered wakes.

The Reynolds number range thus found, viz., $22<Re<43$, defines a specific flow regime that is stable to small disturbances. It is our understanding that this stability is characteristic to planar flows past three-dimensional structures of spheres in a Poiseuille flow under the action of gravity. Pending more detailed experiments, we believe that this effect falls within the same range of Reynolds number of the Segré-Silberberg effect, i.e, $Re\approx30$, since in our experiments the gap size is of the same order of magnitude as the particle diameter used in the definition of *Re*.

Strictly horizontal periodic arrays of spheres have been reported elsewhere [Fortes et al. (1987)]. We believe that these horizontal arrays, being natural occurrences in narrower channels, are a function of the gap ratio defined for gravity shear flows by Bagnold (1974) as

$$B = \frac{y_c - d/2}{d}, \tag{2}$$

where y_c is the transverse coordinate of the center of the sphere. The constriction being more severe, the effects of viscosity are correspondingly more pronounced as a stabilizing factor. Indeed, such horizontal arrays can be formed at fairly high *Re*, well above the range of the present experiments.

To understand the underlying fluid mechanics of the wake architectures we have to investigate the 'dead water' regions behind the spheres, their scale of formation and the diffusion length of the shear layer. As can be seen in Figures 5 and 6, there is a separation bubble, but there is no vortex shedding from the shear layer around the spheres. The apparent asymmetry of the separation bubble is a clear-cut indication of the presence of the lift force towards the undisturbed flow mentioned earlier. The clockwise rotation of the particle builds up pressure on the right side of it, giving rise to a higher pressure that counteracts the suction towards the wake. There is a complex three-dimensional pattern of flow inside the bubble, represented schematically in Figure 5. The arrows show the approximate trajectories of the tiny hydrogen bubbles entrained continuously by the shear layer, confirming the observations of Gerrard (1966). The ensuing mechanism of vortex shedding for higher *Re* breaks down the stability of the architecture due to the propagating disturbances of the oscillatory motion of vortex shedding.

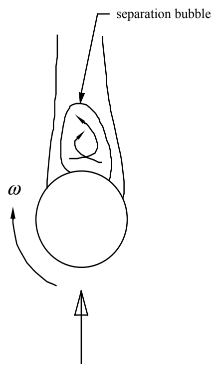

Figure 5. Separation bubble and flow pattern of a spinning sphere caught in the staggered wake.

Figure 6. Visualization of the flow past a spinning sphere caught in the staggered wake. *Re* = 22.

Conclusions

We have made flow visualizations of planar clusters of spheres freely suspended in a Poiseuille flow against gravity between two vertical plane walls. The following preliminary results have revealed some fundamental features of shear flows past these structures:

1. Shear flows past a cluster of up to 4 spheres exhibit a massively stable wake architecture for the particle Reynolds number range given by $22<Re<43$;
2. Out of this range, the particles interact unsteadily, following a dynamic scenario of drafting, kissing and tumbling;
3. The structure of the wakes are such that each downstream particle positions itself in the shear layer of the wake of the upstream particle, thus inducing a rotation that generates a lift force towards the undisturbed flow;
4. There is a sidewards displacement of the cluster as a composite body that we interpret as a Segré-Silberberg effect which contributes to the overall stability of the cluster;
5. The formation of a separation bubble is essential to the understanding of the underlying fluid mechanics. Its length is of the order of magnitude of the particle diameter, but no vortex shedding is observed. We believe that the vortex shedding would break down the stability of the cluster.

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