Vortex rings of one fluid in another in free fall

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Experiments in which vortex rings of one immiscible liquid are created in another from drops falling from rest under gravity are presented and interpreted. These rings are associated with circulations generated by viscosity and, unlike classical vortex rings which occur in miscible liquids at high Reynolds numbers, they can exist even at very low Reynolds numbers. Since the rings do not diffuse, they are well-defined. Nonetheless, there are many similarities in the dynamics of formation and flow of miscible and immiscible rings. Parameters are identified which appear to correlate the authors' observations and photographs of some of the more interesting events are shown.

I. CLASSICAL VORTEX RINGS

By way of comparison, it is instructive to recall that in classical hydrodynamics, it is usual to consider vortical regions embedded in an otherwise irrotational flow. In the case of the ring, a cross section [Fig. 1(a)] is like the "potential vortex" [Fig. 1(b)]: the flow outside a cylinder which rotates rigidly. This is the Taylor problem (flow between two concentric cylinders with the inner one rotating and the outer one fixed) with the outer cylinder moved to infinity. The streamfunction ψ of the flow, with $(\psi_y, -\psi_x)$ for the velocity in the x-y plane, and $r=(x^2+y^2)^{1/2}$, is then $\psi = c \ln r$. Thus, $\Delta\psi = 0$ and the flow is irrotational. The potential vortex satisfies the no-slip boundary condition at the cylindrical boundary, and it is one of only a few potential flow solutions of the Navier–Stokes equations.

Vortex rings can be generated in a number of ways. One way is to impulsively eject a puff of fluid from a circular opening into a bath of the same fluid, as in the smoke ring.[1] Another is to let a drop of liquid fall into a pool of the same liquid.[2] A third method is to force a buoyant fluid into a tank of water (see Sec. 6.3.2 of Ref. 3).[4] These experiments do not involve immiscible liquids. Rings are more easily created in miscible rather than immiscible liquids. Thomson and Newall[5] did an interesting study of ring formation and their stability in miscible and immiscible liquids. They stated the following:

> "It is not every liquid, however, which, when dropped into water, gives rise to rings, for if we drop into water any liquid which does not mix with it, such as chloroform, the drop in consequence of the surface tension remains spherical as it descends. In fact, we may say that, with some few exceptions to be noticed later, rings are formed only when a liquid is dropped into one with which it can mix. This is important, because surface tension has been supposed to play an important part in the formation of these rings; it is difficult, however, to see how any appreciable surface tension can exist between liquids that can mix, and as far as our experiments go they tend to show that it is only the absence of surface tension which is necessary for their production."

On the whole, it is not surprising that a phenomenon which occurs at zero interfacial tension also occurs at small values of interfacial tension. And *small* here means *with respect to viscous effects*, so that the actual numerical value of the coefficient of interfacial tension does not have to be small. The processes are similar whether miscible or immiscible liquids are involved, up to the ring stage. However, interfacial tension does affect the breakup pattern: for example,[6] it can prevent the smaller-sized drops from repeating the sequence of ring formation and breakup; this limits the vortex cascade (see Figs. 1 and 2 of Ref. 5 for a description of this cascade for the case of an ink drop in water) to one or two stages. Membrane *rupture* is another form of breakup. The rupture strength or breaking strength of membranes is not well understood but it may be related to surface tension. We know that the rupture strength generally decreases with surface tension. When the surface tension parameter is small but the surface tension is large, a vortex ring spanned by a tough permanent membrane can form. In other cases in which interfacial tension has been reduced by surfactants, the membrane is blown out and an unstable vortex ring of the type shown in our photographs forms. The difference between strong and weak interfacial tension is illustrated in Figs. 2 and 3. In Fig. 2, a smaller drop of 1000 cS silicone oil is sucked into the wake of an oblate ringlike cap of the same silicone oil falling in a contaminated soybean oil. The membrane in this system is too tough to break. On the other hand, if a surfactant (97% dye, 3% Rhodamine B base powder, Aldrich Chemical Co., Milwaukee, WI) is added (as in Fig. 3), the membrane breaks readily. Figure 1(d) of Ref. 7 illus-

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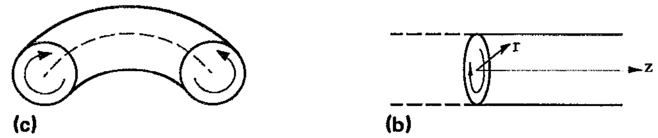


FIG. 1. (a) The two circular cross sections of a ring vortex are shown. The flow is as indicated. The dashed line denotes the axis at the center of the ring. The ring as a whole turns about this axial line. The continual turning of the ring is analogous to the rigid-body rotation of a straight cylinder shown in (b) if the ring were cut and straightened. (b) An infinitely long solid cylinder is rotating with azimuthal velocity Ωr, where Ω is the angular speed. The flow outside the cylinder is irrotational and given by the streamfunction $\psi = c \ln r$.

trates this well. The existence of a spanning membrane in miscible liquids is hard to understand without invoking the idea of transient interfacial tension induced by momentarily strong gradients of composition. Such notions were introduced by Kojima *et al.*,[8] and are considered in more detail in Ref. 9.

II. THE NORMAL STRESS BALANCE

It is probable that the parameters which control the deformations of drops to rings in free fall are associated with the stress balance at the interface:

$$-[\![p]\!]\mathbf{n} + 2[\![\mu \mathbf{D}[\mathbf{u}]]\!]\mathbf{n} + S^*\mathbf{n}2H = 0,$$

where H is the mean curvature at a point on the interface Σ and $\mathbf{D}[\mathbf{u}]$ is the rate of strain. This equation may be decomposed into normal and tangential parts.

We denote

$$[\![\cdot]\!] = (\cdot)_d - (\cdot)_o,$$

to be the jump in the quantity $\cdot$ across the interface, where the subscript d stands for drop and o stands for the outer fluid. In the equilibrium case (that is, no flow), the drop or bubble pulls into a sphere with radius $R_1 = R_2 = a$ and equilibrium pressures satisfying

$$[\![p^e]\!] = p_d^e - p_o^e = 2S^*/a.$$

By using the condition that the velocity is continuous at the interface and the continuity equation, the reader may verify that

$$[\![D_{nn}]\!] = 0, \tag{1}$$

where we denote

$$D_{nn} = \mathbf{n}\cdot\mathbf{D}[\mathbf{u}]\cdot\mathbf{n}.$$

The next step in the reduction of the normal component of the stress balance on the interface Σ is the decomposition of the pressure into an equilibrium part p^e, a hydrostatic part $p^s = \rho g z$, with $p^s = \rho_d g z$ in the drop, and a dynamic part Π due to the motion. We use a coordinate system where the origin is the center of the spherical drop, and we denote the parametrization for the surface of the drop by $z = z_\Sigma(x,y)$. Thus, $[\![p^s]\!] = [\![\rho]\!] g z_\Sigma$, and

$$[\![p]\!] = [\![p^e]\!] + [\![p^s]\!] + [\![\Pi]\!] = (2S^*/a) + [\![\rho]\!] g z_\Sigma + [\![\Pi]\!]. \tag{2}$$

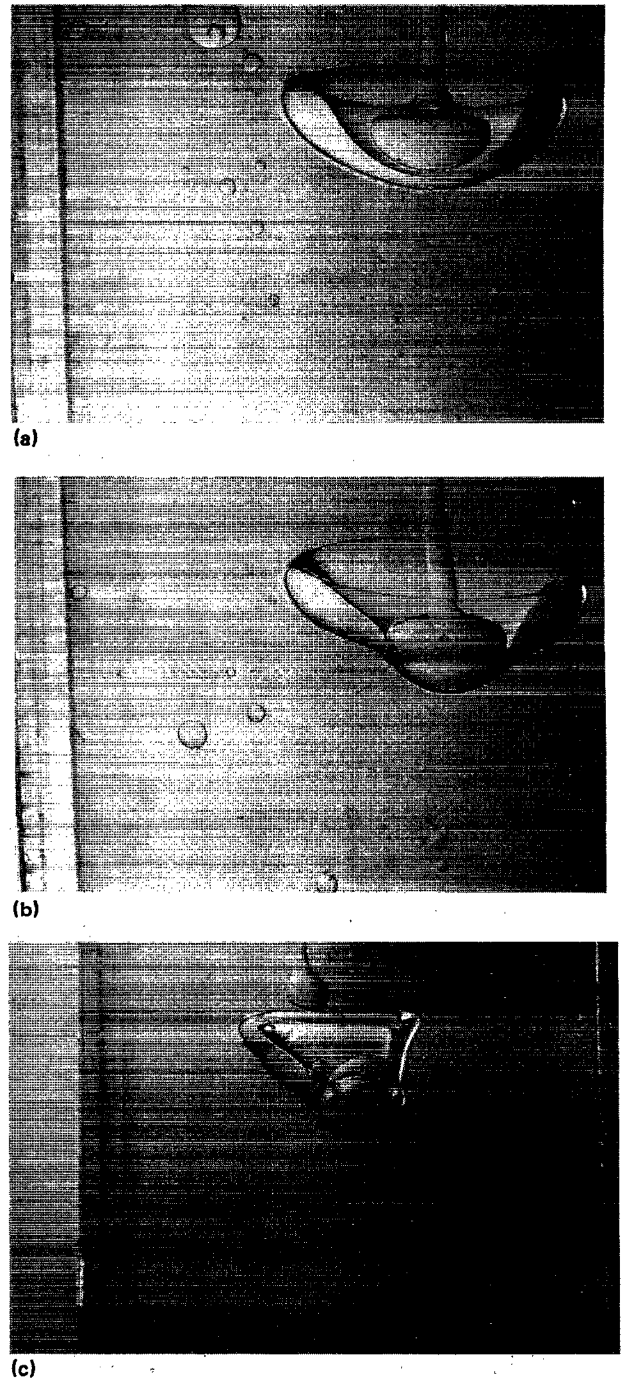


FIG. 2. Failure of poke-through of captured drop of 1000 cS silicone oil in an indented oblate drop of the same silicone oil falling through contaminated safflower oil. (a) The captured drop is sucked strongly into the wake behind the oblate drop. There is a tail drawn out of the captured drop by the motion of safflower oil in the wake which is reminiscent of the tail behind drops in miscible liquids (cf. Fig. 11). (b) The drops are sucked into strong contact. (c) The captured drop decelerates under the restraining action of the silicone oil membrane on the oblate drop which never breaks.

Combining now (1) and (2) with the stress balance equation at the interface, we find that

$$-[\![\Pi]\!] + 2D_{nn}[\![\mu]\!] + [\![\rho]\!] g z_\Sigma + S^*\left(\frac{1}{R_1} + \frac{2}{R_2} - \frac{2}{a}\right) = 0. \tag{3}$$

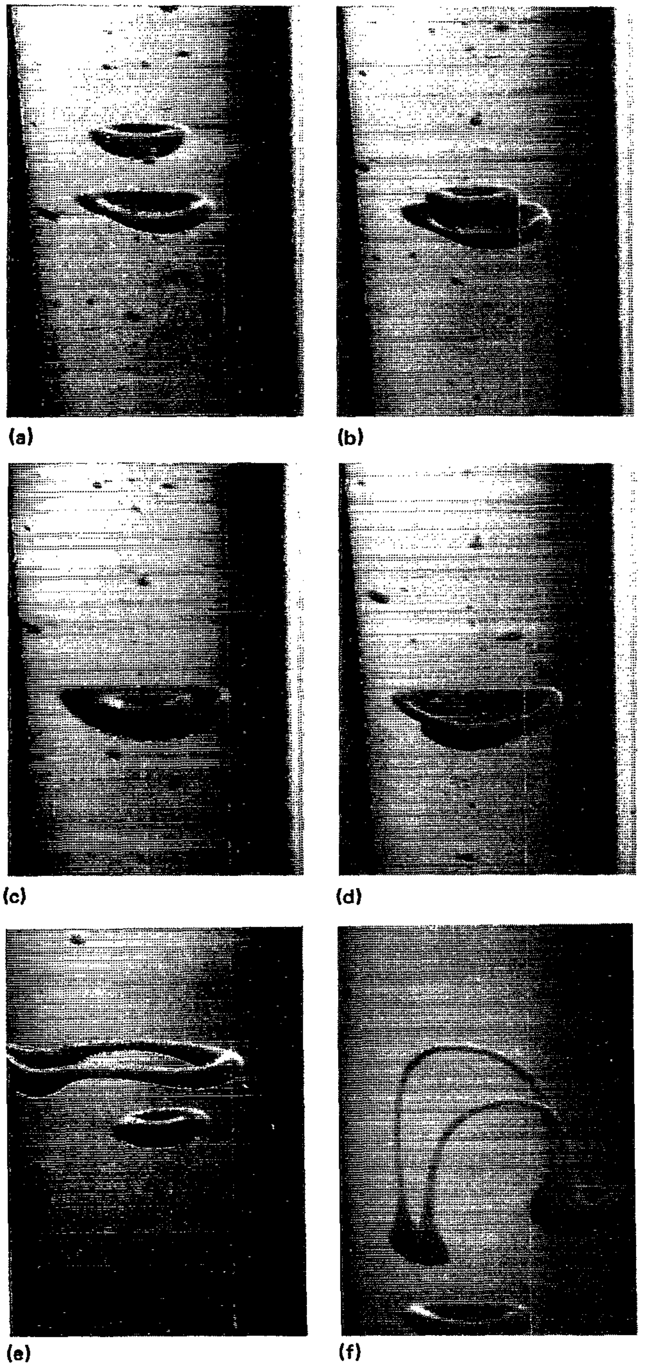


FIG. 3. Ring formation in 1000 cS silicone oil with a surfactant falling in soybean oil. The surfactant is a trace amount of 97% dye with 3% Rhodamine B base powder. (a) One indented oblate sphere accelerates in the wake of another; (b) they come close. (c) Poke-through: the large ring loses its membrane. (d) The small ring never pokes through; it retains the oblate indented shape. (e) Beginning of the two-lobe instability of the Rayeligh–Taylor type. (f) The instability can be compared with Fig. 6(b) where the membrane does not break.

The dynamic pressure is of course an unknown which must be determined from the solution.

III. DIMENSIONLESS PARAMETERS

To identify dimensionless parameters, we scale lengths with a, the radius of the equivalent spherical drop or bubble with the same volume, and velocity with U to be specified later (see next section). The normal stress balance at the interface (see previous section) shows that there are four forces at work: gravity, surface tension, inertia, and viscosity.

The viscosity ratio

$$M=\mu_d/\mu, \tag{4}$$

where μ_d is the viscosity of the drop and μ is the viscosity of the ambient fluid, is very important.

The ratio of inertia to viscous forces is measured by the Reynolds numbers

$$R=Ua/\nu, \quad R_d=Ua/\nu_d, \tag{5}$$

where $\nu=\mu/\rho$, $\nu_d=\mu_d/\rho_d$. To form immiscible vortex rings, inertia is important because the drop will be close to a sphere if R and R_d are sufficiently small.[10]

The viscous part of the normal stress in the drop is scaled by $U\mu_d/a$ and in the exterior fluid by $U\mu/a$. The interfacial tension term in the stress balance is scaled by S^*/a. The ratio of the stress associated with interfacial tension to the viscous part of the normal stress in the outer fluid is

$$S^*/\mu U=J/R, \tag{6}$$

where

$$J=S^*a/\nu^2\rho \tag{7}$$

is Chandrasekhar's capillary number (used in his study of the capillary instability of a jet; see Sec. 111 of Ref. 11) for the outer fluid. Similarly, for the inner fluid we have

$$S^*/\mu_d U=J_d/R_d, \tag{8}$$

where

$$MJ_d/R_d=J/R. \tag{9}$$

For a ring to form, the tendency for interfacial tension to keep the drop spherical should be overcome by the effect of viscosity to distort it. Thus, we expect to see rings when $J/R \ll 1$. In our experiments, we got ring formation only when $M \gg 1$. We did not observe rings in immiscible liquids when $M<5$. The condition $M \gg 1$ may not be universal. Certainly, it is easier to form vortex rings in miscible liquids; for these, $J/R=0$, but evidently when $M \approx 1$ it is possible to form vortices from ink drops falling in water.

We have already mentioned that inertial effects are always important in deforming the drop away from a sphere. These effects can be measured by the Weber number, the ratio of interfacial tension to inertia. The inertial part of the dynamic pressure for the outer fluid is scaled with ρU^2 and the drop with $\rho_d U^2$. The ratio of interfacial tension to inertia in the outside fluid is

$$\frac{(S^*/a)}{\rho U^2}=\frac{J}{R^2}=\frac{1}{W}, \tag{10}$$

and in the drop is

$$\frac{(S^*/a)}{\rho_d U^2}=\frac{J_d}{R_d^2}=\frac{1}{W_d}, \tag{11}$$

where W is the Weber number. Obviously,

TABLE I. Fluid properties. Table notes: Canola oil is also termed rapeseed oil; the glycerin listed is 0.99 pure USP glycerin; the percentages listed for golden syrup and glycerin are dilutions with water; Alconox is an industrial glass cleaner and is used as a surfactant with water.

Fluid	Density (g/cm^3)	Kinematic viscosity (cS)	Viscosity (g/cm sec)
Canola oil	0.915	67	0.61
Glycerin	1.265	656	8.29
0.95 Gly[a]	1.245	244	3.03
0.91 Gly	1.240	113	1.40
0.92 golden syrup	1.400	2 606	36.49
Lyle's golden syrup	1.440	20 804	299.58
Motor oil 30W	0.886	316	2.80
Olive oil	0.914	69	0.63
Palmolive soap	1.05	238	2.50
Safflower oil	0.920	51	0.47
Sesame oil	0.920	64	0.59
Shell Research oil	0.895	2 037	18.23
Sil[b] 5 cS	0.930	5	0.05
Sil 100 cS	0.960	100	0.96
Sil 200 cS	0.970	200	1.94
Sil 300 cS	0.970	300	2.91
Sil 400 cS	0.970	400	3.88
Sil 500 cS	0.971	500	4.86
Sil 600 cS	0.971	600	5.83
Sil 1000 cS	0.971	1 000	9.71
Sil 10 000 cS	0.975	10 000	97.50
Sil 30 000 cS	0.975	30 000	292.50
Soybean oil	0.922	53	0.49
Water	1.000	1	0.01
Water + Alconox	1.080	33	0.36
Walnut oil	0.925	51	0.47

[a] 0.95 Gly = 95% glycerin in 5% water.
[b] Sil = Silicone oil with indicated viscosity.

$$\frac{\rho_d}{\rho}\frac{J_d}{R_d^2}=\frac{J}{R^2}. \tag{12}$$

Since ρ and ρ_d do not differ greatly in our experiments, the Weber number is nearly the same in the outside fluid and the drop. The Weber number W_d for systems that do form rings ranges between 330 and 9600 whereas the W_d for systems that do not form rings ranges between 0.3 and 11 000. The low Weber number drops are spherical.

IV. PHYSICAL AND OTHER PROPERTIES

The physical properties are density, viscosity, and interfacial tension. Other properties used in our discussion are the velocity U and the drop size a. First, we discuss the fluid properties.

Table I lists the fluids used in the experiments. The densities were measured using a Curtin Scientific hydrometer at approximately 21 °C. The viscosities were measured using standard Cannon Fenske tube viscometers. The interfacial tension S^* was measured with the spinning rod tensiometer.

Our early experiments on vortex rings were carried out in a Plexiglas box 3 in. square and 8 ft long. The top of the box is open to allow introduction of the drop and the bottom is closed by a valve. The valve holds the host fluid in and allows the removal of the dropped fluids that collect at the bottom. The Plexiglas is clear to allow good visualization and photographic recording. The apparatus is backlighted by reflecting incandescent light off of a translucent Plexiglas sheet. The most recent experiments were carried out in a tube, which differed from the previous apparatus in that it is made of glass and has a circular rather than square cross section. The tube is 4 ft long, and like the Plexiglas box, it is open at the top, and closed at the bottom with a valve.

The method for introducing the drop into the vortex ring box is as follows. A 10 cm^3 drop of the more dense fluid was carefully placed on top of the host fluid with a calibrated beaker. This gives

$$\tfrac{4}{3}\pi a^3 = 10\ \text{cm}^3 \quad \text{or} \quad a=(2.39)^{1/3}\ \text{cm} = 1.34\ \text{cm}.$$

Care was taken to ensure that the drop was not splashed or accelerated into the host fluid.

A parametric study of drop size was also carried out with volumes other than 10 cm^3. The results of these studies are summarized in Fig. 4.

The velocity scale we use to calculate the Reynolds numbers is U given by (see Sec. 4.9 of Ref. 12)

$$U=\frac{1}{3}\frac{a^2 g}{\mu}(\rho_d-\rho)\left(\frac{\mu+\mu_d}{\mu+\frac{3}{2}\mu_d}\right). \tag{13}$$

Using this U, $R_d=Ua/\nu_d$ depends only on measurable quantities and may be interpreted as the ratio between the buoyancy and viscous forces. The assumptions leading to (13) are that the fluid is a sphere falling at constant speed in Stokes flow, and that if the shear stress and velocity are matched at the interface, then the normal stress is automati-

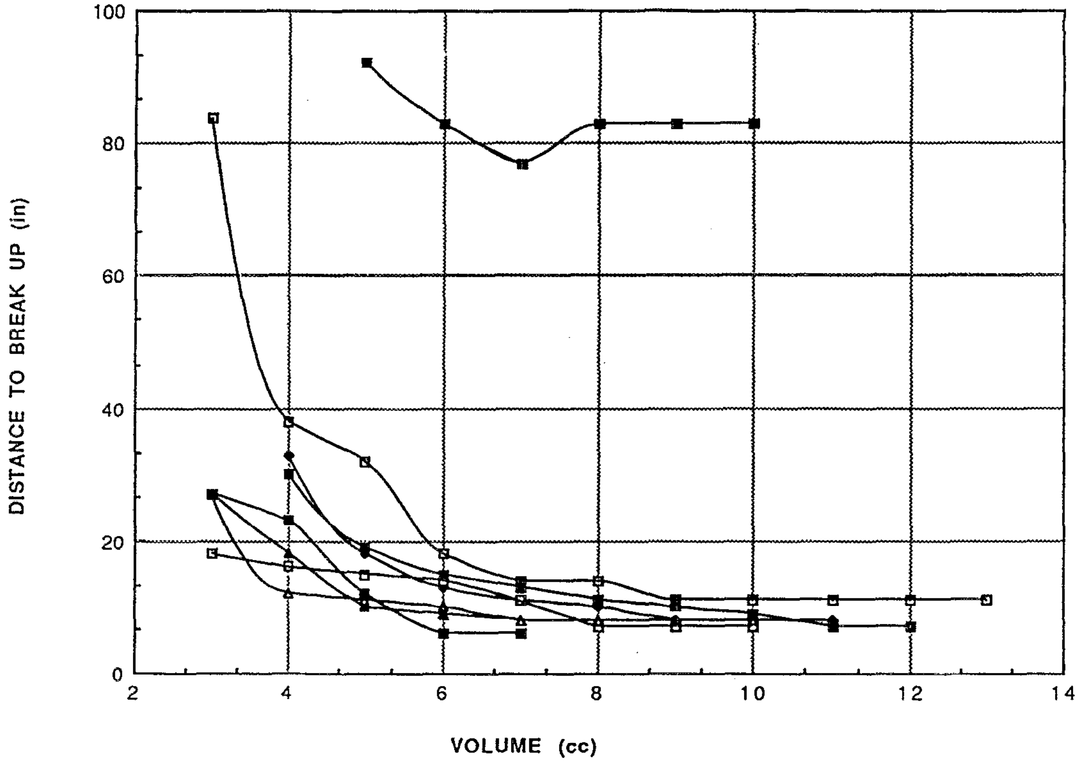


FIG. 4. Distance traveled by a falling drop before a vortex ring forms as a function of drop volume. The formation of the ring occurs when the membrane spanning the ring breaks.

cally matched. The analysis does not say anything about what the drop would do if it were not falling at the terminal speed; for example, in the experiments, the drop starts at rest, some drops do not attain any steady speed, and moreover appear not to reach the speed predicted by this formula. It is difficult to decide *a priori* on a velocity scale because we do not have a formula for predicting the velocity as the drop changes shape. For each experiment, one could measure the maximum speed attained by the drop and use that as U, and this type of data is available for some of the experiments.

The velocity of the drop as it falls in the vortex ring box has been measured for some situations and found to be much smaller than the value from (13). Measurements of the velocity of a falling drop were made by recording the time it took for the drop to cross a 6 in. region of the box. Five such regions were selected to best capture the rate of fall at critical sections. The records were taken ten times for each region and the average velocity was calculated. Figure 5 shows the average velocity versus distance down the tube for three types of glycerin (e.g., 90% glycerin means 90% glycerin in 10% water) and silicone oil falling in soybean oil. (Rings were observed in the 100% glycerin case, but not in the other two cases, which happen to have higher velocities in the figure.) Take, for example, the data in this figure. Compared with this, the value of U from (13) is approximately 110 in./sec, which is about 20 times the actual average velocity. This is consistent with the notion that a spherical drop would fall faster than a flattened spheroid or a ring. The swings in the measured speed reflect the changes in the shape of the drop as it evolves into a ring and decays. We should therefore keep in mind when looking at the tables that the true Reynolds numbers are probably an order of magnitude less than those tabulated.

There are also situations where U from (13) turns out to be large, which is inconsistent with one of the assumptions in the derivation of (13); but since Stokes drag is less than the actual drag at higher Reynolds numbers, we expect that the U is an upper bound on the actual maximum velocity. Thus our tabulated values of J_d/R_d and J/R in the sequel underes-

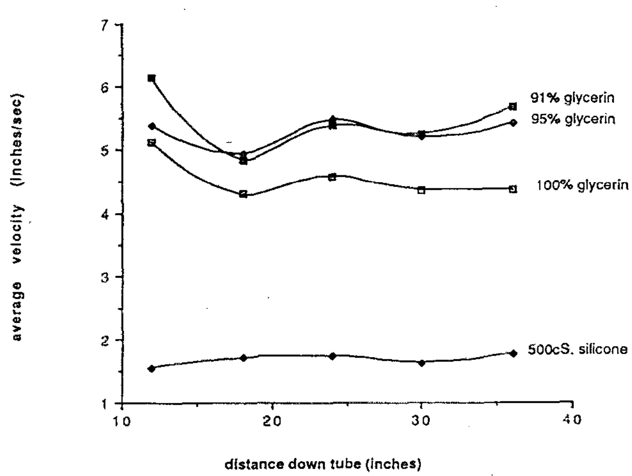


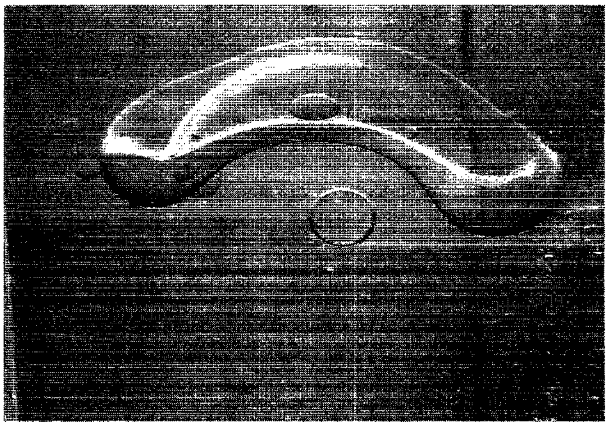
FIG. 5. The average velocity versus distance down the vortex ring tube for the designated liquids falling in soybean oil.

timate the importance of surface tension, but consistently, so that they should probably be an order of magnitude larger than they are. This would imply that the switch in the behavior from ring formation to no ring is actually occurring at a value of J/R of order 1. This, in fact, is what one would expect.

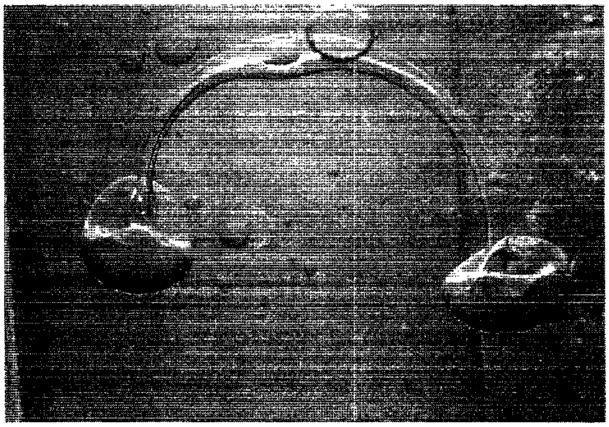
It is interesting that the condition for ring formation (on M and J/R) appears to hold for the entire wide range of Reynolds numbers encountered in the experiments. Why? In the normal stress condition at the interface, the only term we have not really commented on above is the pressure term, which is multiplied by the Reynolds number R. It appears that this term does not affect the ability to give birth to a ring: indeed, the factor R appearing there can be made to disappear just by changing the way the pressure is nondimensionalized.

The formation of vortex rings always involves the breaking of a membrane, by poke-through or blowout, and the breaking strength (toughness) of a membrane is very difficult to control, especially in silicone–vegetable oil systems. Our early experiments were recently repeated with good success except for the breaking of silicon–vegetable oil membranes. Some experiments were carried out with a contaminated safflower oil with various additives. We could never break a membrane in a silicone-contaminated safflower oil system (Figs. 2 and 6). The breaking strength of a membrane may be related to interfacial tension since we could get tough membranes to break by adding certain types of surfactants to the silicone oil (trace amounts of 97% dye, 3% Rhodamine B base powder in Fig. 3; trace amounts of Igepal in Fig. 7). We also had difficulty breaking membranes in a silicone–soybean oil system, even when uncontaminated fluids were used. However, the oils used in the most recent experiments were not exactly the same as those used earlier, and it is possible that the newer oils had an interfacial tension large enough to prohibit vortex ring formation. As was the case for the contaminated-oil system, rings were formed when the above-mentioned surfactants were added to the silicone oil.

FIG. 7. Vortex ring of 1000 cS silicone oil with trace amounts of surfactant (Igepal) falling in soybean oil after blowout.



(a)



(b)

FIG. 6. 1000 cS silicone oil falling in contaminated safflower oil. (a) A vortex ring with circulation has developed but a tough membrane spans the ring. (b) The ring is unstable in the usual way (Rayleigh–Taylor instability) forming the characteristic drops [cf. Fig. 3(d)], but the membrane breaks.

The low values of surface tension in the silicone oil–vegetable oil systems may indicate the possibility of small-scale activity at the interface. This activity could affect the boundary conditions involved, but more research must be done before any definitive statements may be made.

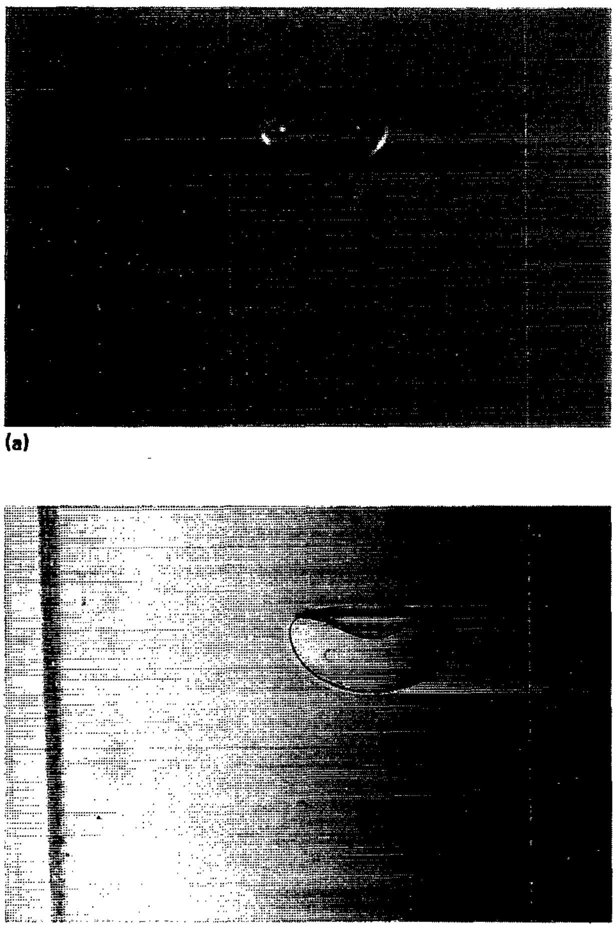
V. DISTORTION OF THE SPHERICAL DROP

It is well known that a spherical drop or bubble, moving slowly, in Stokes flow can keep a spherical shape even when interfacial tension is suppressed.[12,13] There is numerical and experimental evidence to suggest that these solutions are stable to small disturbances[14,15] but not to large disturbances.

When viscous effects win over the effect of interfacial tension, a falling drop cannot maintain a spherical shape. Numerical solutions have been obtained[16] for steady streaming flow past an axisymmetric drop over a wide range of Reynolds numbers, interfacial tension, viscosity ratios, and density ratios. Their results indicate that at lower Reynolds numbers, the shape of the drop tends toward an indented oblate shape with decreasing interfacial tension, and at higher Reynolds numbers the drop becomes more disk shaped with decreasing interfacial tension.

In our experiments, the drop is released at zero speed and undergoes accelerations and decelerations, so that the results mentioned above concerning steady motions cannot strictly be used to infer anything about what our drop is doing. Moreover, as mentioned in the previous section, measurements of the drop speed indicate that it often does not reach the steady speed predicted by the formula (13). However, there are similarities with these analyses and what we have seen.

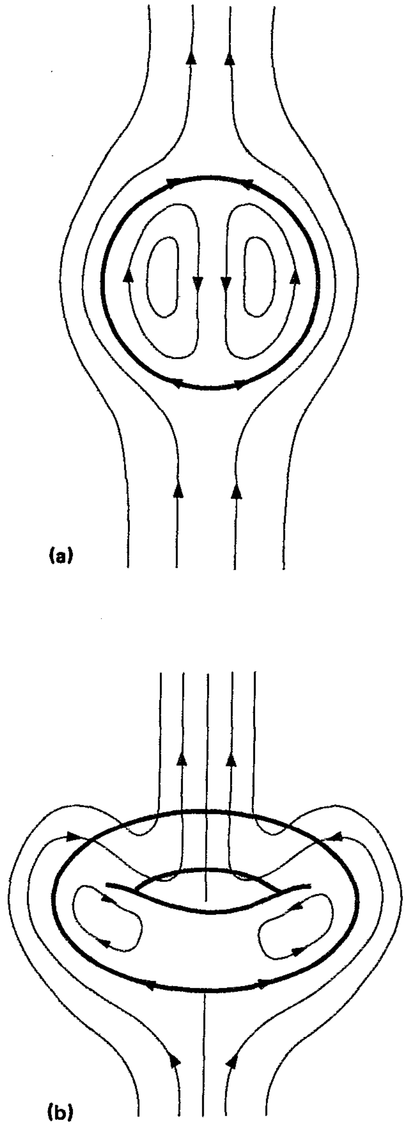
Figure 8 shows an indented oblate drop like those computed in Ref. 15 at low Reynolds numbers (see, for example, their Fig. 3). Experimental observations suggest that the streamline pattern on the concave side of the cap is probably like that of Fig. 9; there are no points of separation or vortices in this guess about the underlying fluid dynamics. The suction in the cap, call it a wake, is large and small drops and even large drops are easily captured by the indented drop, as shown in Figs. 2(a), 10, 11, and 12(a). If the conditions are right, the drop in the wake will poke through the membrane spanning the indentation, as in Figs. 3 and 12, but if the membrane is tough, as in Figs. 2 and 6, the poke-through will fail.



(a)

(b)

FIG. 8. Indented oblate drops falling in safflower oil. (a) Water $M = 0.02$, $J = 4530$, $S^* = 3.39$ dyn/cm. Indentation never develops in water and oil systems without surfactants. (b) 500 cS silicone oil. These are the most common shapes when falling. The high viscosity drop develops a circulation that brings it closer to a vortex ring.



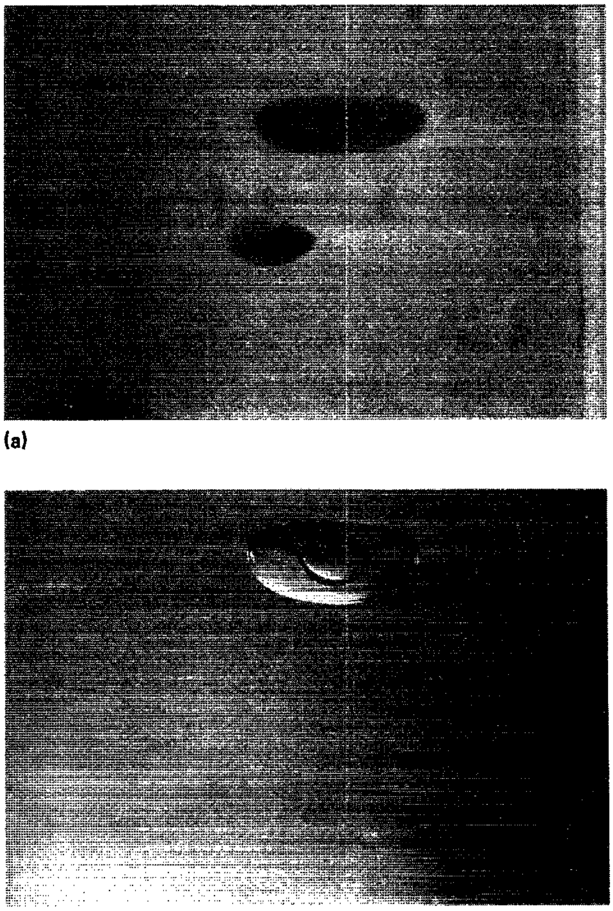
(a)

(b)

FIG. 9. Development of vorticity in a drop falling from rest. The streamlines are sketched in a frame moving with the drops: (a) from experimental observations at sufficiently small velocity; (b) larger velocity.

VI. FORMATION OF RINGS

Stuke[17] performed experiments like those reported here. He cites the work of Northrup[18] who used paraffin in a water bath, where interfacial tension is large. Northrup needed to inject the paraffin at high speed so that viscous forces would create a circulation of sufficient magnitude, as in the case of air injected into water. The larger the interfacial tension, the faster the intrusive speed necessary to create a ring. Stuke showed that rings would form at slower speeds when water



(a)

(b)

FIG. 10. Spheres nested in the wake of an indented oblate drop. (a) Glycerin falling in soybean oil $M = 16.9, J = 0.45, S^* = 18.45$ dyn/cm. (b) 500 cS silicone oil in contaminated safflower oil.

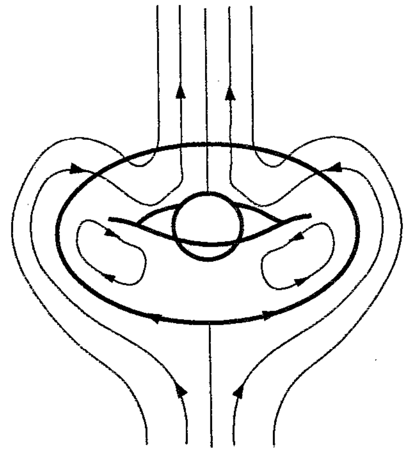


FIG. 11. A streamline pattern for Fig. 2. The flow in the wake could pull out a tail from the nested sphere if the wake were strong as in Fig. 2(b), or the surface tension weak as in the case of miscible liquids.

was replaced with amyl alcohol, with a consequent lowering of interfacial tension. The slower speeds allowed these processes to be recorded in photographs which can be compared with the photographs of this paper. Initially, there is a membrane across the hole of the ring (cf. Figs. 6 and 12) and then the membrane ruptures. Once formed, the ring starts to expand rapidly and the bulk of the paraffin flows into two or three bulges around the ring (cf. Figs. 3 and 7). These heavier bulges fall faster, so that the ring bends and breaks into two or three drops [see his Fig. 5, Fig. 2(b) of Ref. 6, and our figures]. If a drop were large enough, it would form another vortex ring and the sequence repeats itself. An analogous description of the ring instability for miscible liquids at slow speed is given in Fig. 4 of Ref. 8 and in Ref. 7.

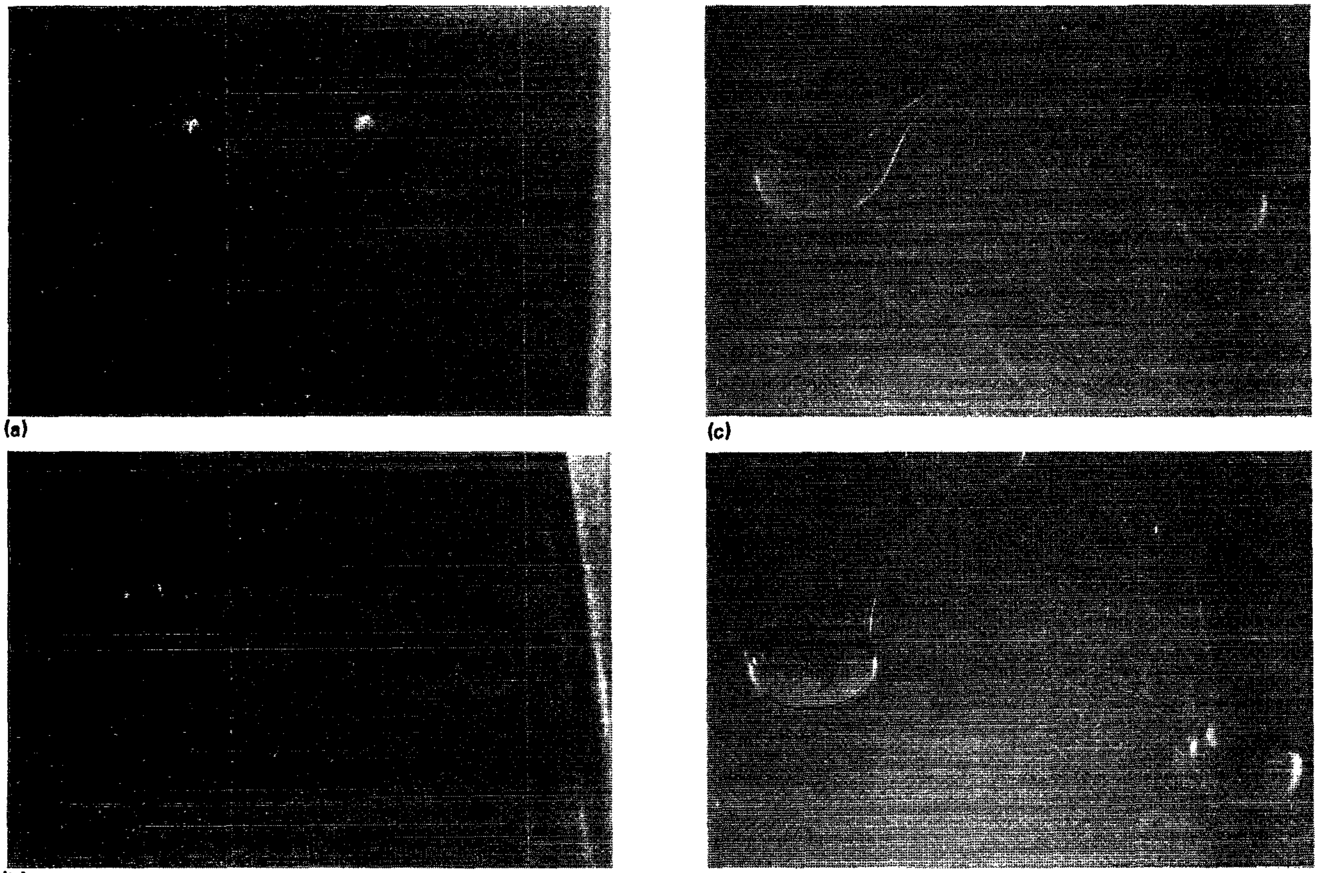
If the conditions are right, if the drop is much more viscous than the host fluid ($M \gg 1$) and the ratio J/R of interfacial to viscous forces is not too large, then the spherical drop will evolve toward a vortex ring. The entries in Tables II–IV for drops of silicone oils in soybean oil exemplify these effects well. The viscosity of silicone oils can be varied through careful mixing without changing their density or surface tension appreciably. It was observed that when the viscosity of the drop was lower than a certain value (about 500 cS here), rings did not form. In particular, when the drop was less viscous than the bath, even with very low interfacial tension, rings did not form, as in Fig. 13.

Inertia alone will not cause a ring to form. Indented oblate drops like those shown in Fig. 8 are the most robust of the falling drops. If conditions are such that the viscous action of the host fluid can create a permanent circulation [like that sketched in Figs. 9(b) and 11] of sufficient strength, the drop will begin to look like a ring, spanned by a membrane, as in Figs. 10, 11, 12(a), and especially 6(a). A free ring will form only if the membrane breaks. The membrane may or may not break. If it breaks, it does so either by poke-through of a smaller drop caught in the wake as in Figs. 6 and 12, or by blowout. Blowout can best be understood by the failure of blowout shown in Fig. 6(a). Blowout can occur only if the membrane is very weak as in miscible liquids or in low interfacial tension systems like those shown in Figs. 7 and 14.

Vortex rings are unstable; whether or not the membrane has broken, the ring will expand rapidly. The rapid extension is a universal characteristic of the instability. If a membrane remains and no drop rests in the wake to poke through, the membrane will stretch and either rupture or fold as in Fig. 6(b).

Because of capillarity, draining, or other causes, bulges develop on the ring; these fall faster than the rest of the ring, and fluid drains rapidly to the heavy bulges, exacerbating the instability. This instability can be considered as a manifestation of the Rayleigh–Taylor instability of the heavy fluid into the light, when the heavy fluid has the shape of a vortex ring. In our experiments, the draining almost always occurs at just two points of the ring, more or less at opposite points on the ring as in Figs. 3(f) and 12(d), and in the figures of Refs. 7 and 8. This type of instability scenario can occur even for ringlike structures like the one shown in Fig. 6(a) in which the membrane does not break and it leads to the folded ring shown in Fig. 6(b). The heavier places fall faster and the ring bends and breaks into drops.

O'Brien,[19] in reviewing her own work and that of Ref. 17, noted that the number of bulges which develop on the ring depends on the Reynolds number and is two for Reyn-



(a) (b) (c) (d)

FIG. 12. Poke-through of 1000 cS silicone oil in safflower oil $M = 19.8$, $J = 0.03$, $S^* = 2.41$ dyn/cm. (a) Silicone oil spheres nested in the wake of an indented oblate drop of the same oil. (b) Poke-through leads to a vortex ring. (c) Vortex ring (Rayleigh–Taylor) instability is the rapid expansion of the ring diameter and the draining of the oil into the falling bulges. (d) Two new indented oblate drops form from the falling bulges in a replication of the dynamic sequence.

TABLE II. Systems that form vortex rings. Gly denotes 100% glycerin; 0.95 Gly denotes 95% glylcerin in 5% water; Shell denotes Shell Research oil; Sil denotes silicone oil with the indicated viscosity; Soy denotes soybean oil. The difference $\Delta\rho$ denotes (density of dropped fluid) − (density of host fluid); $\Delta\nu$ denotes (viscosity of dropped fluid) − (viscosity of host fluid): this difference is negative in some of the systems in the tables.

System	$\Delta\rho$ (g/cm^3)	$\Delta\nu$ (cS)	S^* (dyn/cm)	U (cm/sec)	R_d	ρ_d/ρ	M	J_d/R_d	J/R
Gly/Soy	0.343	606	18.45	278.5	57	1.37	16.9	0.008	0.135
Gly/canola	0.350	589	35.78	227.5	46	1.38	13.6	0.02	0.272
500 cS Sil/canola	0.056	433	2.33	37.0	10	1.06	8.0	0.01	0.080
Gly/olive oil	0.351	588	10.50	242.0	49	1.38	13.2	0.005	0.066
500 cS Sil/olive oil	0.057	431	3.10	58.0	16	1.06	7.7	0.01	0.077
Gly/safflower	0.345	606	15.65	291.5	59	1.38	17.6	0.006	0.106
500 cS Sil/safflower	0.051	450	7.44	43.6	12	1.06	10.3	0.03	0.309
Gly/walnut oil	0.340	605	39.51	285.8	58	1.37	17.6	0.02	0.352
500 cS Sil/walnut oil	0.046	449	4.15	39.1	10	1.05	10.3	0.02	0.206
Gly/sesame oil	0.345	592	15.20	216.3	44	1.38	14.1	0.008	0.113
500 cS Sil/sesame oil	0.051	436	3.40	34.9	9	1.06	8.2	0.02	0.164
500 cS Sil/Soy	0.049	447	1.68	40.3	11	1.05	9.9	0.008	0.079
600 cS Sil/Soy	0.049	547	2.68	40.1	9	1.05	11.9	0.01	0.119
1000 cS Sil/Soy	0.049	947	2.41	39.7	5	1.06	19.8	0.006	0.119
10 000 cS Sil/Soy	0.053	9 947	3.29	42.3	0.6	1.06	199.0	0.0008	0.159
30 000 cS Sil/Soy	0.053	29 947	6.49	42.3	0.2	1.06	597.0	0.0005	0.298
Golden syrup/Soy	0.518	20 751	42.20	413.3	2.7	1.56	611.4	0.0003	0.183
0.92 Golden syrup/Soy	0.478	2 553	28.46	382.6	19.6	1.52	74.47	0.002	0.149
Palmolive/Soy	0.128	185	18.10	108.0	61.0	1.14	5.10	0.0006	0.003

TABLE III. Systems that do not form vortex rings.

System	$\Delta\rho$ (g/cm^3)	$\Delta\nu$ (cS)	S^* (dyn/cm)	U (cm/sec)	R_d	ρ_d/ρ	M	J_d/R_d	J/R
0.95 Gly/Soy	0.323	194	13.43	269.8	148	1.35	6.2	0.02	0.124
0.91 Gly/Soy	0.318	63	11.49	277.3	328	1.34	2.9	0.03	0.87
Water/30W motor oil	0.114	315	9.22	23.7	3172	1.13	0.004	38.85	0.156
Water/Shell	0.105	2036	42.14	3.4	450	1.12	0.0005	1251	0.625
Gly/Shell	0.370	1381	27.83	10.3	2	1.41	0.45	0.34	0.153
500 cS Sil/Shell	0.076	1537	5.82	2.2	0.6	1.08	0.27	0.53	0.143
5 cS Sil/Soy	0.008	−48	1.14	9.2	245	1.01	0.10	2.7	0.27
100 cS Sil/Soy	0.038	47	2.75	34.1	46	1.04	2.0	0.08	0.16
200 cS Sil/Soy	0.048	147	2.16	40.9	28	1.05	4.0	0.03	0.12
300 cS Sil/Soy	0.048	247	2.71	40.2	18	1.05	5.9	0.02	0.118
400 cS Sil/Soy	0.048	347	2.67	39.7	13	1.05	7.9	0.02	0.158
Water/Soy	0.078	−52	3.39	92.3	12 337	1.08	0.02	0.37	0.074
Water + Alconox/Soy	0.158	−20	4.64	156.0	632	1.17	0.73	0.08	0.056
0.60 Golden syrup/Soy	0.342	−27.5	7.42	341.0	1 788	1.37	0.659	0.07	0.046

olds numbers of order one or less. Basically, we observed only two bulges even at Reynolds numbers of order 100, with some very rare exceptions. Perhaps the number of bulges on the ring is related to capillary breakup and is strongly influenced by the value of interfacial tension. We saw many bulges when soap was added to water in soybean oil (see the last entry of Table II). This interpretation is also suggested by the closely similar instability in miscible liquids,[6] in which case the lack of surface tension promotes the formation of many more nodules around the ring, the ring breaks into many drops, and those drops subsequently repeat the cycle and there is a vortex cascade.[5] Surface tension can keep subsequent drops spherical if they are small enough, and thus inhibits the cascade.

The effect of the wall on the drop and ring needs further study.[20] For example, when a ring approaches a wall of the apparatus, it expands considerably before touching it. Also, experiments done with a vortex tube of smaller diameter show that the walls inhibit ring formation. Observations about the way a ring behaves (in the miscible case) at a variety of boundaries is reported in Ref. 18.

The dynamics leading to formation of vortex rings is not well understood. Data presented in the next section show that rings form from drops started from rest when the viscosity of the drop is relatively great and the interfacial forces do not dominate viscous forces.

A falling drop is relentlessly sheared by the host fluid, but only small portions of the host fluid come under the influence of the falling drop, and these only momentarily. If we move with the drop, we can think that we have a uniform flow around the drop as in Fig. 9(b), and this picture is also suggestive of why circulations develop in the drop and not in the host fluid. The flow around the ring would, in the case where the ring fluid is very viscous, resemble that of Fig. 1(a), where the flow is analogous to the rigid-body rotation of Fig. 1(b) and would then be almost potential flow, with potential flow at infinity (uniform flow), and the vorticity localized to the interface region between the fluids.

VII. TWO-FLUID SYSTEMS THAT DO AND DO NOT FORM VORTEX RINGS

We used formula (13) to compute the velocity of a falling drop [with $a = (2.39)^{1/3}$] and rising bubble [with volume 5 cm^3, $a = (1.19)^{1/3}$] and evaluated many of the dimensionless parameters. The parameters are listed in Tables II–IV. Parameters that are not set down explicitly in these tables can be computed readily from the listed values. Table II tabulates the systems that were observed to form rings. The other two tables list the two-fluid pairs that were observed not to form rings.

We find that to form a ring, it is necessary that the drop fluid be much more viscous than the host fluid. Another criterion which appears to be necessary is that the effect of interfacial tension should be smaller than viscous effects, which may be expressed as

TABLE IV. Systems of injected bubbles.

System	S^* (dyn/cm)	ρ_d/ρ	U (cm/sec)	R_d	M	J_d/R_d	J/R
100 cS Sil/water	24.67	0.96	984	1043	96	0.03	2.88
12 500 cS Sil/water	27.11	0.975	621	5.3	12 188	0.0004	4.8
Soy oil/water	3.39	0.922	1917	3829	49	0.004	0.196
30W motor oil/water	9.22	0.886			280		
Olive oil/water	16.42	0.914	2119	3257	63	0.012	0.72

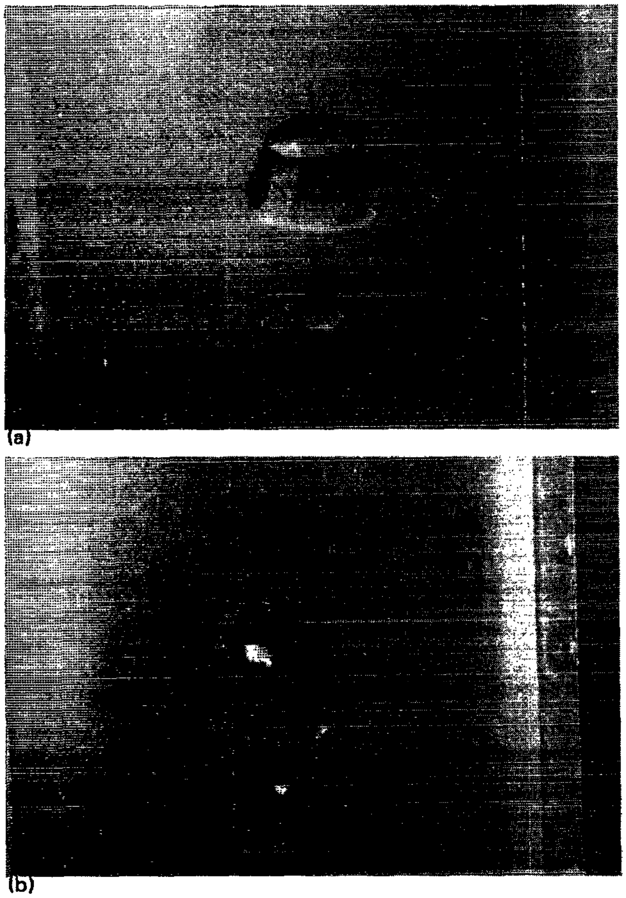


FIG. 13. Crisco rising in a column of water with surfactant (Alconox). The value of interfacial tension has been reduced from 3.39 dyn/cm to 0.158 dyn/cm by the surfactant. The membrane does not break, despite the low tension. (a) A torus is formed inside the water bag. (b) The water in the torus is dragged out in the wake.

$$J/R < O(1), \tag{14}$$

where the notation is defined in (7) and (9). The last column of Table II is comfortably in agreement with (14). However, the velocity U used to compute R was computed from (13) and the true J/R may be an order of magnitude larger than in the table. In general, however, when one inspects the tabulated data, it becomes apparent that since all of the fluids tested have low values of J/R, the factor which distinguishes a system which will not is the value of M. For the most part, the criterion for the fluids tested seems to be that M be greater than 8 or so. However, there is definitely an ambiguous range ($5 < M < 8$) within which we cannot predict whether a ring will form or not.

A very dramatic illustration of the importance of J/R and M is exhibited by data for Palmolive soap dropped in soybean oil (a ring forms; see last entry of Table II) and water dropped in soybean oil (a ring does not form). Palmolive soap is essentially water modified with a surfactant that reduces the interfacial tension enough to move J/R down to a sufficiently small value. The viscosity of Palmolive soap is greater than that of water and this alters M such that the combination of J/R and M produces a vortex ring. The evolution of the ring in this soapy solution is exceptionally rapid and the torus breaks up rapidly into small bubbles, as in the case of miscible liquids.

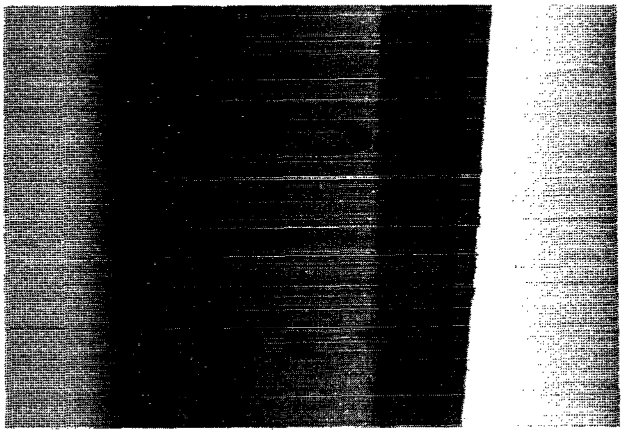


FIG. 14. Vortex ring of dyed glycering falling in soybean oil after blowout.

Table III displays systems that do not form vortex rings. The data show that a modification of the fluids will switch a system that forms a vortex ring to one that does not. An example is the glycerin and soybean oil system. When 9% water is added to glycerin, the resulting diluted solution will not form a vortex ring because the viscosity ratio M has decreased and J/R has increased to the borderline level. A similar adjustment was made for golden syrup by adding water. Of course, we cannot determine in these examples which parameter is truly causing the change of the system, since in both cases M and J/R change simultaneously. However, it is still interesting to observe the effect of altering the relevant parameters.

Table IV lists systems where bubbles of the less dense liquid were released and left to rise through the more dense liquid. In each case, an oil was released into water. No vortex rings were observed. The related case of air bubbles released into water has been shown to yield rings (see Fig. 7 of Ref. 21). The case of 12 500 cS silicone oil was inconclusive because the silicone oil showed an affinity for the Plexiglas box.

In Fig. 4, we display the results of a study of drop size on the distance required for ring formation. This distance decreases monotonically with volume and seems to asymptote to some small value less than 15 in. for large volumes.

ACKNOWLEDGMENTS

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ADDENDUM (by Paul Mohr and D. D. Joseph)

This addendum contains a brief summary of recent findings regarding drop size and surfactant effects on the formation of immiscible vortex rings that were obtained after this paper was submitted for publication. A number of experiments are reported, all of which involved dropping different quantities (1–7 ml) of 1000 cS silicone oil in soybean oil, and allowing them to fall under the influence of gravity. The results of pure silicone drops, as well as silicone drops containing the surfactant Igepal are included.

1. Introduction

The drops of silicone oil are released at the top of a 4 ft tall glass tube that contains soybean oil. They fall under gravity, and observations are made as they traverse the length of the tube. To control the drop sizes, we use a 1 in. diameter (60 ml) plastic syringe with the end almost completely removed. Since small amounts of silicone invariably stick to the syringe and since the modified syringe volume was measured to be 0.8 ml less than with the tip intact, we consistently pull the plunger back 1 ml beyond the desired drop size. With the plunger set, the syringe is filled with the silicone oil and the end is quickly placed into the soybean oil and the plunger is depressed. The syringe is tilted to a sharp angle and twisted to remove the clinging drop. While this method does not allow us to ascertain with a great degree of accuracy the true drop volume, it does provide for drops of very consistent volume.

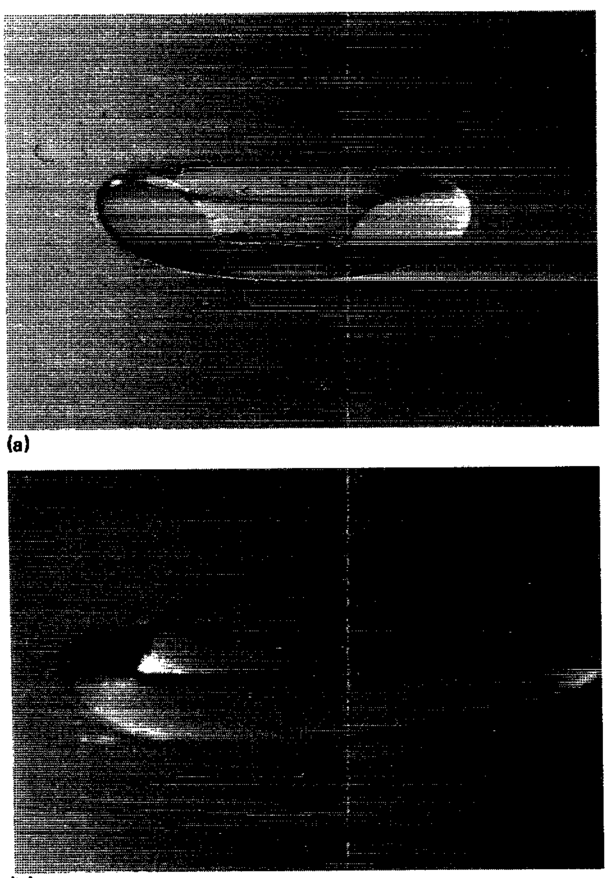

FIG. 15. (a) The center-spanning membrane is rupturing while still at the bottom of this 2 ml drop of pure silicone oil. (b) Here, the membrane has bulged up through the center of the 5 ml, pure silicone drop, forming a large dome.

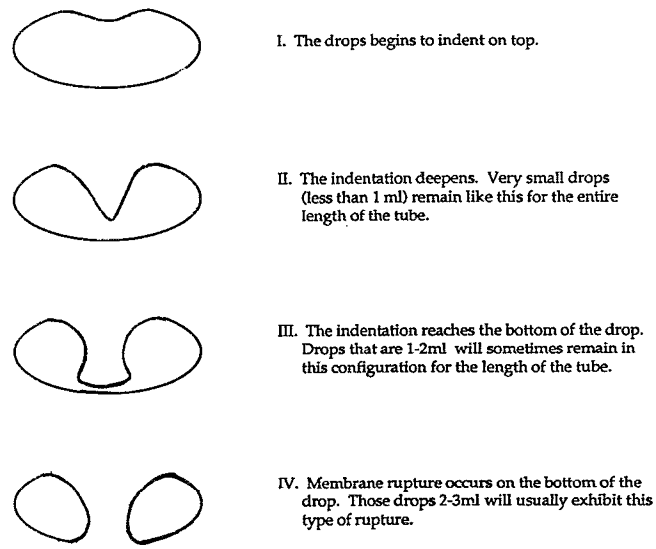

FIG. 16. Drop evolution for volumes of 2–3 ml.

2. Pure silicone oil

Small drops (roughly those <2 ml) were generally observed to remain as indented oblate spheres for the entire length of the tube. When drop sizes were increased to 2 ml, the indentation in the drop deepened, and for many drops

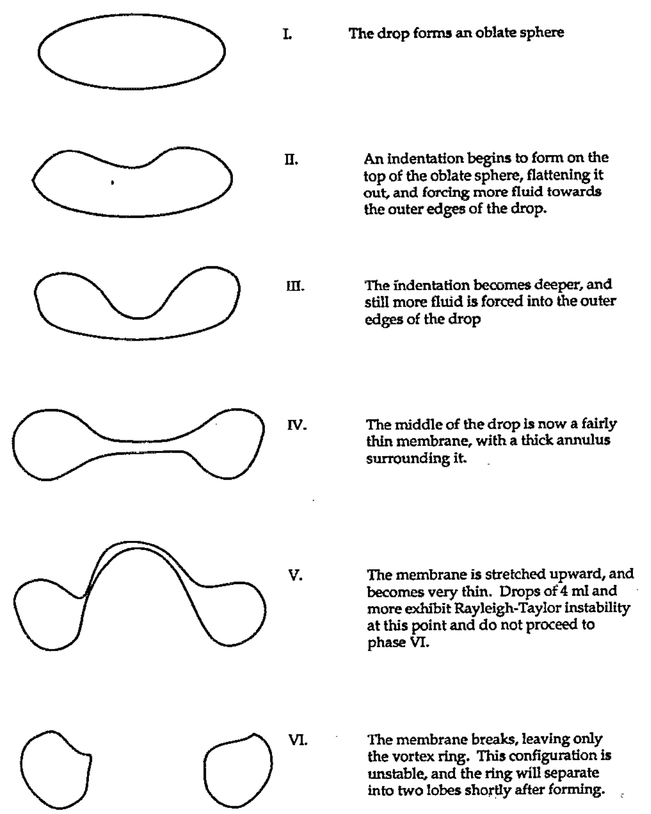

FIG. 17. Drop evolution for volumes of 3–5 ml.

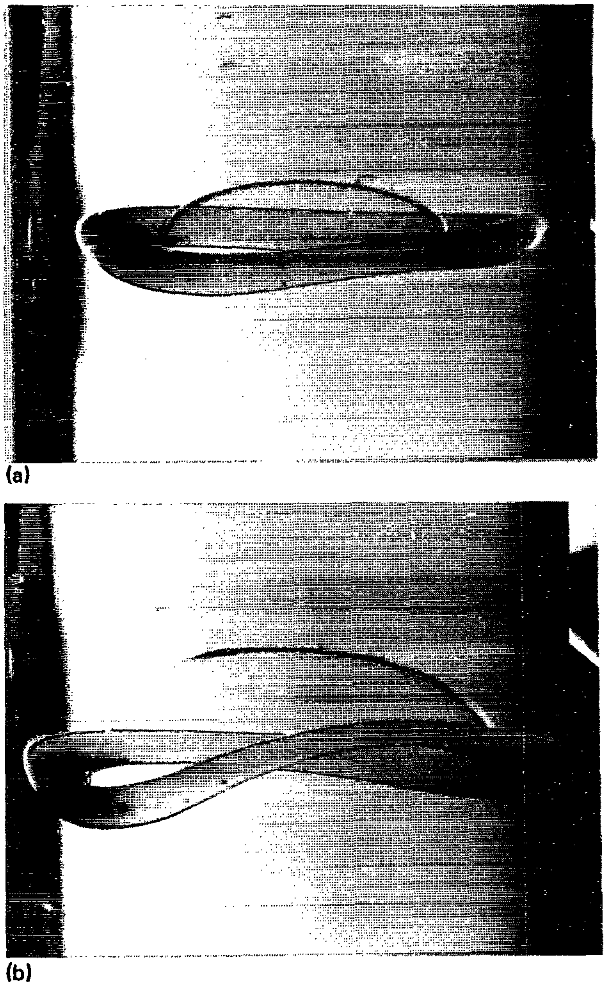

FIG. 18. (a) This 5 ml drop of 1000 cS silicone containing 0.5% Igepal CO-530 has formed a membrane similar to that in Fig. 15(b). (b) The membrane is rupturing from the left side of the drop to the right. Rupture of membranes at this stage of development is extremely rare with pure silicone oil, but occurs frequently when Igepal is added.

(six times out of ten observations) this indentation poked all the way through until a free ring was generated. These rings would always rupture their membranes while they were on the bottom of the drop [see Figs. 15(a) and 16].

Drops of approximately 3 ml also form rings, but their evolution is slightly different than that of 2 ml drops in that the spanning membrane does not always break while it is on the bottom. These drops instead expand horizontally, and the membrane bulges up through the center of the surrounding annulus of fluid [see Figs. 15(b) and 17]. The degree to which it bulges upward is strongly dependent on drop size; the larger the drop, the more extreme the expansion. For drops with volumes of 3 ml, the membrane would occasionally rupture after this expansion. However, those drops of pure silicone oil having a volume greater than 3 ml were not observed to form rings. They underwent the same evolution as the slightly smaller drops, but exhibited Rayleigh–Taylor instability before membrane rupture (see Fig. 10). The instability causes the drop to form two lobes, thus pinching off the center membrane.

TABLE V. Data summary. Quantities listed are pure *1000 cS silicone/1000* cS silicone with 0.5% Igepal. The interfacial surface tension for the pure silicone oil in soybean oil is 2.7 dyn/cm. With 0.5% Igepal in the silicone oil, the interfacial tension is 1 dyn/cm.

Size (ml)	# Trials	# Ruptures	# Pinch-off	# Oblates
1.0	5/5	0/0	0/0	5/5
2.0	10/10	6/4	0/0	4/6
2.5	2/4	2/4	0/0	0/0
3.0	10/10	3/10	7/0	0/0
4.0	10/10	0/9	10/1	0/0
5.0	2/10	0/7	2/3	0/0
7.0	3/-	0/-	3/-	0/-

3. Silicone oil with 0.5% Igepal CO-530

The addition of 0.5% Igepal CO-530 to the 1000 cS silicone oil had a rather dramatic effect on the upper bound of drop size. As mentioned above, large drops (4–7 ml) had the tendency to become unstable, form a tough membrane and pinch off before a ring could be formed. The drops which contained Igepal proceeded in the same fashion except that the membrane usually broke; sometimes before and sometimes after the onset of instability. It was sometimes observed that a membrane would rupture during pinch off. In other words, it appeared that the membrane rupture occurred when the total surface area of the membrane was decreasing. Without Igepal, 4 ml and larger drops would invariably pinch off without membrane rupture, while with Igepal, 4 and 5 ml drops would frequently (nine and seven times out of ten observations, respectively) form free rings [see Figs. 18(a) and 18(b)]. Thus large drops with Igepal experience membrane rupture more frequently than those without, and so it would appear that the membrane strength was in some sense decreased with the addition of the Igepal.

The effect of Igepal on smaller drops sizes is not clear. For 2 ml drops, membranes broke *less* often with the Igepal (four out of ten with Igepal, six out of ten without). However, with 2.5 ml drops membranes broke much more frequently with Igepal (ten out of ten with, three out of ten without). At 3 ml, no effect was observed when Igepal was added (membranes broke ten times in ten trials with or without). It appears that with small drops, size is a more critical factor in determining whether a ring will form than whether or not the drop contains any Igepal.

The raw data are presented in Table V. Under each column two numbers are given; the first is for pure 1000 cS silicone oil, the second for the same containing 0.5% Igepal. The column with the heading "# Ruptures" lists the number of times the membrane was observed to rupture, "# Pinch-off" lists the number of times the instability manifested itself before the membrane could rupture, and "Oblates" lists the number of drops that were observed to remain as indented oblate spheres for the entire length of the tube.

The number of trials is quite small (ten and less) at each drop size, and it could certainly be said that more observations should be made. However, for the most part we found that given the drop size, we could predict quite accurately whether a ring would form.

[1]M. H. I. Baird, T. Wairegi, and H. J. Loo, "Velocity and momentum of vortex rings in relation to formation parameters," Can. J. Chem. Eng. **55**, 19 (1977).
[2]D. S. Chapman and P. R. Critchlow, "Formation of vortex rings from falling drops," J. Fluid Mech. **29**, 177 (1967).
[3]J. S. Turner, *Buoyancy Effects in Fluids* (Cambridge U.P., Cambridge, 1979).
[4]G. A. Simons and R. S. Larson, "Formation of vortex rings in a stratified atmosphere," Phys. Fluids **17**, 8 (1974).
[5]J. J. Thomson and H. F. Newall, "On the formation of vortex rings by drops falling into liquids, and some allied phenomena," Proc. R. Soc. London **39**, 417 (1885).
[6]V. O'Brien, "Why raindrops break up—vortex instability," J. Meteorol. **18**, 549 (1961).
[7]F. T. Arecchi, P. K. Buah-Bassuah, F. Francine, C. Pérez-Garcia, and F. Quercioli, "An experimental investigation of the break-up of a liquid drop falling in a miscible fluid," Europhys. Lett. **9**, 333 (1989).
[8]M. Kojima, E. J. Hinch, and A. Acrivos, "The formation and expansion of a toroidal drop moving in a viscous fluid," Phys. Fluids **27**, 19 (1984).
[9]D. D. Joseph, "Fluid dynamics of two miscible liquids with diffusion and gradient stresses," Eur. J. Mech. B/Fluids **9**, 565 (1990).
[10]R. Clift, J. R. Grace, and M. E. Weber, *Bubbles, Drops and Particles* (Academic, New York, 1978).
[11]S. Chandrasekhar, *Hydrodynamic and Hydromagnetic Stability* (Dover, New York, 1981).
[12]G. K. Batchelor, *An Introduction to Fluid Dynamics* (Cambridge U.P., Cambridge, 1970).
[13]J. Happel and H. Brenner, *Low Reynolds Number Hydrodynamics* (Martinus Nijhoff, Boston, 1983).
[14]C. J. Koh and L. G. Leal, "An experimental investigation on the stability of viscous drops translating through a quiescent fluid," Phys. Fluids A **2**, 2103 (1990).
[15]C. Pozrikidis, "The instability of a moving viscous drop," J. Fluid Mech. **210**, 1 (1990).
[16]D. S. Dandy and L. G. Leal, "Buoyancy-driven motion of a deformable drop through a quiescent liquid at intermediate Reynolds numbers," J. Fluid Mech. **208**, 161 (1989).
[17]B. Stuke, "Zur Bildung von Wirbelringen," Z. Phys. **137**, 376 (1954).
[18]E. Northrup, "A photographic study of vortex rings in liquids," Nature **88**, 463 (1912).
[19]V. O'Brien (private communcation, 1985).
[20]A. M. D. Amarakoon, R. G. Hussey, W. J. Good, and E. G. Grimsal, "Drag measurements for axisymmetric motion of a torus at low Reynold number," Phys. Fluids **25**, 1495 (1982).
[21]J. K. Walters and J. F. Davidson, "The initial motion of a gas bubble formed in an inviscid liquid. Part 2. The three-dimensional bubble and the toroidal bubble," J. Fluid Mech. **17**, 321 (1963).