Nonlinear Mechanics of Fluidization of Spheres, Cylinders and Disks in Water

D. D. JOSEPH*, A. F. FORTES*, T. S. LUNDGREN* AND P. SINGH*

Abstract. Experiments on fluidization with water of spherical particles falling against gravity in columns of rectangular cross section are described. All of them are dominated by inertial effects associated with wakes. Two local mechanisms are involved: drafting and kissing and tumbling into stable cross stream arrays. Drafting, kissing and tumbling are rearrangement mechanisms in which one sphere is captured in the wake of the other. The kissing spheres are aligned with the stream. The streamwise alignment is massively unstable and the kissing spheres tumble into more stable cross stream pairs or doublets which can aggregate into larger relatively stable horizontal arrays. The stability of cross stream arrays in beds of spheres constrained to move in two dimensions is amazing. These arrays may even coalesce into aggregations of close packed spheres separated by regions of clear water. A somewhat weaker form of cooperative motion of cross stream arrays of rising spheres is found in beds of square cross section where the spheres may move freely in three dimensions. Horizontal arrays rise where drafting spheres fall because of greater drag.

Experiments using cylinders of different length to diameter ratios were also carried out. All objects float with their broad side perpendicular to the stream, broadside on. The broadside on position of particles is due to a turning couple at the front of bodies of the same nature as the couple which turns canoes broadside on in a stream and an explanation can be given from potential flow. Kissing spheres are a composite body equivalent to a long cylinder; they tumble because of the turning couple on the composite body. Broadside on position of single particles seems to be more stable than other positions.

---

* Department of Aerospace Engineering and Mechanics
University of Minnesota
Minneapolis, Minnesota 55455
This work was supported by the Fluid Mechanics branch of the National Science Foundation and by the Mathematics Division of ARO. The financial support from CNPq - Brazil given to one of us (Fortes) is gratefully appreciated.

Wakes behind blunt bodies, long cylinders and flat objects of various kinds are very strong. Single nonspherical particles oscillate around a mean horizontal position with a frequency that may be determined by vortex shedding. Other particles are sucked into wakes, tending to suppress the oscillations. Clusters of particles, held together in the wake of larger particles, move as a group. Interesting particle clustering can be achieved by manipulation of wakes.

1. Introduction. In this paper we describe some experiments on the fluidization of spherical particles between two parallel glass plates and in a persplex column with a rectangular cross section. Some experiments between parallel plates were performed in beds of different width with spheres whose diameters were slightly smaller than the gap between the plates. We say that this fluidization of a single layer of particles is "two-dimensional". To our knowledge, the experiments here and in the papers of Volpicelli, Massimilla and Zenz [1966] and Garside and Al-Dibouni [1973] are the only ones using two-dimensional beds of a single layer of particles. Some of the observations which we shall make about the dynamics of fluidization in these beds can also be inferred by careful examination of photographs presented by Volpicelli, et al, though they did not make these inferences. They showed that fluidization in two-dimensional beds satisfies the fluidized bed correlations of Richardson and Zaki [1954] with a shifting of curves of correlation due to wall effects.

The fluid dynamics of two-dimensional fluidization of spheres i of intrinsic interest and is also useful as a diagnostic tool for three dimensional fluidization. It is certain that there are some wall effects in two-dimensional beds. However, the dominant features o fluidization so clearly evident in the two-dimensional beds were als observed in three-dimensional beds at all Reynolds numbers for which fluidization was possible. The three-dimensional beds are the parallel plates with smaller spheres (two-layer beds) and columns with rectangular and square cross sections, using spheres of different densities and sizes.

Different flow regimes were observed in the experiments. All o them are dominated by inertial effects associated with wakes. A basic mechanism of fluidization involves pairwise coupling of spheres whose line of centers is parallel to the stream. We describe the dynamic scenario associated with this pairwise coupling as drafting, kissing and tumbling. The second sphere is drafted into the wake of the first sphere; they kiss, then tumble. The falling motion of kissing spheres whose centers are parallel to the stream is unstable to couples of the type which turn streamlined bodies broadside on. Neighboring sphere with centers aligned cross stream which appear to have the greatest stability are also a characteristic feature of the fluid dynamics o fluidized beds. The robust stability of the cross stream alignments are particularly dramatic in a "two-dimensional" bed of a single layer of particles. In this way drafting and kissing can lead to aggregation The aggregates are closely packed arrays of spheres which can align i contact along vertical lines through their centers. Such aggregates are a characteristic feature of a two-dimensional fluidized bed, an they can take form as propagating or even as standing "shock waves" of aggregates separated by clear water. Propagation of stable cross stream arrays also occurs in the three-dimensional case. We see cases where the propagation occurs as a result of drafting, kissing and tumbling spheres collecting at the top and leaving the bottom of the aggregates.

The clear water regions between aggregates will be called voidage cracks. Voidage cracks, called "bubbles", are a characteristic feature of fluidization of beds of particles with air. Bubbles look much different than void cracks and the dynamics of fluidization with air must be hugely different than fluidization with water. The change of momentum in the liquid of liquid-solid systems is as important as the changes in the momentum of the particles, because their densities are not greatly different. The literature on fluidization is abundant with empirical and theoretical analysis of crack formation in fluidized beds. The latest works on the subject, such as those by El-Kaissy and Homsy [1976], Homsy, El-Kaissy and Didwania]1980] and Liu [1982 and 1983] hold that the crack formations can be traced to voidage instability waves. Though we are not yet committed to any theoretical model of a fluidized bed, our observations indicate that aggregation associated with drafting, kissing and tumbling can give rise to the propagation of voidage cracks. This type of propagation can be viewed as the shock waves of the effective density of spheres which were described in the works by Wallis [1962], Verloop and Heertjes [1970] and Fanucci, Ness and Yen [1979]. In the fluidization of a single layer of particles, suction in the wakes of spheres in relative motion lead to aggregation due to repeated capture of spheres and even to standing shock waves of closely packed spheres which percolate along vertical lines, as in Figure 1.

The topology of sphere aggregation in fluidization is a functional of the dynamics. Rowe [1961] and Rowe and Henwood [1961] studied drag forces on spheres in fixed aggregates. Unfortunately, they pushed their flow through the unnatural array of spheres shown in Figure 1 when U is turned through 90°. The hexagonal arrangements of similar spheres used by Richardson and Meikle [1964] are also misleading, because they artificially constrain the spheres.

Many investigators have reported the formation of natural clustering of particles in both gas and liquid-solid systems. As early as in the work of Wilhelm and Kwauk [1948], in the sedimentation experiments of Kaye and Boardman [1962], the formation of particle clusters have been reported. Experimental evidence of the importance of aggregates formed from drafting, kissing and tumbling was given by Happel and Pfeffer [1960]. Their study was restricted to free falling spheres along the axis of a cylindrical glass column of liquid with Reynolds numbers in the range of 0.3 and 0.7. Even at these low Reynolds numbers wake effects are important, even dominant. Happel and Pfeffer conclude their experimental study with the observation that

> "...it is possible the spheres suspended in random orientation may not maintain their positions relative to each other. Perhaps the formation of doublets (kissing spheres), and their corresponding higher velocities of fall, has been one of causes for the wide discrepancies in the presently available fluidization data."

Recent experimental work of Tsuji, Morikawa and Terashima [1982] on the hydrodynamic interaction between two spheres is relevant to our investigation. They used the pendulum method to measure the drag on the sphere and flow visualization. The pendulum method restrains the sphere to move like a pendulum at the end of a string. The method was extended to study flow interaction: a dummy sphere is set up in the front or rear of the test sphere, or a group of spheres connected by a rod in the transverse direction is hung by two strings. They used both streamwise and crosswise arrangements and determined the corresponding effects on the drag and vortex patterns. Their results agree with our findings on the existence of the suction in the wakes of spheres in relative motion even at low Reynolds numbers, when a single sphere does not shed vortices, and with our observation of increased drag on the cross stream arrays.

The compelling experimental demonstrations of wake dominated nonlinear mechanisms for fluidizing beds of particles ought to be addressed in the hydrodynamic modeling of fluidization and sedimentation.

2. Experimental parameters. The two-dimensional bed exhibited in Figure 2 was made of two parallel glass plates 0.285 in. apart with a 75 cm test section. The width was adjustable. The glass plates were framed in U-shaped beams, and changes of the bed thickness within ±0.007 in. were made possible with the use of adjustment screws so as to ensure an even distribution of the water flow throughout the bed. The working fluid was water supplied from a pressurized water tank through a needle valve. This valve gives fine control of the flow rate so that a fairly steady working pressure was available at the entrance of the bed. The maximum total throughput of the 5 HP centrifugal pump water supply system was approximately 50 ℓ/min., where ℓ stands for liter. An equalizing section consisting of a 6 in. layer of packed glass beads followed by a 2 in. layer of metal foam was placed upstream of the test section. Pressure taps attached to the rear plate of the bed were located right above the equalizing section and at the top of the test section. The water flowing from the top of the bed was collected in a trough open to the atmosphere. The flow rates were measured with a Micro Motion Mass Flow Meter, and the pressure drop across the entire bed was measured with piezometer tubes. The two-dimensional bed could be tilted at any angle from the vertical. This allows one to adjust the effective force of gravity.

The three-dimensional bed is exhibited in Figure 3. The rectangular cross sections used in this Perspex plate apparatus were 3 in. by 2 in. and 3 in. by 3 in. The square cross section, used for plastic beads, did not allow through flow of a magnitude sufficient to fluidize the heavier glass beads. The smaller cross section was used in the experiments on fluidization of glass beads. This bed had an equalizing section composed of a 4 in. layer of honeycomb aluminum followed by a 1 in. layer of metal foam.

Both beds could be illuminated from behind as well as from the front with 4 high intensity 650 watt quartz flood lamps. Illumination of the two-dimensional bed was made through an opaque Perspex plate graduated with a metric scale on its left side for measuring the bed height. The three-dimensional beds were illuminated from the front.

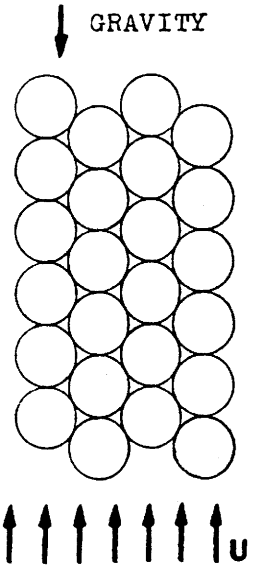


Figure 1. Close packed spheres contact on vertical lines.

Figure 2. Two-dimensional apparatus shown in an inclined position.

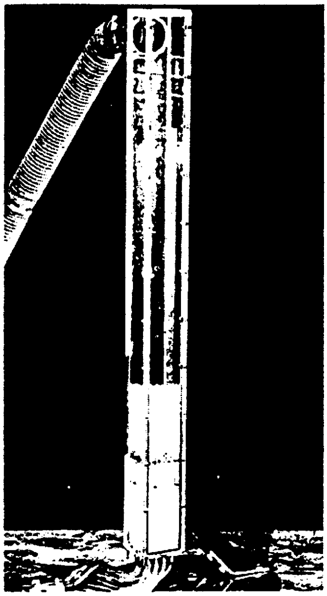


Figure 3. Three-dimensional apparatus.

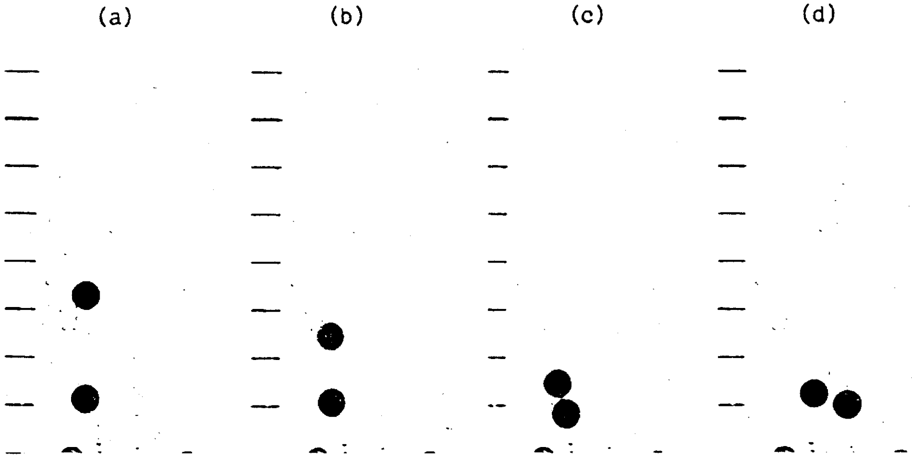


Figure 4. Drafting, kissing and tumbling of plastic spheres in a bed inclined 23° from the vertical, Re = 700. (a) The line of centers lies along the stream. (b) Both spheres are falling, but the second is falling faster than the first. (c) Kissing after drafting. (d) Tumbling after kissing.

Flow visualizations were recorded using still 35 mm photography and conventional video-cassette recording. Motion analysis was made possible with the help of a Spin Physics Motion Analysis System which incorporates a high speed video camera that can take up to 2000 frames per second. In order to enhance the contrast against a black background, the glass beads were etched in a solution of hydrofluoric acid. Some plastic beads were dyed in a solution of red oil dye and acetone in order to be better able to follow the motion of one single particle in the bed. The bed width was adjustable.

Average properties of the spheres are given below:

Particles	Diameter D, in.	Density ρ, g/cm^3
Glass	0.231	2.462
	0.117	2.462
Plastic	0.251	1.119

In the two-dimensional bed, bed widths of 3 in., 6 in. and 8 in. were used. The flow rates used varied from 5.2 ℓ/min. to 21.5 ℓ/min. for fluidization with glass beads and from 1 ℓ/min. to 3.1 ℓ/min. for the plastic beads. In the three-dimensional bed, the maximum flow rate used was about 48 ℓ/min. for fluidization of glass spheres. Smaller flow rates were sufficient for the fluidization of plastic spheres.

The voidage ϕ is the fraction of water in the total volume occupied by the fluidized bed. The total volume is the box which is defined by the portion of the bed between the two planes perpendicular to the flow direction and tangent to the highest and lowest sphere in the bed. When using the smaller glass particles in the two-dimensional bed and for all the experiments in the three-dimensional bed, the void fractions were calculated from the volume of the particles determined by a careful measurement of the water added up to the least possible static bed height and from the known total volumes. The total volume changes as the bed expands.

The measured vertical free-fall velocities in water of the glass and plastic beads, given below, were measured within a precision better than 1%:

Particles	Diameter, in.	Bed	Velocity, m/s
Glass	0.231	2-D	0.341
	0.117		0.337
		3-D	0.551
			0.372
Plastic	0.250	2-D	0.094
		3-D	0.154

The Reynolds number is $Re = DU_s/\nu$ where D is the diameter of the sphere, ν is the kinematic viscosity and U_s is the superficial velocity of the fluid based on the net volume flux of water across a plane perpendicular to the flow direction. The Reynolds numbers of our experiments were always greater than 270, making our flow regimes definitely non-Stokesian. Potential flow analysis of the dynamics of fluidization is equally deficient, because the dynamics is controlled by suctions in the wakes of spheres in relative motion. For example, analysis shows that two spheres in steady potential flow will attract one another when they move perpendicular to their line of centers and they will repel one another when they move parallel to their line of centers

(Lamb [1938], p. 191). In fact, these predictions do not apply to flows with wakes. It seems probable that many features of the flows we observed could be explained by a proper introduction of vortical regions (wakes) into otherwise potential flow. In our experiments, spheres moving broadside on do not attract and spheres following along their line of centers do, a result already known even for weak inertial effects, as the Oseen's equations show (Oseen [1927], pp. 199 ff.).

3. Analysis of drafting, kissing and tumbling. Fundamental features of the fluid dynamics may be observed by fluidizing two spheres. Figures 4a and 4b show two plastic spheres suspended in a rising stream in the two-dimensional bed. The bed was inclined 23° 35' from the vertical. This inclination introduced a gravity force component acting on the spheres normal to the walls of the channel, which has increased the stabilizing effects of the wall friction. We will show later that this wall friction does not significantly change the particle interactions. The Reynolds number was 730. In Figure 5a the same plastic spheres were suspended in a vertical channel. The Reynolds number was slightly smaller than in the inclined channel. A single sphere is in a force equilibrium between the net weight of the sphere and the drag on it. If the two spheres do not interact, they are nearly in equilibrium and drift slowly from place to place. One sphere always drifts into the wake of the other. The second sphere which is now on the line of centers parallel to the stream begins to accelerate, first slowly, then rapidly, to the falling first sphere. This can occur when the centers of spheres are even five or six diameters apart. This is the drafting part of the drafting, kissing and tumbling scenario as shown in Figures 4a, 4b and 5a. The second sphere is very rapidly sucked into contact with the first, then kissed. This is shown in Figures 4c and 5b. The falling motion of contacting spheres aligned in the direction of motion is very unstable. As soon as they kiss, they tumble and are thrown apart as shown in Figures 4d and 5c. A slight angular displacement of the line of centers of two contacting spheres will induce an unsymmetric wake which evidently gives rise to a strong destabilizing couple. Drafting, kissing and tumbling can occur repeatedly and with great frequency. Moreover, since the wake of spheres is determined by the direction of streaming, the events of this scenario are controlled locally and also occur in the three-dimensional bed, as can be seen in Figures 6a, 6b and 6c. The still picture camera sequence of Figures 7a, 7b and 7c taken 1/3 seconds apart, shows the drafing, kissing and tumbling occurring with the dyed pair of plastic spheres on the right side of the pictues, in the 3 in. by 3 in. cross section channel.

The stabilizing effect of the wall friction accounts for the higher Re in the inclined channel, but the evidence shows that the mechanisms of drafting, kissing and tumbling are affected neither by inclination nor by wall effects. Generally speaking, the dynamics in the regions near the wall is not inertially dominated. Thus, the particle interactions near the wall could not be controlled by inertial suction, which contradicts blatantly the experimentally observed drafting of one sphere in the wake of another. Walls can have an important quantitative effect on frictional drag without altering the qualitative balance with opposing forces associated with inertial suction.

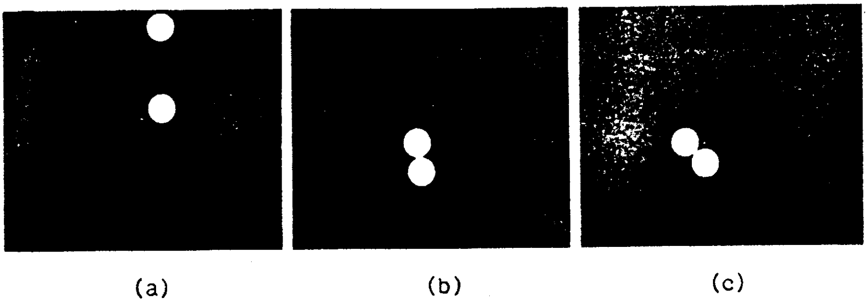


Figure 5. Drafting (a), kissing (b) and tumbling (c) in the vertical two-dimensional bed. The mechanisms are not affected by the inclination of the bed.

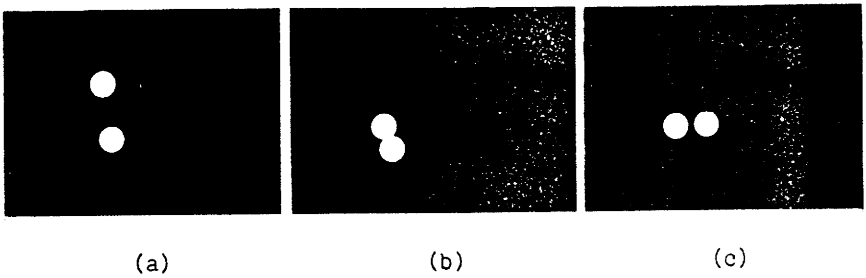


Figure 6. Drafting (a), kissing (b) and tumbling (c) in the three-dimensional bed.

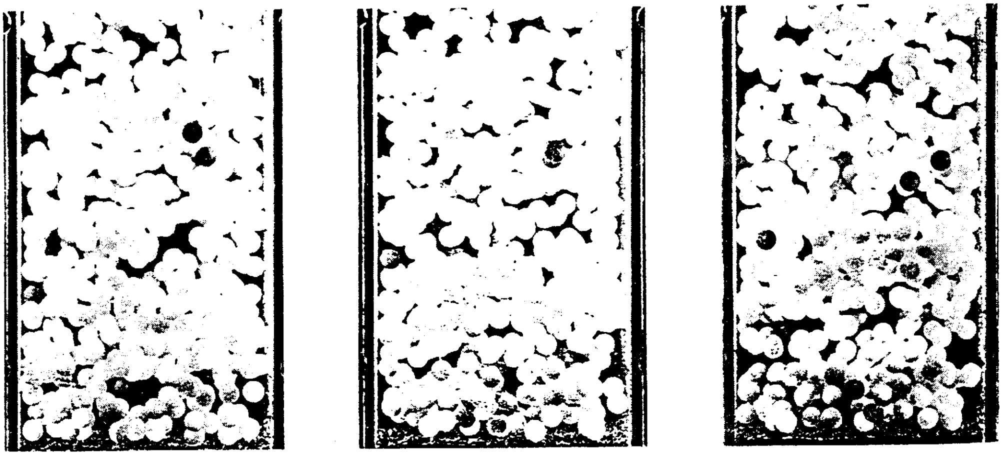


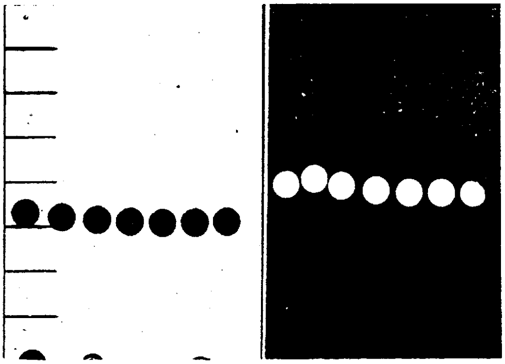
Figure 7. The interaction of two dyed plastic spheres in the three-dimensional apparatus. Rising cross stream arrays of spheres are also apparent. Re = 850. Lubrication layers of clear fluid at the side walls are evident.

Since it appeared impossible to maintain a water flow of absolute uniformity over the cross sectional area of the three-dimensional bed, the increased level of the fluid turbulence reduced the distance of interaction of the spheres. In fact, the shrinking of the wake region behind bluff bodies with increasing Reynolds number has been reported in the physical explanations of Gerrard [1966], as well as in the experimental work of Achenbach [1974]. Turbulence does not overcome the capture phenomenon, though it does reduce the distance of interaction.

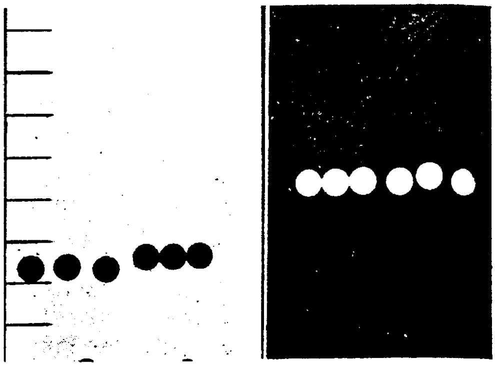
In Figures 8a and 8b we have exhibited one more feature of fluidization seen in two-dimensional flows with many particles in purer form. The total number of spheres in Figure 8a is eight; eight spheres can span the 2 in. width of the channel with some play. The picture shows seven spheres standing stationary in the upward stream of water. The eighth sphere is stationary, standing on the equalizing section. Figure 8b shows the same configuration in the vertical two-dimensional bed. They are stable, for a time, in the configuration in which their lines of centers are perpendicular to the stream. Since they rise, they are being accelerated in their wakes, pulled along by the suction. We call attention to the fact that spheres aligned cross stream do not touch, but they are evidently spaced to produce stability. There are many different stable spacings. These cross stream lines of spheres can be found in the many photographs of the two-dimensional beds of spheres shown in Section 5, especially Figures 15a and 15b.

To explore the stability of the cross stream alignments, we decided to cement three spheres together. The cemented spheres are massively stable when their line of centers is perpendicular to the stream. In a striking display of this basic feature of fluidization, Figures 9a and 9b show that the stability of the cross stream arrays are indeed unaffected by the inclination of the walls. The increased drag experienced by the spheres in this configuration (Tsuji, et al [1982]) testifies to the fact that the cemented triplet and the free spheres rose abreast only when spaced in stable configurations. The factors which enter into the breakdown of these configurations are not perfectly understood. However, it was obvious that interparticle collisions associated with drafting, kissing and tumbling and fluid turbulence play an important role. This breakdown may be more or less identified with the propagation of banded aggregates of rising cross stream arrays of spheres which gain spheres from the tumbling of kissing spheres drafted into the aggregate from the top and lose spheres from the collapse of the aggregate from the bottom. This regulating mechanism for propagation of aggregates is evident in Figures 7 through 14.

4. Experimental results for the fluidization of many spheres. Figures 10a, 10b and 10c show the onset and propagation of void cracks in a channel 8 in. wide using glass beads in the inclined channel. They are separated by time intervals of about 1/3 second. The cracking of closely packed structures is called void cracking and can be viewed as a density perturbation or even as a shock wave of density (Wallis [1962], Verloop, et al [1970] and Fanucci, et al [1979]). The average void fraction in these figures is 0.53 and R = 1080. Under these conditions, the bed is nearly stationary and nearly packed. Essentially, the



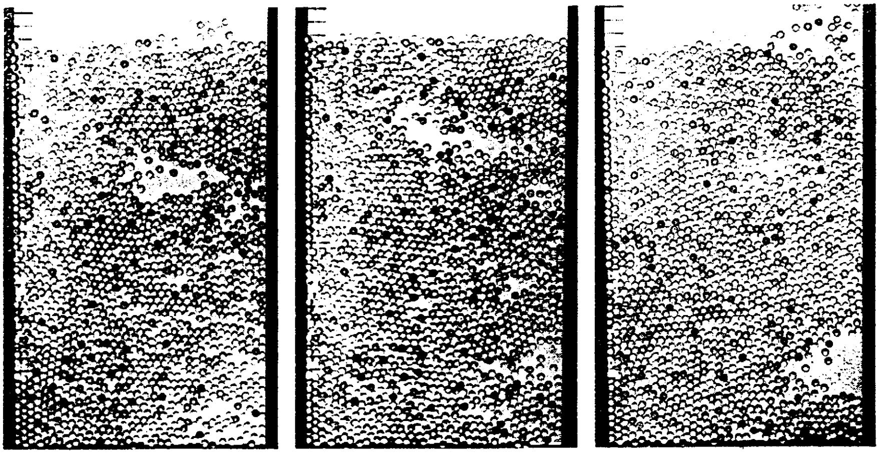
(a) (b)



(a) (b)

Figure 8. Rising stable cross stream array of spheres in the two-dimensional bed. Different spacings are stable. (a) Seven distinct spheres in the inclined channel. (b) Seven distinct spheres in the vertical channel.

Figure 9. A different stable cross stream array. (a) Three distinct and cemented spheres in the inclined channel. (b) Three distinct and three cemented spheres in the vertical channel.



(a) (b) (c)

Figure 10. Fluidization of glass beads at 0.53 average voidage and Re = 1080 in the two-dimensional inclined bed. The channel is 8 inches wide. Readers should focus on the motion of voids. This motion occurs as a withdrawal of spheres at the void roof and the collection of spheres at the void floor.

ame structure can be seen in the vertical channel, although not with he same stability. We might expect that the pressure drop across the tationary closely packed bed is related to the superficial velocity by he well-known formula for the pressure drop-velocity relation in the tate of incipient fluidization, like the Ergun equation (Richardson 1971]).

The effects of inertia of the liquid in flow through a porous edia appear as a drag proportional to the square of the velocity. The dea is that this drag is due to the "dead water" region behind partiles (Joseph, Nield and Papanicolau [1982] derive the quadratic drag (in he equation below) from this idea). The same idea seems to operate in luidized beds, i.e., the contributions of the inertial hydrodynamic ineraction between particles to the bulk properties of the flow can be xpressed as

$$\mathbf{f} = g(\phi)\,|\mathbf{u}|\mathbf{u}\ ,$$

here $\mathbf{u}$ is the local fluid velocity relative to the particle and $g(\phi)$ is n unknown function of the voidage ϕ. Although a discussion of constiutive relations is beyond the scope of the present work, the quadratic rag term suggested here is physically consistent with the observed henomena.

Packed beds of fluidized spheres are different from porous materal because the drag forces may move the spheres. This type of local ensity perturbation usually takes form as void cracking of the packed tructure. The upper surface of the crack moves upward, widening the rack. This type of cracking occurred only in the two-dimensional eds. The cracks are always perpendicular to the bulk flow direction, ore or less, and never parallel to the flow. The flow across the oof of the crack thus creates a local pressure gradient that drives he packed structure above the crack upwards. The relief of this presure gradient in the final bursting of the ascending void crack in the pper layer of the bed shown in Figure 10c is indicative of its presence s a driving force. The existence of this local pressure gradient is onsistent with the more general observation of Harrison and Davidson 1963, 1971] that for a fluidized bed of given voidage the pressure rop at a given velocity is less than for a randomly packed fixed bed t the same velocity and voidage.

Another mechanism for the propagation of cracks is the continuous islodging of spheres from the void roof. This occurred whenever there as a local defect in the otherwise stable lattice of the aggregate. he rising velocities of the void roof which propagates by depletion of pheres from the roof were always smaller than the velocity of single articles in free fall and the velocity of the flow across the crack as never sufficient to keep the dislodged particles fluidized.

Figures 11a and 11b show fluidization of glass beads in the 3 in. ide inclined channel with Re = 1250. The presence of large voids, the ig clear water regions, is not well described by specifying a void raction. The slugging flow of packed spheres displaces regions of lear water. The same cracks or shocks observed in the wider channel

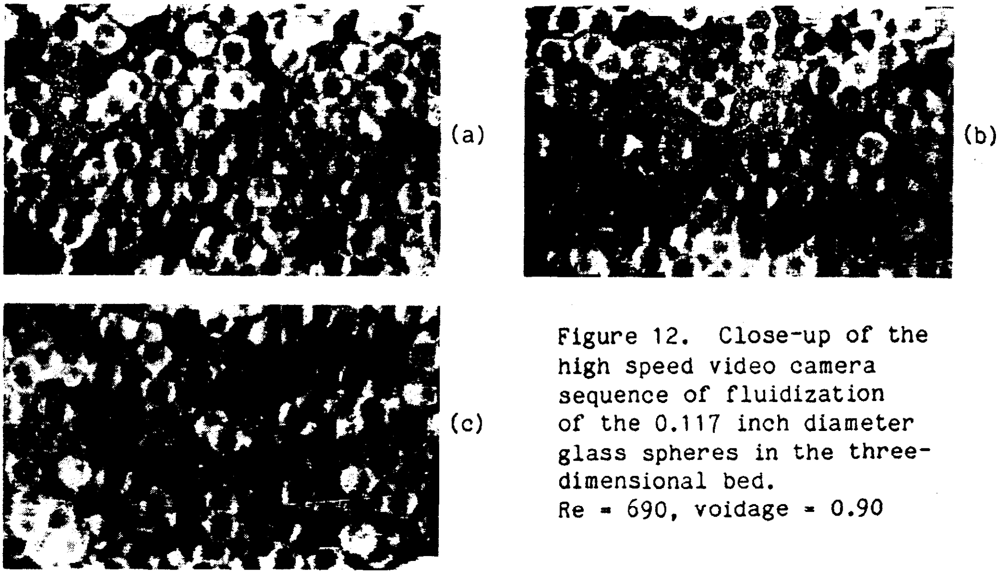


Figure 12. Close-up of the high speed video camera sequence of fluidization of the 0.117 inch diameter glass spheres in the three-dimensional bed.
Re = 690, voidage = 0.90

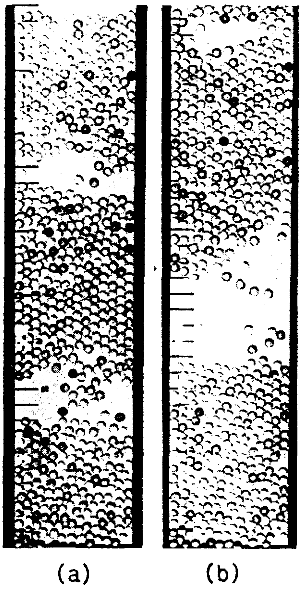


Figure 11. Fluidization of glass beads at Re = 1250 in a 3-inch wide two-dimensional inclined bed.

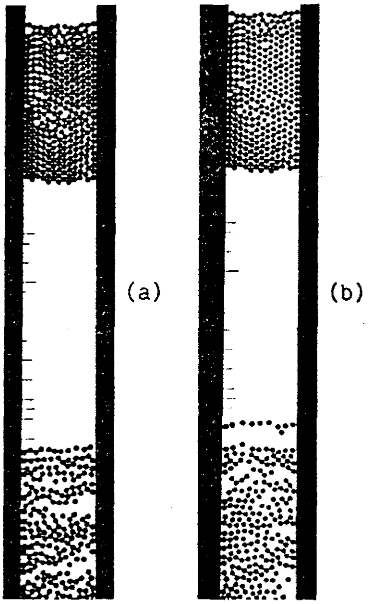


Figure 13. Fluidization of plastic beads in the 3-inch wide two-dimensional inclined bed at Re = 290. The spheres at the top of the bed are stable and close packed with contacting vertical lines of centers, as in Figure 1. The bottom configuration with stable cross stream row is nearly stationary.

now span the whole channel. Also, the expansion mechanism, as described before, is revealed here in the same way. The different sequences taken with the high speed video camera shown in Figures 12a, 12b and 12c, separated by time intervals of 0.05 seconds, reveal the distinctive features referrred to thus far. First, the characteristic topological structure of closely packed spheres whose line of contact lies along the stream, as is vividly depicted in the aggregates at the top of Figure 13a and 13b, manifests itself in the preferential way the spheres collide with one another, i.e., the top sphere collides with the bottom one following in its wake. This topological feature is compatible with the dynamics associated with drafting and kissing. The lateral collisions can be attributed to the eddying motion of the liquid, since there is no possible lateral attraction between two spheres, as pointed out earlier. Secondly, even for these highly turbulent flows, the uprising motion is entirely determined by cross stream arrays of spheres. The fluid is pushed upward through the channels around the vertical line of centers of contacting spheres. There is a wake above each sphere, providing the glue. Isolated particles drift into these stream channels and are dragged into their own wakes, gluing more spheres to form the aggregates.

The remarkable line of particles at the top of the bottom part of the beds shown in Figures 13a and 13b was a characteristic feature of the flow in both the vertical and inclined channels.

Further increase of the Reynolds number does not change the basic mechanisms, and although the eddying motion sets up the lateral collisions of the particles, the expanded bed reveals essentially the same structure.

5. Nonspherical particles float broadside on. We did some fluidization experiments with nonspherical bodies, long cylinders with rounded and square ends, disks and a raft of glued long cylinders. No matter what, these bodies float broadside on. This is explained by the classical potential flows analysis of translating motions of a solid in a liquid (Thompson and Tait [1879], Lamb [1945, Section 124], Milne-Thompson [1960]). For any body there are three directions in which permanent translations are possible, but only one of these three, with the broadside perpendicular to the stream, is stable. The position of the stagnation points in any perturbed equilibrium is such as to turn the body broadside on, as in Figure 14.

The flows in our experiments are not potential flows but potential flow is probably a good approximation for the flow on the forward side of the body. Behind the body there is essentially a dead water region of low pressure whose main effect is to produce a drag. Evidently the turning couples on the forward side are large enough to turn the body broadside on. Many examples of the broadside on positioning of moving bodies have been put in evidence. For example, Milne-Thompson [1960] writes that

> "Thus a ship has to be kept on her course by the helmsman, an elongated airship requires similar attention. A sailing ship will not sail permanently before the wind with the helm lashed, but tends to set itself at right angles to the

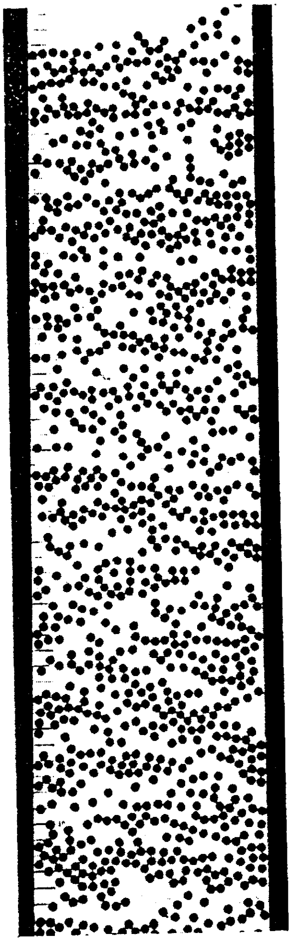


Figure 15. Fully expanded fluidized bed of plastic beads Re = 300, voidage = 0.83. Most spheres line up in horizontal arrays. Vertical pairs are drafting.

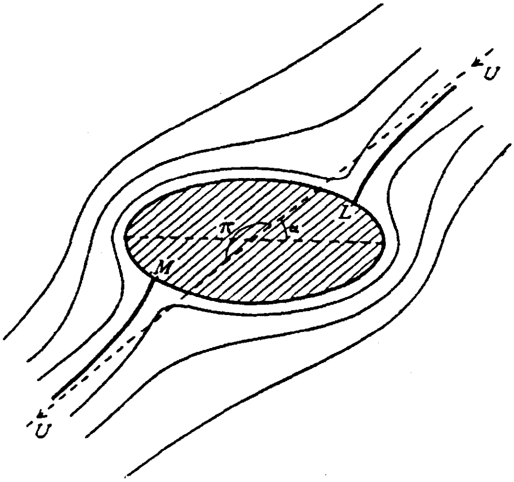


Figure 14. Potential flow solution for uniform flow past a slender body. The high pressure at the points of stagnation give rise to a couple causing the body to turn broadside on. In real flows this mechanism operates only on the front side, with a wake on the back.

wind. A body sinking in liquid tends to sink with its longest dimension horizontal."

6. **Arrays of spherical particles tend to float broadside on.** It seems to us that broadside on orientations of distinct spheres are more stable than other orientations (see Figures 7 through 13, 15).

We are aware that this sweeping statement is an interpretation of our experiences and not an established fact. Relatively greater stability of the cross stream doublets can be supported by an analysis of kissing and tumbling of the type shown in Figure 4, 5 and 6. Drafting is obviously a phenomenon which is dominated by wakes, and not by potential flow. When two spheres kiss, they instantaneously form a single body, equivalent to cemented spheres nearly aligned with the vertical plus the attached wakes at the back end and between the spheres. The front part of this body might be thought to be in potential flow. The upward force at the forward stagnation point of the composite body will give rise to a turning couple causing the kissing spheres to tumble. It is obviously impossible for the tumbling composite body to align itself broadside on without first separating the kissing spheres, and the separated spheres tend to align broadside on. The result of the tendency in the expanded bed shown in Figure 15, and in other figures, is a stratified structure in which at any moment most spheres are aligned in cross stream doublets and larger cross stream arrays.

7. **Fluidized cylinders.** Experiments were carried out with two different kinds of cylinders, round ends with length/diameter = 1.0 in./0.255 in. and square ends with length/diameter = 0.504 in./0.255 in. The diameter of these cylinders was slightly smaller than the gap of 0.285 inch between the parallel plates of the narrow gap apparatus. The motion of cylinders in the narrow gap apparatus is constrained to two dimensions. The specific gravity of the cylinders was 1.2. The Reynolds numbers for these experiments was in the range of 500 to 800. A single long particle was never in equilibrium under its weight and drag no matter whether it was fluidized in the two- or three-dimensional bed or even if it was sedimenting. The single cylinders would always oscillate around a mean horizontal position in a rocking mode which we think is associated with vortex shedding. Infrequently a single cylinder would rock momentarily into a vertical position and suddenly drop several inches before being turned to the horizontal. This shows that the major fluidizing force on long particles is a pressure drag due to wakes.

There are very strong wakes behind fluidized cylinders. Other particles which are sucked into the wakes seem to suppress unsteadiness. Some interesting figures of equilibrium consisting of long cylinders with round ends glued together by wake forces are shown in Figure 16a and 16b. These curious configurations were stable and stationary for minutes. A similar, but slowly changing configuration of long particles is shown in Figure 16c. In Figure 17 we have exhibited two examples of shorter cylinders with squared ends in the two-dimensional bed. Wake interactions are very evident in these figures and take form as piles of cylinders in contact along their long sides, and in a characteristic inverted T formation in which the top cylinder stands end on in the wake of the bottom cylinder. There are a few

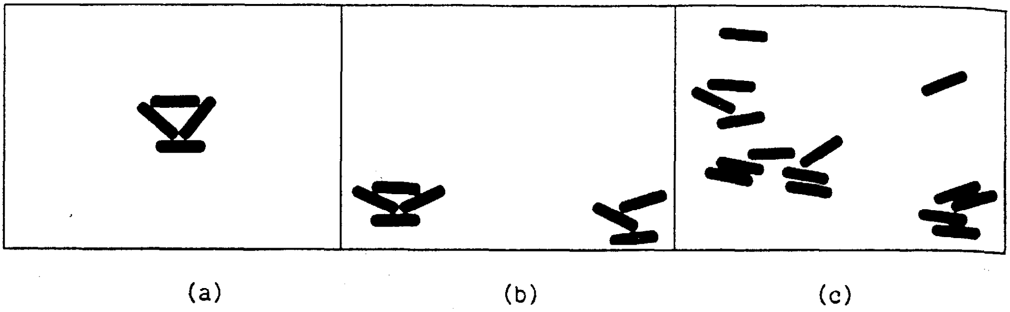


Figure 16. Fluidized cylinders in the vertical two-dimensional bed, Re = 650. The flow is up, against gravity. In (a) and (b) the cylinders are stationary. In (c), some of the cylinders are in motion.

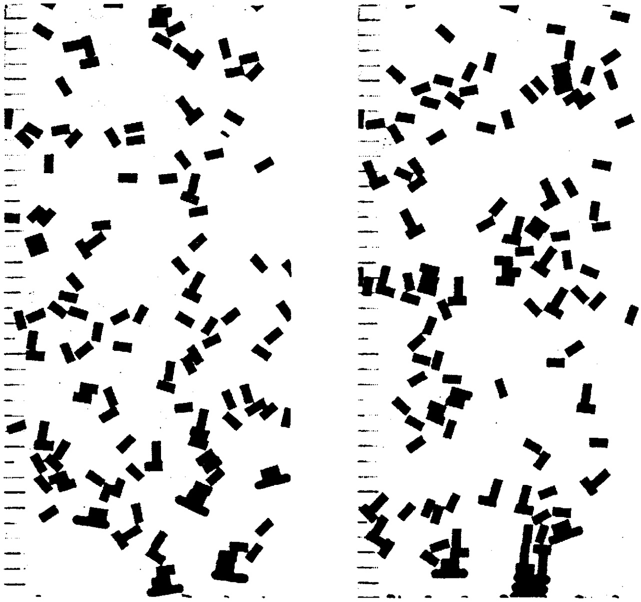


Figure 17. Fluidization of cylinders in the two-dimensional vertical bed. Most of the cylinders have squared ends. They are together in wakes and touch either on their long sides or edgewise in an inverted T. Short cylinders stack on long cylinders as a natural arrangement induced by the flow.

longer cylinders with round ends in the beds. These long particles collect shorter ones in their wakes creating a kind of fluidized bed architecture.

A different kind of inverted T is shown in Figure 18. In this figure one sees an inverted T formed from three spheres. The two on the bottom are glued and the one on top is held tenaciously in place by strong suction in the wake.

It is not possible to obtain all of the structured configurations which occur in the two-dimensional beds when the cylinders are not constrained to move in two-dimensions. As in two-dimensions, the dominant orientation of the cylinders is broadside on. There are very strong wake interactions in three-, as well as two-, dimensional beds. The cylinders cluster in horizontal arrays separated by clear water. Clear water regions are also evident at the side walls as in the case of the plastic spheres shown in Figure 7.

8. Fluidized disks. We made disks from the same stock as the cylinders of diameter d = 0.255 inch. The length/diameter ratio of these disks is approximately one half. In Figure 20 we have exhibited a photograph of the fluidized disks and some cylinders in the two-dimensional bed. The disks draft, kiss and tumble in much the same way as spheres. Most disks float stably broadside on in an oscillating rocking mode but disks which float edgeside on do appear and can be stable for seconds. Disks and small cylinders collect in the wakes of large cylinders. Some disks stand on the cylinders in edgeside on position, forming the characteristic inverted T shown in Figure 17. Inverted T's can occasionally be seen in three-dimensional beds. An example is exhibited at the top of Figure 21a. There are very strong wake effects in three dimensions; drafting, kissing and tumbling are commmon and tend to produce the clusters shown in Figure 21b and 21c.

9. Stacking experiments in two and three dimensions. We built a raft by glueing five long cylinders together. We fluidized the raft in the three-dimensional bed. It floated broadside on, on the average, whilst rocking in the characterisitc oscillation which we suppose is associated with vortex shedding. We matched the drag on the raft with the drag on a small cylinder length/diameter = 0.25 in./.0255 in. and fluidized many such cylinders with the raft. The cylinders are sucked into the wake of the raft like debris behind a fast moving truck, as shown in Figure 22. The composite body, raft plus entrained cylinders, is very stable, the oscillations are entirely suppressed.

Conclusions. The following results are suggested by the experiments discussed in this paper.

1. The dynamics of beds of particles fluidized by water at moderate and high Reynolds numbers is dominated by local mechanisms associated with wakes.

2. The results of Happel and Pfeffer [1960] indicate that even at Reynolds numbers as small as 1/2 the fluidization of spheres is strongly influenced by nonlinear effects of wakes and can not be described by models based in Stokes equations.

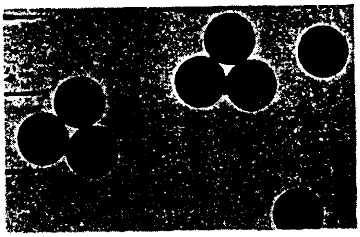


Figure 18. Fluidized spheres in a two dimensional vertical bed. There are three triplets. The bottom pair of the three is glued. The top sphere is nested tenaciously in place by the wake.

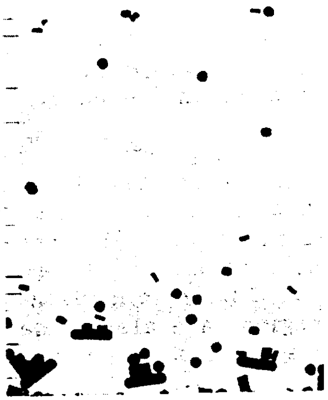


Figure 20. Fluidized disks and cylinders in the vertical two-dimensional bed.

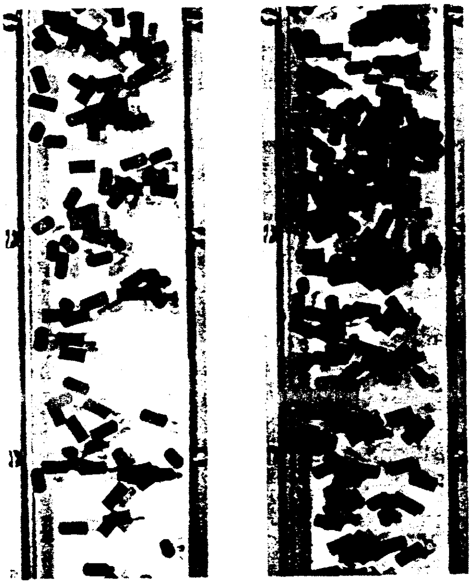
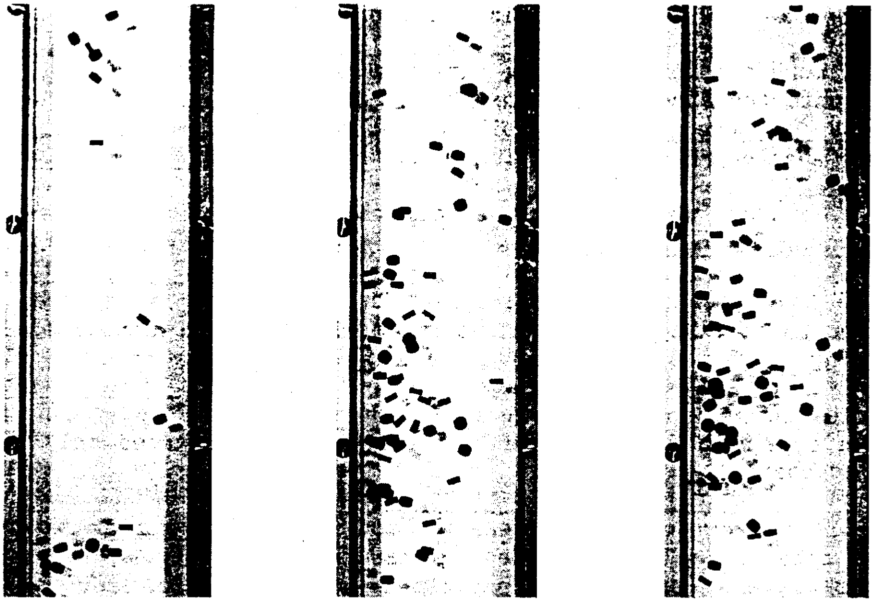


Figure 19. Fluidization of cylinders in the three-dimensional bed. These are the same cylinders as in Figure 17. The wake stacking and inverted T configurations are unstable in three-dimensions. The dominant orientation of the cylinders is broadside on. There are horizontal clusters separated by clear water and clear water is on the side walls (cf. Figure 17).



(a) (b) (c)

Figure 21. Fluidized disks in the three-dimensional bed. Strong wake effects are evident in the inverted T shown in (a) and in the clusters shown in (b) and (c).

Figure 22. Short cylinders are sucked into the wake of the raft.

3. Drafting, kissing and tumbling appears to be the major rearrangement mechanism in the fluidization of beds of spheres with water.

4. Long particles and long arrays of single particles line up broadside onto the stream. The instability of streamwise pairs of kissing particles and the relative stability of crosswise pairs (doublets) seem to play a major role in aggregation.

5. Drafting particles fall and cross stream arrays of particles rise in a stream in which single particles are neutrally buoyant.

6. The stability of cross-stream arrays in beds of spheres constrained to move in two dimensions is amazing. Single horizontal lines of rising particles, even with different spacings, are robustly stable in the vertical and inclined channel. Single stable stationary lines of particles in horizontal arrays and stable aggregates of close packed spheres separated by regions of clear water were observed only in the inclined channel.

7. A weaker form of cooperative motion of cross stream arrays of rising spheres is found in beds of square cross section where the spheres may move freely in three dimensions. In a fluidized bed some particles must go up and some must come down. The going up and the coming down are very different. Falling spheres move fast. They accelerate in the wakes of other falling spheres. Rising arrays are generated from falling spheres by tumbling into cross stream doublets after drafting and kissing. Cross stream arrays rise because they have a greater drag.

8. It was not possible to fluidize a single long particle in a stationary position, even in the two-dimensional bed. Single long particles always oscillate in a rocking motion about the broadside on position. Single sedimenting long particles also oscillate in a rocking motion. The oscillation is possibly produced by vortex shedding.

9. The oscillation of single long particles tends to be suppressed by the presence of other particles. This suppression takes form by interacting wakes of cooperating particles. The effect is greatly enhanced in beds where the motion of the particles is confined to two dimensions. In these cases it was possible to fluidize stationary structures glued together by wakes.

10. Fluidized disks always align broadside on. Single disks oscillate in a rocking motion around the preferred alignments. Many disks draft, kiss and tumble like fluidized spheres.

11. Big particles always suck small paticles into their wake.

12. There was a lubrication layer of clear water at the walls of the three-dimensional beds of particles of all shapes.

Acknowledgement. A portion of this work is taken from the forthcoming Ph.D. thesis, "Nonlinear Mechanics of Fluidization of Beds of Spherical Particles", by Antonio Fortes and the forthcoming paper by Fortes, Joseph and Lundgren to appear in J. Fluid Mech.

REFERENCES

[1] E. ACHENBACH, Vortex shedding from spheres, J. Fluid Mech., 62, Part 2 (1974), pp. 209-221.

[2] M. M. EL-KAISSY and G. M. HOMSY, Instability waves and the origin of bubbles in fluidized beds, Int. J. Multiphase Flow, 2 (1976), pp. 379-395.

[3] J. B. FANUCCI, N. NESS and R. H. YEN, On the formation of bubbles in gas-particulate fluidized beds, J. Fluid. Mech., 94, Part 2 (1979), pp. 353-367.

[4] J. GARSIDE, and M. AL-DIBOUNI, Behavior of liquid fluidized beds containing a wide size distribution of solids, Fluidization and Its Applications, Toulouse, 1973, pp. 53-62.

[5] J. H. GERRARD, The mechanics of the formation region of vortices behind bluff bodies J. Fluid Mech., 25, Part 2 (1966), pp. 401-413.

[6] J. HAPPEL and R. PFEFFER, The motion of two spheres following each other in a viscous fluid, A. I. Ch. E. Journal, 6, 1 (1960), pp. 129-133.

[7] D. HARRISON and J. DAVIDSON, Fluidized particles, Cambridge University Press, 1963, pp. 15-19.

[8] D. HARRISON and J. DAVIDSON, Fluidization, Academic Press, 1971, pp. 47-50.

[9] G. M. HOMSY, M. M. EL-KAISSY and A. DIDWANIA, Instability waves and the origin of bubbles in fluidized bed, II, Int. J. Multiphase Flow, 6 (1980), pp. 305-318.

[10] D. D. JOSEPH, D. A. NIELD and G. PAPANICOLAU, Nonlinear equation governing flow in a saturated porous medium, Water Res. J., 18, 4 (1982) pp. 1049-1052; Erratum 19 (1983), pp. 591.

[11] B. H. KAYE and R. P. BOARDMAN, Cluster formation in dilute suspensions, Symp. Interact. Fluids and Particles, London (1962). p. 17.

[12] H. LAMB, Hydrodynamics, Dover Publications, 1945, p. 191.

[13] J. T. C. LIU, Notes on a wave-hierarchy interpretation of fluidized bed instabilities, Proc. Roy. Soc., London, **A** 380 (1982), pp. 229-239.

[14] J. T. C. LIU, Nonlinear instabilities in fluidized beds, Mechanics of Granular Materials: New Models and Constitutive Relations, J. T. JENKINS and M. SATAKE, eds., 1983, pp. 357-364.

[15] L. M. MILNE-THOMPSON, Theoretical Hydrodynamics, 4th Ed. Macmillan, 1960, p. 530.

[16] C. W. OSEEN, Hydrodynamik, Akademishe Verlagsgesellschaft M.B.H., 1927, pp. 199 ff.

[17] J. F. RICHARDSON, Incipient Fluidization and Particulate Systems, In Fluidization, J. F. DAVIDSON and D. HARRISON, eds., Academic Press, 1971, pp. 50-51.

[18] J. F. RICHARDSON and R. A. MEIKLE, Sedimentation and fluidization, Part IV: Drag force on individual particles in an assemblage, Trans. Inst. Chem. Eng., 39 (1964), pp. 357-362.

[19] J. F. RICHARDSON and W. ZAKI, Sedimentation and fluidization, Part 1, Trans. Inst. Chem. Eng., 32 (1954), pp. 35-53.

[20] P. N. ROWE, Drag forces in a hydraulic model of a fluidized bed, Part II, Trans. Inst. Chem. Eng., 39 (1961), p. 175.

[21] P. N. ROWE and G. A. HENWOOD, Drag forces in a hydraulic model of a fluidized bed, Part I, Trans. Inst. Chem. Eng., 39 (1961), p. 43.

[22] W. THOMPSON and P. G. TAIT, Natural Philosophy, 2nd Ed., Cambridge, 1879.

[23] Y. TSUJI, Y. MORIKAWA and K. TERASHIMA, Fluid-dynamic interaction between two spheres Int. J. Multiphase Flow, 8, 1 (1932), pp. 71-82.

[24] J. VERLOOP and P.M. HEERTJES, Shock waves as a criterion of the transition from homogeneous to heterogeneous fluidization, Chem. Eng. Sci., 25 (1970), p. 825.

[25] G. VOLPICELLI, L. MASSIMILLA and F. S. ZENZ, Nonhomogeneities in solid-liquid fluidization, Chem. Eng. Symp. Series, 62, 67 (1966), p. 42.

[26] G. B. WALLIS, A simplified one-dimensional representation of two-component vertical flow and its applications to batch sedimentation, Symp. Interact. Fluids and Particles, London 9 (1962), p. 17.

[27] R. H. WILHELM and M. KWAUK, Fluidization of solid particles, Chem. Eng. Progress, 44, 3 (1948) p. 201.