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Tall Taylor cells in polyacrylamide solutions

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The behavior of Taylor cells in a polyacrylamide solution contained between rotating cylinders is described. As the rotational speed increases, the cell aspect ratio changes from about 1 to 4. Hysteresis of the 4-cell configuration is observed.

It is well known that Couette flow between concentric cylinders (radii a and b, $b > a$) induced by the rotation of the inner cylinder (at a constant angular velocity Ω) gives up its stability to secondary motions when a critical value of the Taylor number is exceeded.[1] In Newtonian liquids with kinematic viscosity ν, this dimensionless Taylor number may be taken as $\Omega(b-a)^2/\nu = T$, and the secondary motions which are stable when $T > T_c$ are in the form of toroidal vortices which are approximately square in cross section (Taylor vortices). Further increases of speed Ω will induce further instabilities; the first of these appears in the form of undulations which travel around the vortices and destroy the axisymmetry of the motion.[2] Still further increases in Ω lead to yet more complicated motions.

Steady axisymmetric vortices in Newtonian liquids are stable for an interval of angular velocities above the first critical value. The "squareness" of the cross-section of the vortices barely changes as the Taylor number is varied across the stable interval; moreover, the hysteresis effects, if present, are not strong when the Taylor number does not exceed the second critical value where axisymmetric flow becomes unstable. Above this second critical value, hysteresis effects and nonuniqueness are the rule[2]; now the number of cells and the number of waves around a cell depend on the history of the speed changes of the inner cylinder. In Coles'[2] experiments in the Newtonian liquids there are nearly 28 square cells when the angular velocity is slightly above the first critical value; the maximum number of cells achieved is 32 and the minimum number (a rare event), is 18. Also, the cells in Coles' experiments never have height–width ratios greater than 16/9, even when the motion is doubly periodic.

Taylor vortex flow also arises through instability and bifurcation of Couette flow of viscoelastic liquids. Although it appears that the property of near "squareness" of the cross sections of cells holds in the experiments[3-7] when the angular velocity is near its first critical value, the stable axisymmetric flows which do appear differ greatly from the axisymmetric motions in Newtonian liquids. Theoretical studies of the stability of viscoelastic liquids between coaxial rotating cylinders are reported in Refs. 8–20.

The rotating cylinder apparatus employed in these experiments was constructed along the lines of the one used so successfully by Coles.[2] Both cylinders were made of glass, and the over-all height of the liquid-filled annulus between the cylinders was 17.4 cm. The outer diameter of the inner cylinder was 10.0 cm and there was a 1.1 cm clearance between the two cylinders.

The fluid used in our experiments was a polyacrylamide solution consisting of 54.80% by weight glycerine, 43.72% water, and 1.48% polyacrylamide crystals. This polyacrylamide solution is more pseudoplastic than most other viscoelastic fluids that have been used in the previously reported experiments. Data on the viscoelastic characteristics of polyacrylamide solutions are given in Refs. 21 and 22. With this fluid we have observed some new properties of Taylor vortices. The most important of these is that at a given rate of shearing the number of steady, axisymmetric Taylor vortices is not unique, and may vary between 14 and 4.

At low values of the rotational speed the instability in the polyacrylamide solution appears in the form of toroidal vortices of almost square cross section, comparable in size to the cells for Newtonian fluids near the critical Taylor number. In Fig. 1 we have compared the initial instability configurations for a Newtonian fluid, [the 16 cells in Fig. 1(a)] and the polyacrylamide solution [the 14 cells in Fig. 1(b)]. It is evident from these photographs that the structure of the Taylor cells in the polyacrylamide solution is different from the structure in the Newtonian fluid. Flow visualization, using tracer particles, is outstandingly better for the flow field which develops in the polyacrylamide solution.

When the rotational speed of the inner cylinder is increased beyond that which gives the configuration of Fig. 1(b), very dramatic changes in the cell pattern are observed. The number of cells decreases as the speed increases, going first to 12, then 10, 8, 6, and finally 4. When a stable 4 cell configuration has been reached, it will retain its stability as the speed is then decreased. This hysteresis is strikingly demonstrated in Figs. 2(a) and 2(b). Figure 2(a) shows a stable eight-cell configuration at a rotational speed of 22 rev/sec, whereas Fig. 2(b) shows a stable four-cell configuration at the same speed. The Taylor-cell structure of Fig. 2(b) is obtained from that of Fig. 2(a) by first increasing the speed to transfer the stability of the eight-cell configuration to the four-cell

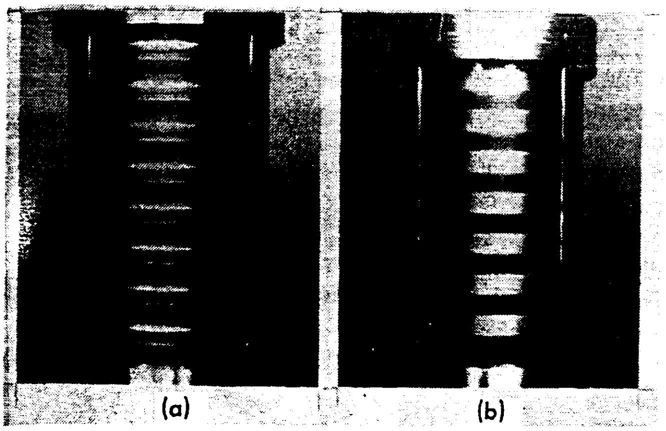

FIG. 1. Taylor cells of approximately square cross section near the initial instability point: (a) Newtonian fluid (light oil), $\Omega = 1.2$ rev/sec, $\nu = 0.36$ cm²/sec, $\Omega/\nu = 3.34$, 16 cells; (b) polyacrylamide solution, $\Omega = 6$ rev/sec, $\nu =$ approximately 10.8 cm²/sec, $\Omega/\nu = 5.5$, 14 cells.

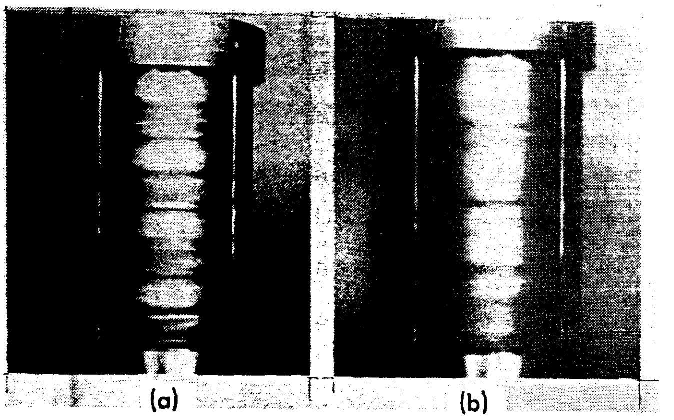

FIG. 2. Two Taylor cell configurations in polyacrylamide solution at a rotational speed $\Omega = 22$ rev/sec. Temperature in the fluid = 118°F, $\nu =$ approximately 8 cm²/sec. (a) 8 cells; (b) speed increased from (a) to give 4 cells and then returned to 22 rev/sec; four-cell configuration remains: $\Omega/\nu = 2.75$.

configuration, followed by a decrease in the speed.

The distinct change from a stable 14-cell configuration, with a cell aspect ratio of about 1, to a stable four-cell configuration, with a cell aspect ratio of 4, seems not to have been observed before. We believe that the tall Taylor cells are another manifestation of normal stress effects in viscoelastic fluids.

The difference in behavior and cell character between non-Newtonian and Newtonian fluids can be further emphasized by comparing the tall cells in the polyacrylamide solution, Fig. 2(b), with the 16 cells in the Newtonian oil, Fig. 1(a). Since the Taylor numbers for the two flows are approximately the same, the cause of the difference observed must be sought elsewhere.

One further interesting observation is that when the speed of rotation was raised well above that required to produce the four-cell configuration, and sustained at the high value for several minutes, there appeared to be a degradation of the polymer, and in all subsequent tests with the same fluid sample, it was never possible to produce a configuration with less than six cells.

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