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COMPETITION BETWEEN INERTIAL PRESSURES AND NORMAL STRESSES IN THE FLOW INDUCED ANISOTROPY OF SOLID PARTICLES

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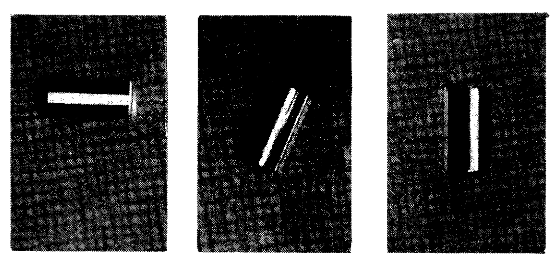
1. INTRODUCTION

It is well known that a long body settling in a viscous liquid will turn its broadside to the stream. The same long body settling in a viscoelastic liquid will turn its broadside parallel to the stream at small speeds [1,2], but heavier long bodies which fall faster again turn broadside. Sedimenting spheres in a fluid filled channel will arrange themselves so that the line of centers between neighboring spheres is across the stream in a viscous liquid and parallel to the stream in a viscoelastic liquid when the fall velocity is small but across the stream again when the fall velocity is large. In both cases the anisotropy is associated with wakes; drafting, kissing and tumbling in a viscous liquid and drafting and kissing but no tumbling in a viscoelastic liquid. Spheres falling close to a wall of the channel rotate as if rolling down the wall in the intuitive way in a viscous liquid, but rotate as if rolling up the wall against intuition in a viscoelastic liquid. Provisional explanations of the peculiar observations can be framed as a competition between inertia and normal stresses with cross stream arrays preferred when inertia dominates and streamwise arrays preferred when normal and extensional stresses dominate.

Certain flows of viscoelastic liquids can be usefully described as a competition between inertia and normal stresses [3] with inertia scaling with U^2/L and normal stresses with U^2/L^2, where U is a typical speed and L a characteristic length for gradients. A similar kind of competition appears to be responsible for the flow induced anisotropies in sedimenting and fluidized suspensions of solid particles which we have observed recently in our laboratory and are reporting here. Elsewhere, it has been noted that small spheres (~70 μm) in an oscillating liquid sheared by the back and forth motion of parallel plates[4] or cone and plates [5,6,7] align in the direction of shear when the liquid is viscoelastic and across the direction of shear when the liquid is Newtonian[7]. It is necessary ultimately to understand why one kind of structure is stable in a flow dominated by inertia and another structure in a flow dominated by viscoelasticity.

2. OBSERVATIONS ABOUT THE TILT ANGLE OF SEDIMENTING CYLINDERS

In this paper we are putting forward the idea that flow induced anisotropy of spherical particles finds its explanation in the flow induced turning couples on long bodies. The streamwise orientation of a settling or fluidized long body is unstable in a viscous liquid, and will always turn its broadside to the stream as in Figs. 1a and 2a. An explanation[8] for this can be found in the couples which are produced by high pressures at the stagnation points on the long body shown in Fig 3. Potential flow is probably a good approximation for viscous flow on the forward side of the body. If the pressures outside a thin boundary layer at the stagnation points were reversed, the long body would not put its broadside into the stream, but instead would put its broadside parallel to the stream, as is in fact the case in the settling of long cylinders in various viscoelastic liquids shown in Figs. 1c and 2c. In practice, the wakes which develop on the back side of bodies give rise to a drag. The turning couples of such wake are self-equilibrating in symmetric cases (α=0 and α=90° in Fig. 3) when the Reynolds number is small and the flow is steady and self-limiting when the Reynolds number is large, due to vortex shedding.[9]



(a) (b) (c)

Fig 1. Cylinders settling under gravity in 0.75% solution of Polyox (WSR-301) in water. The cylinder diameter and length are 0.25 in. and 0.6 in. respectively. The container is 25 in. high, 5 in. wide and 0.44 in. deep. The tilt angle α of the cylinder axis from horizontal is a function of the fall speed. (a) Stainless steel (ρ=0.283 lb/in^3). (b) Aluminum (ρ=0.0975 lb/in^3). (c) Plastic (ρ=0.0476 lb/in^3).

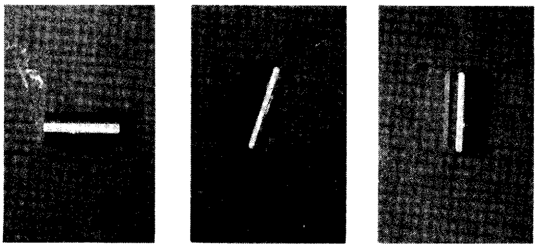


Fig. 2. Aluminum cylinder, (D,L)=(0.25,0.6) in., settling in aqueous Polyox (WSR-301) of different mass concentration ϕ. The tilt angle α is a function of concentration. (a) ϕ=0.5%. (b) ϕ=0.75%. (c) ϕ=1.25%.

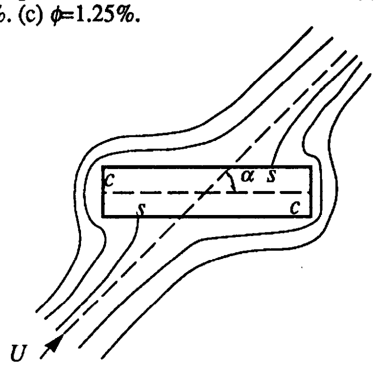


Fig 3. Potential flow past a cylinder. The pressure at stagnation points s will turn the broadside of the body into stream as in Figs. 1a and 2a. If the extensional stress at s were reversed, as may be possible in a viscoelastic liquid, the body would line up with the stream as in Figs. 1c and 2c. The same type of turning with the longside parallel to the stream could be provided by normal stresses caused by strong shears at the corners c. In practice, viscosity will lead to boundary layers and wakes whose effects are not yet understood.

The angle α of tilt between the direction of steady fall and the cylinder axis is evidently determined by a competition between the normal stresses and inertia when neither dominates, as in Figs. 1b and 2b. The normal stresses discussed in the caption of Fig. 3 and in the theory to be discussed in §5 are extensional stresses at points of stagnation. This is clearly at best a partial explanation because forces act all over the body, not just at points of stagnation. The stresses are influenced by the shape of particles, as is seen in Fig. 4.

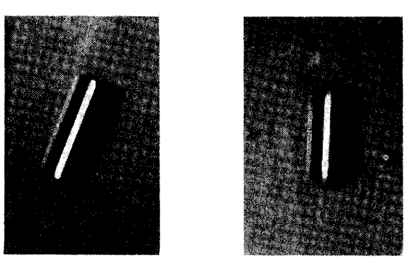


Fig 4. Brass cylinder, (D,L) = (0.3,0.8) in. and ρ=0.306 lb/in^3, falling in 1.25% aqueous Polyox (WSR-301). The tilt angle is much smaller when the ends are rounded as in (b). Viscoelastic effects are more pronounced in cylinders with rounded ends.

3. TURNING COUPLES ON ROLLING SPHERES

We have observed a new phenomenon which is evidently associated with turning couples around the point of contact between a sphere and plane wall as the sphere slides and rolls down the plane wall of a liquid filled container. The sphere rotates as if rolling down the wall in a viscous liquid, but will turn as if rolling up the wall in a viscoelastic fluid (Fig 5.) The couple which produces the counter clockwise motion in the glycerin shown in Fig 5a changes sign in the viscoelastic liquids shown in Fig 5b.

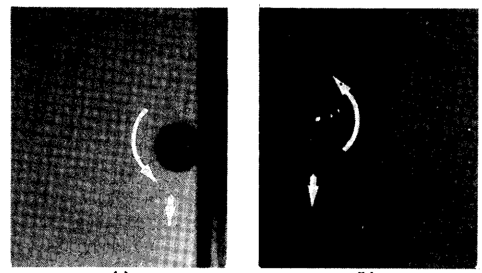
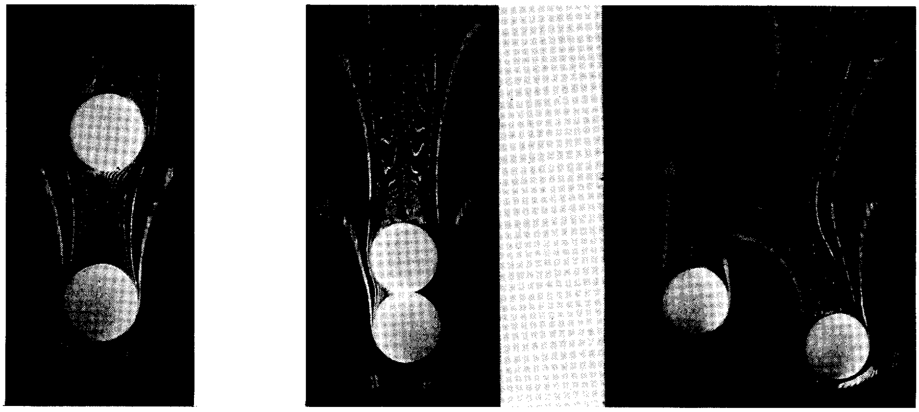


Fig 5. Spheres falling close to a channel wall rotate as if rolling down the wall in viscous fluids, (a) 50/50 glycerin/water solution, and up the wall in viscoelastic fluids, (b) 1.2% aqueous polyacrylamide. The channel in these pictures is tilted slightly to keep the sphere close to the wall.

4. FLOW INDUCED ANISOTROPY OF SPHERICAL PARTICLES

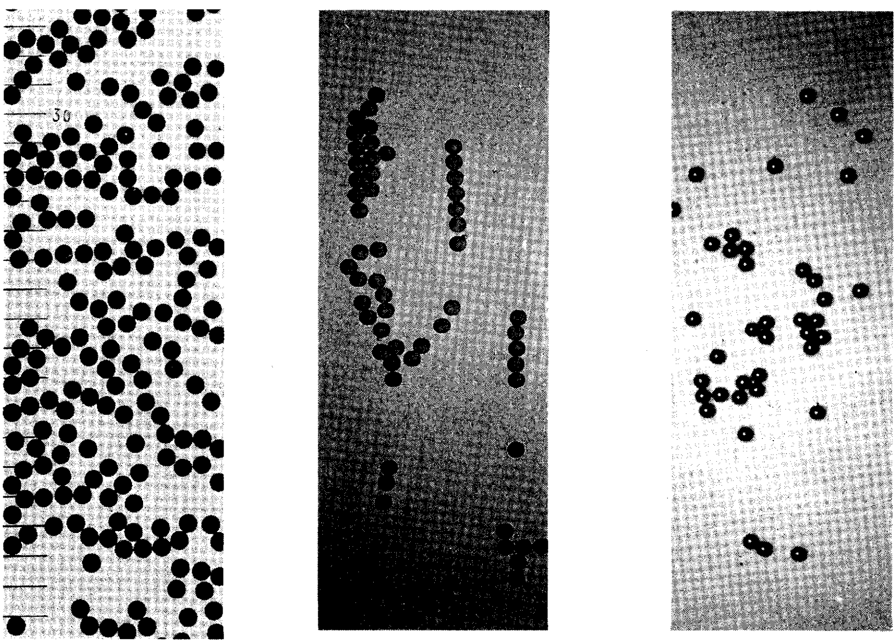
The angle α between the direction of steady fall and the cylinder axis (Fig. 3) is determined by the competition of normal stress and inertia when neither dominate. Broadside on configurations occur in the same Polyox solution for heavier cylinders which fall faster. Inertia dominates in Newtonian liquids, in dilute solutions with small normal stresses, and in more concentrated elastic solutions when the Reynolds number is large enough (Figs. 1a, 2a).

Flow induced anisotropy of sedimenting and fluidized spheres is associated with wakes and surprisingly, with turning couples on long bodies. In viscous liquids this anisotropy is associated with a mechanism called drafting, kissing and tumbling (Fig.6). One sphere is accelerated into the wake of another, they kiss and tumble. The kissing spheres are momentarily a long body which is unstable to the kind of forces shown in Fig.3 which turn long bodies broadside on; so they tumble. This is a local mechanism which implies that globally the only stable configuration is the one in which, on the average, the line of centers between spheres is perpendicular to the stream as seen in Fig. 7a. Stable cross stream arrays of spheres fluidized by water in channels whose small gap confine the motion to two dimensions are a dominant structure which can be observed in nearly every regime of flow.[5,6,7,11,12,13,14] It is apparent from Fig. 7b that a different and even orthogonal anisotropic structure is generated in the viscoelastic liquids in which long bodies settle longside on. The solid spheres in these liquids draft and even kiss, but not tumble. When the fall velocity is large the line of centers between spheres is perpendicular to the stream again as in Fig.7c.



(a) (b) (c)

Fig. 6. Spheres (Delrin, D=0.25in.) in viscous fluid (Glycerin/water) interact through the mechanism of drafting, kissing and tumbling (Joseph, Singh and Fortes[17]). (a) Drafting: the top sphere is caught in the wake of the lower sphere and accelerated downwards. (b) Kissing: the top sphere contacts the lower sphere, creating a long body. (c) Tumbling: the long body arrangement is unstable and experiences a torque which causes the spheres to tumble into a stable cross stream arrangement. Wake effects work also in a viscoelastic liquid, but if the velocity is not too great the spheres will not tumble.



(a) (b) (c)

Fig. 7. Flow induced anisotropy in suspensions of spheres fluidized in water (a), and in a 0.6% solution of Polyox (WSR-301) in water (b) and (c). The plastic spheres in (a) are fluidized by an upward flow of water but the plastic and steel spheres in (b) and (c) are falling under gravity. The cross stream arrangement of particles shown in (a) is characteristic of fluidized suspensions of spheres confined by walls to move in two-dimensions when the suspending fluid is Newtonian. Sedimenting spheres tend to disperse through the dynamics of tumbling; turning couples on long bodies tend to give rise to cross stream structures. Pairs of spheres falling along the line of centers of a tube attract each other in viscoelastic liquids[15]. The normal stress at points of contact between the chains of spheres in (b) must pull the particles together. When the fall velocity is large the line of centers between spheres is perpendicular to the stream again (c).

5. SOME THEORETICAL CONSIDERATIONS

Leal [1] has studied the sedimentation of slender bodies in a second order fluid with inertia neglected. He considers only those non-Newtonian effects resulting from the disturbance velocity field generated by the lowest order geometry independent approximation of the Stokeslet distribution used in slender body theory. He finds that freely translating particles with fore-aft symmetry exhibit a single stable orientation with the axis of revolution vertical. This may suggest that the angle α of tilt observed in experiments may be determined in a competition between inertia and viscoelasticity. The mechanism which aligns the slender body with the stream is not easy to extract from Leal's analysis.

Brunn[16] studied the interaction of two spheres in a second order fluid with inertia neglected and he found an attractive force which draws the spheres together. Riddle, et al [15] discovered that if the initial separation of two spheres settling along their line of centers in a viscoelastic fluid is larger than a certain critical separation the spheres will diverge, whereas it is smaller than this separation they will converge. The analysis of Brunn[16] does not give rise to a critical separation and it cannot treat close approach because it has been assumed that the distance between sphere centers is large compared to the radius.

Joseph[10] has shown that every potential flow is a solution of the equation of motion for second order fluid with stresses given by

$$\sigma_{ij} = -[\, C + \hat{\beta}\, \phi_{,i\ell}\, \phi_{,i\ell} - \rho\phi_{,t} - \rho|\mathbf{u}|^2/2]\, \delta_{ij} + 2[\mu + \alpha_1(\partial_t + \mathbf{u}\cdot\nabla)]\, \phi_{,ij} + 4(\alpha_1+\alpha_2)\phi_{,i\ell}\, \phi_{,\ell j} \qquad (1)$$

where

$$\sigma_{ij} = T_{ij} + \rho\mathbf{g}\cdot\mathbf{x}\, \delta_{ij} \qquad (2)$$

is the active dynamic stress, $\mathbf{g}$ is gravity, C is a Bernoulli constant, $\hat{\beta}$ is the climbing constant, ρ is the density, μ is the viscosity, α_1 and α_2 are the quadratic constants, $\alpha_1=-n_1/2$ and $\alpha_2=n_1+n_2$ with n_1 and n_2 being the first and second normal stress coefficient. In general, potential flow cannot satisfy no slip conditions at solid walls. In this theory the streamlines are determined say by the prescribed values of the normal component of velocity. You cannot see the effects of changing the values of the material parameters in the values or distribution of velocity which are given by potential flow. In the case of a rod rotating in a sea of second order fluid, the potential flow solution also satisfies the boundary conditions at rod surface and the potential flow is exact. Potential flows of viscous fluids exist outside boundary layer regions and separated regions at the back of bluff bodies. We have learned how to use potential flows in viscous flows and we must learn how to use them in viscoelastic flows.

The case of flow at the stagnation points of a body in steady flow, in an arbitrary direction is of special interest. The steady streaming past a stationary body is equivalent, under a Galilean transformation, to the steady motion of a body in an otherwise quiet fluid. The potential flow of a fluid near a point $(x_1, x_2, x_3) = (0,0,0)$ of stagnation is a purely extensional motion with

$$[\lambda_1, \lambda_2, \lambda_3] = \frac{U}{L}\dot{S}\, [2, -1, -1] \qquad (3)$$

where $\dot{S}$ is the dimensionless rate of stretching in the direction x_1, L is the scale of length and

$$[u_1, u_2, u_3] = \frac{U}{L}\dot{S}\, [\, 2x_1, -x_2, -x_3]. \qquad (4)$$

In this case

$$\begin{bmatrix} \sigma_{11} & 0 & 0 \\ 0 & \sigma_{22} & 0 \\ 0 & 0 & \sigma_{33} \end{bmatrix} = \frac{\rho}{2}U^2\,[\,\dot{S}^2\frac{4x_1^2+x_2^2+x_3^2}{L^2} - 1\,]\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + 2\mu\frac{U}{L}\dot{S}\begin{bmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} + 2\frac{U^2}{L^2}\dot{S}^2\begin{bmatrix} -\alpha_1+2\alpha_2 & 0 & 0 \\ 0 & -7\alpha_1-4\alpha_2 & 0 \\ 0 & 0 & -7\alpha_1-4\alpha_2 \end{bmatrix} \qquad (5)$$

At the stagnation point itself

$$\sigma_{11} = -\frac{\rho}{2}U^2 + 4\mu\frac{U}{L}\dot{S} + 2(2\alpha_2-\alpha_1)\frac{U^2}{L^2}\dot{S}^2. \qquad (6)$$

Since $\alpha_1 < 0$, $2\alpha_2-\alpha_1=\frac{5}{2}n_1+2n_2 > 0$, the normal stress term in (6) is positive independent of the sign of $\dot{S}$ but $4\mu\dot{S}$ is negative at the front side of a falling body and is positive at the rear. This is a new manifestation of the competition between inertia and normal stress, which may play a role in the flow induced anisotropy. In practice we would not expect the symmetrical streamlines predicted by potential flow, but the normal stresses that are generated in the non-separated regions of flow around bodies may play an important role in turning long bodies and chaining of spherical bodies.

Dimensionless groups may be formed from the ratios of inertia $\rho U^2/2$, viscosity $4\mu U/L$ and normal-extensional stress $(5n_1+4n_2)U^2/L^2$. We could again speak of an inertial radius for the competition between inertial and normal-extensional stress with inertia dominant when $L>L_c$ and normal stress dominant when $L<L_c$ where

$$L_c^2 \approx \frac{10n_1+8n_2}{\rho} \qquad (7)$$

is a material property. This seems agree with the experimental results of Riddle et al [15]. Their experiment should give rise to a potential flow at early times and their critical separation may correspond to our inertial radius.

The concept of an extensional viscosity is a very special one since it is based on the assumption that the flow on which the extensional stress difference $\sigma_{11} - \sigma_{22}$ as defined is exactly the potential flow (4). For a second order fluid

$$\sigma_{11} - \sigma_{22} = 6\mu\frac{U}{L}\dot{S} + 12\,(\alpha_1+\alpha_2)\frac{U^2}{L^2}\dot{S}^2 \qquad (8)$$

The extensional viscosity is useless for computing the forces and moments on bodies because the isotropic part of the stress which contains viscoelastic terms in general has been subtracted off.

6. CONCLUSIONS

To our knowledge, the following observations documented in this paper are novel.

(1). It is well known that a long body settling in a viscous liquid will turn its broadside to the stream. The same long body settling in a viscoelastic fluid will turn its broadside parallel to the stream but heavier long bodies which fall faster again turn broadside.

(2). There is a regime in which normal-extensional stresses and inertia compete. This competition evidently decides the tilt angle which the axis of a long body makes with the direction of fall in steady flow.

(3). The tilt angle can be controlled by changing the concentration of the solution using the same long particle or by changing the weight of the particle in the same solution.

(4). The shape of the ends of the particle has an effect on the tilt angle, with rounded ends giving a smaller angle of tilt as in the viscoelastic case.

(5). The natural orientation of a long body in a fluidized suspension is the key to understanding anisotropic structures which develop in sedimenting and fluidized spheres.

(6). Sedimenting spheres in a fluid filled channel will arrange themselves so that the line of centers between neighboring spheres is across the stream in a viscous liquid and parallel to the stream in a viscoelastic liquid when the fall velocity is small but across the stream again when the fall velocity is large.

(7). Spheres falling close to a wall of the channel rotate as if rolling down the wall in the intuitive way in a viscous liquid, but rotate as if rolling up the wall against intuition in a viscoelastic liquid.

(8). Provisional explanation of the peculiar observations can be framed as a competition between inertia and normal stresses with cross stream arrays preferred when inertia dominates and streamwise arrays preferred when normal and extensional stresses dominate.

Quantitative documentation of these conclusions is being prepared for a forthcoming publication.

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