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White–Metzner models for rod climbing in A1

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JOURNAL OF NON-NEWTONIAN FLUID MECHANICS

The journal will appeal to people working on basic rheological science and applications.

Subjects considered suitable for the journal include the following (not necessarily in order of importance):

(1) The development of rheological equations of state for non-Newtonian liquids from both a continuum and microstructure starting point, but the outlook will always be practical in the sense that abstract developments with no hope of application will not be welcomed.

(2) The use of rheological equations of state in the solution of meaningful flow problems involving one or more fluid phases. Analytic and/or numerical solutions would be in order. These should be problems which involve the equations of continuity and motion and energy in a non-trivial way.

(3) Experimental work associated with non-Newtonian liquids, including

(i) confirmation or otherwise of predictions from rheological equations of state developments and the confirmation of predictions from the solved flow problems

(ii) novel flow situations which would suggest the need for further analytical work

(iii) rheometry

(iv) practical situations of flow which are in need of systematic theoretical and experimental research. These are likely to arise in the polymer processing, pharmaceutical, food, detergent and petroleum industries, but this list is not meant to be exhaustive.

(4) Problems involving heat and mass transfers and mixing in flowing non-Newtonian liquids.

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Abstract

Measurements of rod climbing in A1 give rise to an apparent linear relation between the height rise h and the angular velocity Ω of the rod. We use a White–Metzner model to fit the data and we find that the height rise on the rod deviates from the quadratic dependence on the angular velocity when the viscosity or relaxation time vary with shear rate. When both the relaxation time and the viscosity change simultaneously with the shear rate, the climbing profile on the rod deviates more from the standard theory for rod climbing. A simplified argument based on the upper-convected Maxwell model and using power laws for the viscosity and the relaxation time gives rise to $h(\Omega) \propto \Omega^{n+m}$, where n and m are power-law indices which can be chosen to fit the data. The height rise data for A1, which is linear rather than quadratic in the shear rate at low shears, can be fitted to a White–Metzner model using measured values for the viscosity function and a long time of relaxation which decreases strongly with the rate of shear. This result suggests that a shear-decreasing relaxation time function may be useful for describing the rheology of fluids with small quadratic range.

Keywords: A1; rod climbing; Weissenberg effect; White–Metzner models

1. Introduction

It is well known that many non-Newtonian fluids will climb a rotating rod (the Weissenberg effect). The climb is associated with normal stresses which do not exist in Newtonian fluids. The climbing property of non-Newtonian fluids can be used to characterize important rheological parameters

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in such fluids. Mathematical descriptions of the Weissenberg effect have been developed using the domain perturbation method by Joseph and Fosdick [1] and Joseph et al. [2]. The theory gives a relationship between the climbing profile and the viscoelastic properties of the fluid which at lowest (quadratic) order is determined by the climbing constant $\hat{\beta}$ of the fluid. Values of the climbing constant for various liquids were measured by Beavers and Joseph [3,4], Joseph and Beavers [5], Beavers et al. [6], and Kolpin et al. [7]. These values and others were tabulated by Joseph et al. [8]. Yoo et al. [9] examined the development of secondary motions in rod-climbing flow and their effects on the climbing profile by carrying out the perturbation analysis to the fourth order. The aforementioned results are summarized in the book on fluid dynamics of viscoelastic liquids by Joseph [10]. Recently, Debbaut and Hocq [11] numerically simulated the climbing flow for both Oldroyd-B and Johnson–Segalman fluids. They also examined the shape of the free surface and the development of secondary motions.

The operational theory for the measurement of the climbing constant is given by Joseph et al. [8]. The shape of the free surface can be expressed in the form

$$h(r, \Omega) = h_0(r) + h_2(r)\Omega^2 + O(\Omega^4) \tag{1}$$

where r is the coordinate in the radial direction, Ω is the angular velocity of the rod, $h_0(r)$ is the static climb, and $h_2(r)$ is to be determined by the following problem

$$\frac{T}{r}(rh_2')' - \rho g h_2 = -2\frac{a^4}{r^4}\hat{\beta} + \frac{\rho a^4}{2r^2}, \tag{2}$$

with $h_2'(a) = 0$ and $\lim_{r\to\infty} h_2(r) = 0$, where T is the surface tension, ρ is the density of the liquid, a is the radius of the rotating rod, g is the acceleration due to gravity and $\hat{\beta}$ is the climbing constant which is related to the parameters of a simple fluid of grade two, or is related to the coefficients of the first and second normal stress differences at zero shear. If the surface tension is neglected, from (2) we get

$$h_2(r) = \frac{1}{\rho g}\left(\frac{2a^4}{r^4}\hat{\beta} - \frac{\rho a^4}{2r^2}\right). \tag{3}$$

When the surface tension is not negligible, which is always the case, it is impossible to find an analytical solution to the problem (2). However, an approximate solution was derived by Joseph and Fosdick [1], which gives

$$h_2(a) \approx \frac{a}{2T\sqrt{S}}\left(\frac{4\hat{\beta}}{4+\lambda} - \frac{\rho a^2}{2+\lambda}\right), \tag{4}$$

when evaluated at the rod, where $\lambda = a\sqrt{S}$ and $S = \rho g/T$.

Beavers and Joseph [3] proposed to use (1) and (4) as a basis for a rotating rod rheometer. The value of $\hat{\beta}$ is determined from the measured values of the height of climb on the rod. The rod is rotated slowly, at a speed for which a measurable height of climb can just be distinguished. The climb is measured as a function of increasing rotational speed Ω, and the slope of the plot of $h(a, \Omega)$ vs. Ω^2, or $h_2(a)$, is computed for $\Omega \to 0$. Then from (4), the value of $\hat{\beta}$ is determined.

For many viscoelastic liquids, for example, STP, TLA-227, and fluids M1 and S1, there is a well-defined region where the climbing height $h(a, \Omega)$ changes quadratically with the angular velocity Ω, and the theory described above works well. However, for other liquids the agreement between prediction of the theory and experimental data is not satisfactory, and in some cases a quadratic region cannot be seen. One example is A1 which is a test fluid consisting of polyisobutylene in decalin. A sample experimental measurement for A1 is shown in Fig. 1. The experimental data show no sign of the quadratic region, instead they indicate that the height rise on the rod is almost linearly proportional to the angular velocity of the rod. One might think that this linear relation is due to the development of secondary motions. For example, in Fig. 2 we have plotted the climbing height vs. the dimensionless Deborah number for an upper-convected Maxwell fluid, which is calculated using the finite element method by Debbaut and Hocq [11]. Except at very small angular velocities, this figure shows that the climbing height changes linearly with the angular velocity. The plots of

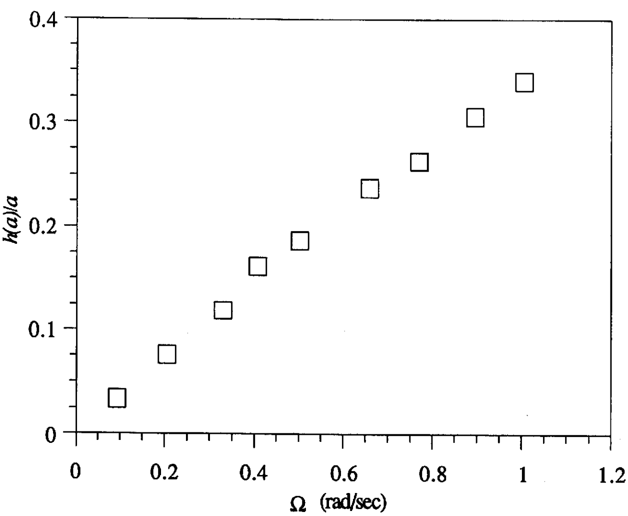

Fig. 1. Experimental data of height rise on the rod vs. the angular velocity of the rod for A1 at 24.5°C.

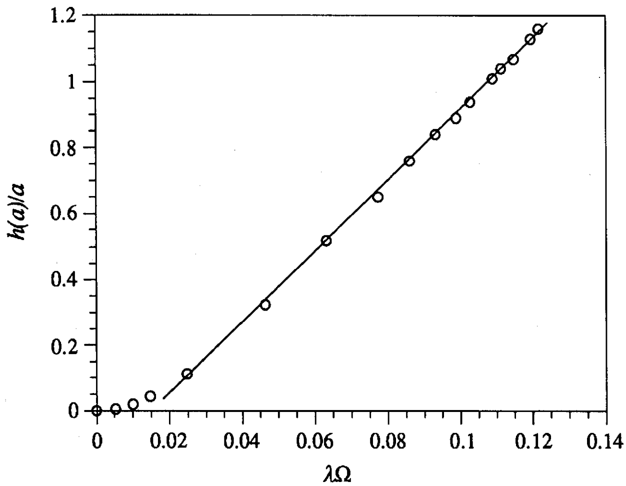

Fig. 2. (After Fig. 3(a) in Debbaut and Hocq [11].) Climbing height as a function of the Deborah number ($\lambda\Omega$) for an upper-convected Maxwell fluid, where λ is the relaxation time of the fluid.

streamlines and vortex intensity indicate that secondary motions are present at those large angular velocities. Therefore, the higher-order term in the expansion (1) becomes important. Furthermore, a simple plot of the function $f(x) = x^2 - x^4$ around $0.2 \leq x \leq 0.6$ reveals the apparent linear relation between f and x. However, this may not be the explanation to the data shown in Fig. 1, because the linear relation shown there persists to very small angular velocities where the effect of secondary motions should be negligible.

One reason for the disagreement between theory and experiment was suggested by Beavers et al. [6]. They measured the climbing constants for STP, TLA-227, and polyacrylamide, and reported that while the theory works well for STP and TLA-227, it does not appear to work for polyacrylamide. They noticed that both STP and TLA-227 have a nearly constant shear viscosity at different shear rates, whereas polyacrylamide is quite pseudoplastic (shear thinning). They suggested that shear thinning which does not enter into a second-order theory might be a reason for the disagreement between theory and experiment. We also noticed that M1 is a Boger fluid (Hudson and Ferguson [12]) whose apparent shear viscosity is almost independent of shear rate, and for M1 the theory works well (Hu et al. [13]). On the other hand, for A1 the apparent shear viscosity decreases strongly with shear rate in the range where the rod climbing experiment is performed (Hudson and Jones [14]).

In this paper we study the effects of shear thinning and the dependence of the relaxation time on the shear rate of a White–Metzner model in rod-climbing flow, and give an explanation for the linear relation between the height rise on the rod and the angular velocity of the rod shown in Fig. 1. In the White–Metzner model the viscosity and the relaxation time depend on the second invariant of the rate of strain (or the shear rate for the present problem). Because the rotational speed of the rod is assumed slow, secondary motions are neglected in the analysis. We shall first formulate the problem and list the governing equations, the boundary and free surface conditions. The equations are solved numerically by a shooting method and various choices of the viscosity and relaxation functions are presented and discussed. Finally, a comparison is made with the experimental data for A1 and the model parameters are selected to fit the data.

2. Formulation of the problem

Consider a circular rod of radius a being rotated with angular velocity Ω in an infinite sea of liquid. Choose a cylindrical coordinate system (r, θ, z) where z is along the axis of the rod with $z = 0$ located at the unperturbed free surface of the liquid. The velocity components of the flow are denoted as $\boldsymbol{u} = u\boldsymbol{e}_r + v\boldsymbol{e}_\theta + w\boldsymbol{e}_z$. We assume that the flow is incompressible, steady and axisymmetric. The governing equations of the flow are

$$\nabla \cdot \boldsymbol{u} = 0 \tag{5}$$

and

$$\rho(\boldsymbol{u} \cdot \nabla\boldsymbol{u}) = -\nabla(p + \rho g z) + \nabla \cdot \boldsymbol{S} \tag{6}$$

where p is the pressure and ρ and g are as defined above. The extra stress tensor $\boldsymbol{S}$ is given by constitutive equations for the liquid. For a White–Metzner fluid, we have

$$\lambda\overset{\triangledown}{\boldsymbol{S}} + \boldsymbol{S} = 2\eta\boldsymbol{D} \tag{7}$$

where $\boldsymbol{D}$ is the symmetric part of the velocity gradient $\boldsymbol{L} = \nabla\boldsymbol{u}$, and the upper-convected derivative of the extra stress $\boldsymbol{S}$ is expressed as

$$\overset{\triangledown}{\boldsymbol{S}} = (\boldsymbol{u} \cdot \nabla)\boldsymbol{S} - (\boldsymbol{L}\boldsymbol{S} + \boldsymbol{S}\boldsymbol{L}^{\mathrm{T}}). \tag{8}$$

The relaxation time λ and the viscosity η depend on the second invariant of the rate of strain $\Pi = \frac{1}{2}\,\mathrm{tr}(\boldsymbol{D}^2)$, or the shear rate of the flow $\dot{\gamma} = \sqrt{2\,\mathrm{tr}(\boldsymbol{D}^2)}$, which are fitted into a Carreau model

$$\eta(\dot{\gamma}) = \eta_0[1 + (\tilde{\kappa}\dot{\gamma})^2]^{(n-1)/2} \quad \text{and} \quad \lambda(\dot{\gamma}) = \lambda_0[1 + (\tilde{m}\dot{\gamma})^2]^{(m-1)/2} \tag{9}$$

with η_0, $\tilde{\kappa}$, n and λ_0, $\tilde{m}$, m being coefficients in the model.

Assume that secondary motions can be neglected, which is justified from the results of the perturbation analysis when the angular velocity of the rod is small enough. Then the velocity of the flow has a simple form

$$\boldsymbol{u}(r) = v(r)\boldsymbol{e}_\theta. \tag{10}$$

Denote the extra stress tensor as

$$\boldsymbol{S} = \begin{bmatrix} S_{rr} & S_{r\theta} & S_{rz} \\ S_{r\theta} & S_{\theta\theta} & S_{\theta z} \\ S_{rz} & S_{\theta z} & S_{zz} \end{bmatrix}. \tag{11}$$

With the velocity profile (10), the constitutive equation (7) yields a solution

$$S_{rr} = S_{rz} = S_{\theta z} = S_{zz} = 0, \qquad S_{r\theta} = \eta\left(\frac{\mathrm{d}v}{\mathrm{d}r} - \frac{v}{r}\right), \qquad S_{\theta\theta} = 2\lambda\eta\left(\frac{\mathrm{d}v}{\mathrm{d}r} - \frac{v}{r}\right)^2. \tag{12}$$

Substituting the above stresses into the momentum equation (6), we get

$$\frac{\partial p}{\partial r} = \rho\frac{v^2}{r} - \frac{S_{\theta\theta}}{r}, \qquad \frac{\partial S_{r\theta}}{\partial r} + 2\frac{S_{r\theta}}{r} = 0, \qquad \frac{\partial p}{\partial z} = -\rho g. \tag{13}$$

The shear stress $S_{r\theta}$ can be obtained from the second equation (13) and combined with the second equation (12), giving rise to

$$\eta(\dot{\gamma})\left(\frac{\mathrm{d}v}{\mathrm{d}r} - \frac{v}{r}\right) = \frac{C_\tau}{r^2} \tag{14}$$

where C_τ is an integration constant to be determined and

$$\dot{\gamma} = \left|\frac{\mathrm{d}v}{\mathrm{d}r} - \frac{v}{r}\right| = \frac{|C_\tau|}{\eta(\dot{\gamma})}\frac{1}{r^2}. \tag{15}$$

The pressure can then be solved from the first and third equations (13), using the third equation (12) and (14),

$$p(r, z) = 2C_\tau^2 \int_r^\infty \frac{\lambda(\dot{\gamma})}{\eta(\dot{\gamma})}\frac{1}{r^5}\,\mathrm{d}r - \int_r^\infty \rho\frac{v^2}{r}\,\mathrm{d}r - \rho g z + p_{\mathrm{a}} \tag{16}$$

where p_{a} is the atmospheric pressure above the free surface.

Let the free surface of the liquid be described as $z = h(r, \Omega)$. The profile of the climb can be determined from the normal stress condition at the free surface of the liquid,

$$-p(r, h) + p_{\mathrm{a}} = \frac{T}{r}\frac{\partial}{\partial r}\left[\frac{rh'}{(1 + h'^2)^{1/2}}\right] \tag{17}$$

where T is the surface tension and $h' = \partial h/\partial r$. Using the pressure from (16), we have

$$h(r) - \frac{1}{\rho g}\int_r^\infty \left[2C_\tau^2\frac{\lambda(\dot{\gamma})}{\eta(\dot{\gamma})}\frac{1}{r^5} - \frac{\rho v^2}{r}\right]\mathrm{d}r = \frac{T}{\rho g r}\frac{\partial}{\partial r}\left[\frac{rh'}{(1 + h'^2)^{1/2}}\right]. \tag{18}$$

The shear stresses at the free surface must vanish. However, for the velocity field specified in (10) the azimuthal component of the shear stress is not zero when the free surface is not flat, which will induce secondary motions in the flow. However, the effect of secondary motions is negligible if the angular velocity is small enough.

The boundary conditions for the velocity field and the free surface profile are

$$v(r) = \Omega a \text{ at } r = a, \qquad \lim_{r\to\infty} v(r) = \lim_{r\to\infty} v'(r) = 0 \tag{19}$$

$$h'(r, \Omega)\big|_{r=a} = 0 \quad \text{and} \quad \lim_{r\to\infty} h(r, \Omega) = \lim_{r\to\infty} h'(r, \Omega) = 0. \tag{20}$$

Evaluating (14) at $r \to \infty$, we find

$$v \approx -\frac{C_\tau}{2\eta_0}\frac{1}{r} \text{ as } r \to \infty. \tag{21}$$

3. A special case—upper-convected Maxwell (UCM) fluid

When both the viscosity and the relaxation time of the fluid are independent of shear rate, i.e. $\eta = \eta_0$ and $\lambda = \lambda_0$, the White–Metzner model reduces to the upper-convected Maxwell model. The velocity can be integrated from (14),

$$v(r) = \frac{\Omega a^2}{r} \tag{22}$$

and the constant C_τ is given by

$$C_\tau = -2\eta_0 \Omega a^2. \tag{23}$$

Moreover, (15) and (16) reduce to

$$\dot{\gamma} = 2\Omega \frac{a^2}{r^2} \tag{24}$$

and

$$p(r, z) = \frac{2\lambda_0 \eta_0 \Omega^2 a^4}{r^4} - \frac{\rho \Omega^2 a^4}{2r^2} - \rho g z + p_a. \tag{25}$$

When the surface tension is ignored, the free surface is given by

$$h(r, \Omega) = \frac{\Omega^2}{\rho g}\left(\frac{2\lambda_0 \eta_0 a^4}{r^4} - \frac{\rho a^4}{2r^2}\right). \tag{26}$$

Compared with the climbing height for a second-order fluid without surface tension, eqn. (3), the climbing constant for a UCM fluid can be defined as $\hat{\beta} = \lambda_0 \eta_0$.

In a White–Metzner fluid, even if the viscosity and the relaxation time do change with shear rate, when the parameters $\tilde{\kappa}$ and $\tilde{m}$ in the viscosity and relaxation functions (9) are small enough such that the shear thinning and reduction of the relaxation time occur at shear rates larger than those experienced in the rod-climbing experiment, the results for the UCM fluid would still hold.

4. Numerical solution

Given the viscosity and relaxation time functions (η and λ) for the liquid, this problem is fully described by the velocity profile equation (14) and the free surface equation (18). The velocity equation does not depend on the free surface profile, and can be solved separately. Once the velocity profile is known, the free surface profile can be determined for a given value of the contact angle, which is here taken to be zero. These two equations are solved numerically using a shooting method.

To solve the velocity equation (14), the slope of the velocity dv/dr at $r = a$ is selected to target the asymptotic value of v at $r \to \infty$, relation (21). In each shooting, given a value of dv/dr at $r = a$, the value of C_τ is calculated by evaluating (14) at $r = a$. The solution is then marched to larger r and at a sufficiently large r, the asymptotic value of v is checked; if (21) is not satisfied, the value of dv/dr is adjusted for the next shooting.

A similar shooting method is used to solve the free surface profile equation (18). The height rise on the rod $h(a)$ is used as the adjusting parameter, and the conditions at $r \to \infty$ are the targets. In the method, numerical integration is implemented to evaluate the integral in (18) at different locations.

5. Results

We have already noted that in an upper-convected Maxwell fluid where the relaxation time and viscosity are constants, the climbing profile is approximated by eqn. (26) when the surface tension is zero or by eqn. (4) when the surface tension is not zero. For most cases when the climbing constant $\hat{\beta} = \eta_0 \lambda_0$ is not too small, the second term in these two equations can be neglected. Then the height rise on the rod is proportional to $\hat{\beta}$ and Ω^2, or

$$h(\Omega) \propto \eta_0 \lambda_0 \Omega^2. \tag{27}$$

Thus at a given angular velocity, the height rise on the rod is proportional to the values of the relaxation time and the viscosity.

First we examine the effect of letting the relaxation time vary with $\dot{\gamma}$ when the viscosity is constant. In this case, the velocity and the shear rate are identical to those of the UCM, and the relaxation time may be expressed as

$$\lambda(\dot{\gamma}) = \lambda_0\left(1 + \frac{4\Omega^2 a^4 \tilde{m}^2}{r^4}\right)^{(m-1)/2}. \tag{28}$$

Figure 3 shows the computed height rise on the rod for different power indices in the relaxation time function (28). In the computation we take $\rho = 1.0$, $a = 1.0$, $g = 980$, $T = 40$, $\eta = \eta_0 = 10$ and $\tilde{m} = 10$. For each curve in the figure, the relaxation time at zero shear rate λ_0 is chosen such that its value remains the same at a fixed shear rate, say $\lambda(2) = 1.0$. In this way the height rise on the rod at the angular velocity corresponding to this specified shear rate will be approximately constant. This figure shows that when the power index m decreases (the relaxation time decreases more rapidly with the shear rate) the climbing profile on the rod deviates from the quadratic dependence on the angular velocity.

We next examine the effect of changing the viscosity function when the relaxation time is constant. The viscosity function is given in the first eqn. (9). The fluid is shear thinning when the power index n is less than one. Figure 4 gives the computed height rise on the rod for different power indices. In the computation we take $\rho = 1.0$, $a = 1.0$, $g = 980$, $T = 40$, $\lambda_0 = \lambda = 1.0$ and $\tilde{\kappa} = 20$. The viscosity functions shown in Fig. 4 have been selected so that the rise on the rod will be nearly the same at some particular value of $\dot{\gamma}$, here taken as $\dot{\gamma} = 2$ with $\eta(2) = 10$ for each curve in Fig. 4. The value $\dot{\gamma} = 2$ corresponds approximately to $\Omega = n\dot{\gamma}/2 = n$ (see eqn. (31)).

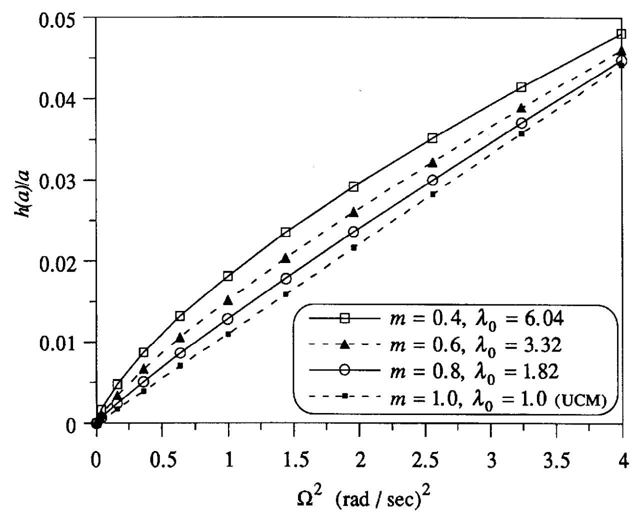

Fig. 3. Effect of the index in the function of relaxation time on the height rise on the rod with a constant viscosity.

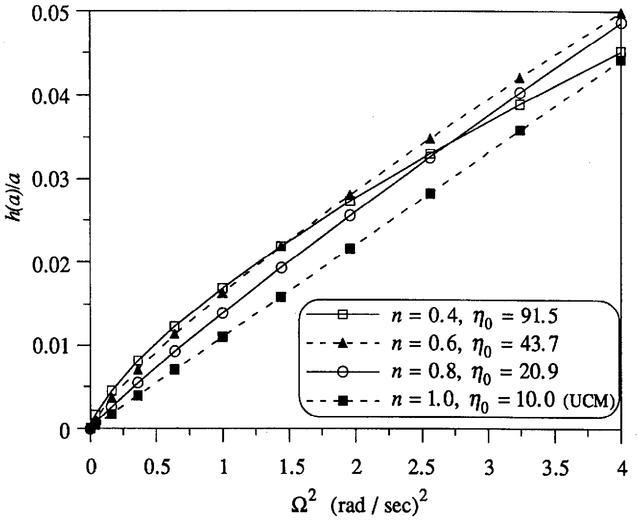

Fig. 4. Effect of the index of shear thinning on the height rise on the rod with a constant relaxation time.

Figure 4 shows that the more shear thinning the fluid (decreasing power index n), the greater is the departure from the quadratic dependence on the angular velocity.

When both the relaxation time and the viscosity are allowed to change with the shear rate, we expect to get greater deviation from second-order theory. This can be understood by the following simplified argument. In a White–Metzner fluid, when the viscosity and the relaxation time do change at the shear rate of the rod-climbing experiment, because the most important region for climbing is the region next to the rod, we may use the viscosity and the relaxation time evaluated at the shear rate in this region to calculate the height rise on the rod. Thus we may locally freeze the viscosity and relaxation time to use the result for a UCM fluid (27):

$$h(\Omega, \dot{\gamma}) \propto \eta(\dot{\gamma})\lambda(\dot{\gamma})\Omega^2, \tag{29}$$

where $\dot{\gamma}$ is the shear rate on the rod. Supposing now that at the given $\dot{\gamma}$ (still small) the viscosity and relaxation function have entered into the power-law region, we have

$$\eta(\dot{\gamma}) = \eta^* \gamma^{n-1} \quad \text{and} \quad \lambda(\dot{\gamma}) = \lambda^* \dot{\gamma}^{m-1} \tag{30}$$

where η^* and λ^* are constants. The shear rate at the surface of the rod is given by

$$\dot{\gamma} = \frac{2}{n}\Omega \tag{31}$$

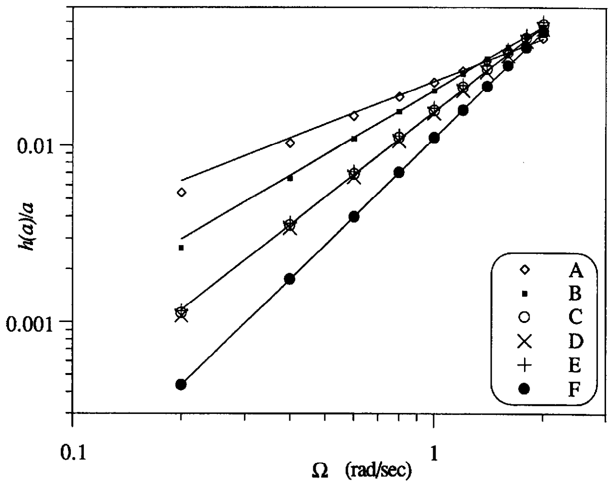

Fig. 5. The comparisons between the profile of numerical calculation and the simplified power-law relation (32). The dots are numerical calculations and the solid lines are the corresponding power-law relations. The parameters used are listed below.

Cases	η_0	n	λ_0	m	C	$n+m$
A	91.48	0.4	6.04	0.4	0.0230	0.8
B	43.74	0.6	3.32	0.6	0.0205	1.2
C	20.91	0.8	1.82	0.8	0.0155	1.6
D	10.0	1.0	3.32	0.6	0.0155	1.6
E	43.74	0.6	1.0	1.0	0.0155	1.6
F	10.0	1.0	1.0	1.0	0.0110	2.0

$\rho = 1.0$, $a = 1.0$, $g = 980$, $T = 40$, $\tilde{\kappa} = 20.0$, $\tilde{m} = 10.0$.

(see eqn. (2.15) on p. 30 of Ref. 15). Substituting (30) and (31) into (29), we obtain

$$h(\Omega) \approx C\Omega^{n+m} \tag{32}$$

where C is a new constant. Figure 5 shows the comparisons between the results of the numerical computation and the simplified equation (32) which shows that the power-law argument presented above works excellently.

6. Comparison with experimental data for A1

A1 is a 2% (w/v) polyisobutylene ($M_w = 4.3 \times 10^3$ kg mol^{-1}, $M_w/M_n = 2$) solution in decalin (a mixture of *cis*- and *trans*-decahydronaphthalene). It was prepared and sent to a number of laboratories around the world. A workshop was held in France to compare the results of experiments on this fluid in shear and extension.

The physical properties of A1 are summarized by Hudson and Jones [14]. Its density $\rho = 880\ \mathrm{kg\,m^{-3}}$ at 25°C. The surface tension for A1 was not measured; however, it is estimated to be around $T = 30 \times 10^{-3}\ \mathrm{N\,m^{-1}}$, since for Vistanex (polyisobutylene) in decalin solution the surface tension is about $T = 31.2\text{–}39.2 \times 10^{-3}\ \mathrm{N\,m^{-1}}$ at different concentrations (Joseph et al. [8]), and for M1, which is also a polyisobutylene dissolved in a complex solvent, the surface tension is $T = 30 \times 10^{-3}\ \mathrm{N\,m^{-1}}$ (Hu et al. [13]).

The viscosity of A1 was fitted to a Carreau–Yasuda model [14]

$$\eta = \eta_\infty + \frac{\eta_0 - \eta_\infty}{[1 + (3.01\dot{\gamma})^{1.1}]^{0.6}}$$

with $\eta_0 = 19.4$ Pa · s and $\eta_\infty = 0.018$ Pa · s at 25°C which we refitted into our viscosity function (9) with $\eta_0 = 19.0$ Pa · s, $\tilde{m} = 3.0$ and $n = 0.34$, that is,

$$\eta(\dot{\gamma}) = \frac{19.0}{(1 + 9.0\dot{\gamma}^2)^{0.33}}\ \mathrm{Pa \cdot s} \tag{33}$$

for shear rates less than 100.

A relaxation time at zero shear rate λ_0 can be estimated from the data of the storage modulus G' and the dynamic viscosity η' given in Fig. 7 of

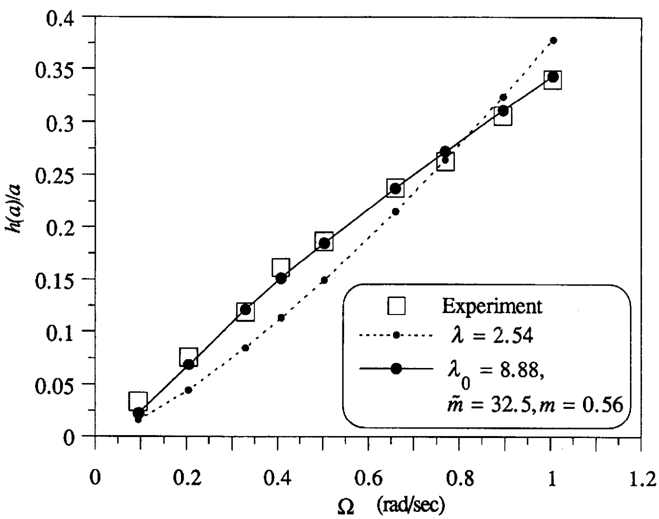

Fig. 6. The comparison between the experimental data and the numerical results. In the experiment, the radius of the rod was 0.16 cm, and the measured height of climbing is scaled by this radius. The solid line is the numerical result with a relaxation time function (second eqn. (9)) whose parameters are fitted using the rod-climbing data.

Hudson and Jones [14]. Extrapolating to the limit of $\omega = 0$, we find that

$$\lambda_0 = \lim_{\omega \to 0} \frac{1}{\eta'} \frac{G'}{\omega^2} \approx \frac{172}{19.4} \approx 8.88 \text{ s}. \tag{34}$$

This is a long relaxation time representing the relaxation of low frequency deformations. Assuming now that the low frequency response of A1 is governed by a White–Metzner model, we see first if the climbing data can be made to fit a White–Metzner fluid with $\eta(\dot{\gamma})$ given by (33) and a constant λ. The best fit is obtained with $\lambda = 2.54$. We next put $\lambda_0 = 8.88$, $\tilde{m} = 32.5$, and $m = 0.56$ in the second eqn. (9) for best fit. This second method works well (see Fig. 6).

7. Discussion

It is perhaps necessary to say that there are probably other modeling assumptions which could be developed to describe the strong departure of the A1 climbing data at low shears from that predicted by second-order theory. The results displayed in this paper suggest that the decay of memory with shear rate at low frequencies is important. In recent work on the settling of particles, Joseph and Liu [16] discussed corridors of reduced viscosity in which the effects of shear thinning are remembered for a time. This kind of memory is long; much longer than that associated with the propagation of shear waves in fluids at rest. It is probable that the corridors of reduced viscosity are produced by evanescent destruction of molecular conformations in the presence of strong shears. The healing time for evanescent destruction should depend on the degree to which the molecular conformation has been taken away from its state of equilibrium or rest value by large rates of shear $\dot{\gamma}$. As a stimulus to further thinking, a disturbance of the equilibrium might be imagined to decay something like $\exp[-t/\lambda(\dot{\gamma})]$ with λ given by the second eqn. (9), with fast decay for small λ (large $\dot{\gamma}$). This kind of theory has yet to be developed. We may think of limiting cases in which the rest conformation is so easily destroyed by shear that the relaxation time is effectively zero at most shear rates except the smallest ones. Such fluids would exhibit elastic responses at very low shear rates, but with no normal stresses. Such fluids would not climb a rod. The 3% solutions of aqueous Xanthan studied by Joseph and Liu [16] have the properties just described. Xanthan is a semi-rigid polymer whose rest conformations are readily upset by shearing.

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