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Lubrication of a Porous Bearing —Reynolds' Solution

The problem of lubrication of a journal in a porous bearing is considered. A Reynolds' equation modified to accommodate mass transfer with the fluid-saturated bearing is solved, and the influence of the permeability and radius ratio of the bearing is examined. The effects of the bearing flow are such as to reduce the magnitude of the pressure and shift the maximum away from the position of minimum gap. In extreme cases, the integrated resultant of the pressure forces is so reduced as to become comparable in magnitude with the normally negligible shear stress resultant. This latter resultant has an opposing sense so that the total load capacity of the bearing is greatly reduced as a result of bearing flow.

Historical Introduction and Statement of Purpose

IN THIS REPORT, we discuss the unique features which are introduced into the theory of hydrodynamic lubrication of bearings when the bearings are permeable and tolerate a flow of oil. The subject of the analysis is not entirely new, and a few aspects of the problem have been treated by several authors [1, 2, 4, 5, 9].[1]

Porous bearings are extensively used in technological applications. The chief advantage of these bearings is that they require no exterior oil supply. Such bearings, after long service, are not hydrodynamically lubricated in that the load is not supported by a fluid film. Since our interest in this work is confined to hydrodynamically lubricated bearings, our analysis applies only to the initial "break-in" period (in which the oil supply is plentiful) and to the occasional pressurized system (in which oil losses are absent or annulled by a continuous supply). The bearings analyzed in this work necessarily operate at higher rotational speeds for a given load and have a potential for service in applications where light loads and high speeds are characteristic. The pressure which would develop in a solid bearing at a given journal speed is reduced when the bearing is permeable and the journal speed is unchanged. This fact has importance for applications in which lubricant cavitation is to be avoided.

[1] Numbers in brackets designate References at end of paper.

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The goal of this investigation is the prediction of bearing behavior as a function of bearing parameters. This is essentially a specification of the interrelations of permeability, bearing thickness, clearance, and the eccentricity of the journal. The nonpermeable counterpart of this problem is the subject of the classical theory of hydrodynamic lubrication. The bearing assembly of infinite length, with which we are concerned, is completely described for the nonpermeable bearing by the theory of Reynolds [10] and Sommerfeld [14]. There is a classical short-length bearing approximation developed by Ocvick [9] which has been extended by Morgan and Cameron [1] and Cameron, Morgan, and Stainsby [2] for applications to porous bearings. In these approximations, the circumferential variations of the pressure gradient, which may dominate the flow in longer bearings, are entirely neglected. An analysis of the porous bearing problem which starts from the low Reynolds number equation of Stokes has been constructed by Joseph and Tao [4]. This analysis, however, neglects effects of the bearing thickness. The journal is represented by a rotating circle of the bipolar family, and the inner bearing surface, by another circle of this family. The bearing itself is presumed infinite in extent and models, as is shown in the sequel, a thick bearing (radius ratio $> 3/2$).

Our present effort starts from a Reynolds' equation modified to account for the through flow (as suggested by Cameron, et al. [1, 2]) and matched to the bearing flow by matching conditions at the common boundary. The bearing itself is presumed to be encased in a solid so that the outer bearing surface is impermeable. The bearing has a finite thickness and is infinite in length. The pressure in the bearing is harmonic, and its first derivative (which gives the normal velocity) vanishes at the outer solid-bearing surface. At the inner bearing surface, the harmonic functions must satisfy the modified Reynolds' equation.

The results of this investigation support conclusions of [4]

Nomenclature

c = radial clearance
e = eccentricity
$\mathbf{F}$ = body force
h = film thickness
k = permeability
$K = 12Rr_1/c^3$
M = moment on bearing or journal
$\hat{p}_a$ = absolute pressure in porous bearing
$\hat{p}_o$ = absolute pressure in porous bearing measured at $\theta = 0$ and π
$\hat{p}$ = relative pressure ($= \hat{p}_a - \hat{p}_o$)
p_a = absolute pressure in free fluid region
p_o = absolute pressure in free fluid region measured at $\theta = 0$ and π
p = relative pressure ($= p_a - p_o$)
$\hat{P}$ = pressure parameter
P = pressure parameter
$\mathbf{Q}$ = filter velocity in porous bearing
Q = volume rate of flow
r = radial variable
r_1 = inner radius of porous bearing
r_0 = outer radius of porous bearing
$R = r/r_1$
u = circumferential velocity of fluid
v = radial velocity of fluid
V = circumferential velocity of journal
X = x-component of force on bearing or journal
Y = y-component of force on bearing or journal
ϵ = eccentric ratio ($= e/c$)
$\alpha = (1 - \epsilon^2)^{1/2}$
β = radius ratio ($= r_0/r_1$)
$\eta = h/c = 1 + \epsilon \cos \theta$
θ = angle measured counterclockwise from top of the bearing
μ = viscosity
ϕ = permeability parameter ($= kr_1/c^3$)

and extend these to accommodate the effects of the finite thickness of the porous bearing. The chief effect of the permeability is the reduction of pressures which would otherwise develop. The force resultants are lower as a result of the diminution of the pressure, and the shear-stress resultant may be nonnegligible relative to the pressure-force resultant. Effects of changing the thickness of the bearing are pronounced as the bearing thickness is small, and they rapidly diminish as the ratio of the outer to inner bearing is increased beyond 3/2.

Statement of Problem, Including Mathematical Formulation

Consider a solid journal rotating in a porous bearing of a finite constant thickness contained in a solid housing with end effects neglected. The cross sections of the journal and the bearing are nonconcentric circles, Fig. 1.

The assumptions of conventional lubrication theory retained in this analysis are:

1 The lubricant is Newtonian and incompressible.

2 The film is so thin, compared with the ratio of kinematic viscosity to linear velocity, that the motion of the fluid is laminar, inertial terms and bearing curvature can be neglected, and pressure and viscosity are uniform across the film.

3 No slip occurs between fluid and bearing surfaces.

4 There is no end leakage from the bearing (problem is two-dimensional).

The motion of the viscous fluid is induced by the rotation of the journal. The behavior of the fluid outside the porous bearing is obviously influenced by the flow within the porous bearing. The solutions to the governing equations valid in different adjacent regions must be matched on the boundaries.

Briefly, the problem is characterized by the requirements that the flow satisfy:

1 The appropriate specialization of the Reynolds' equation in the free region (outside the porous region).

2 The Laplace equation for the pressure in the porous region.

3 No-slip conditions immediately outside the porous region (see [4] for full discussion).

4 A condition requiring the radial filter velocity to vanish on the outer (solid) boundary of the porous bearing.

5 Continuity of pressure across the porous bearing.

6 Continuity of the normal component of the velocity across the porous boundary.

In the porous-bearing ring (assumed homogenous and isotropic), the filter velocity $\mathbf{Q}$ is related to the relative pressure $\hat{P}$ by Darcy's law

$$\mathbf{Q} = \frac{k}{\mu} \nabla \hat{p} \tag{1}$$

From mass conservation, $\mathbf{Q}$ is solenoidal, implying that

$$\nabla^2 \hat{p} = 0 \tag{2}$$

It is required that $\hat{P}$ be a single-valued harmonic function, the normal derivative of which vanishes on the outer radius of the ring and which satisfies a Reynolds' equation along the inner circumference of the ring.

We next follow Morgan and Cameron [1] in a derivation of Reynolds' equation modified to account for mass exchanges with the oil-saturated bearing. As is usual in lubrication theory, we unwrap the gap between journal and bearing. It is assumed then that lengthwise variations are dominated by transverse variations so that the equation

$$u = \frac{1}{2\mu} \frac{dp}{dx} (y^2 - yh) + \frac{V}{h} y \tag{3}$$

where $h(x)$ is the local channel height, is locally valid. Conservation of mass requires that (Fig. 2)

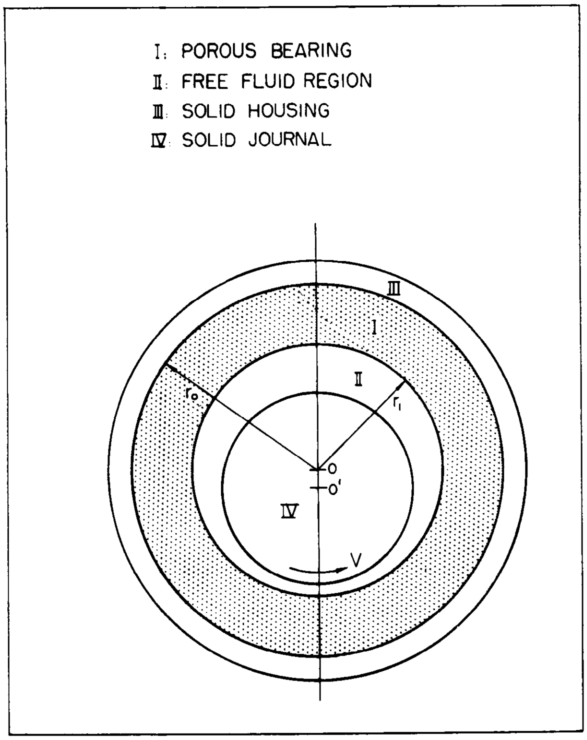

Fig. 1 Bearing configuration

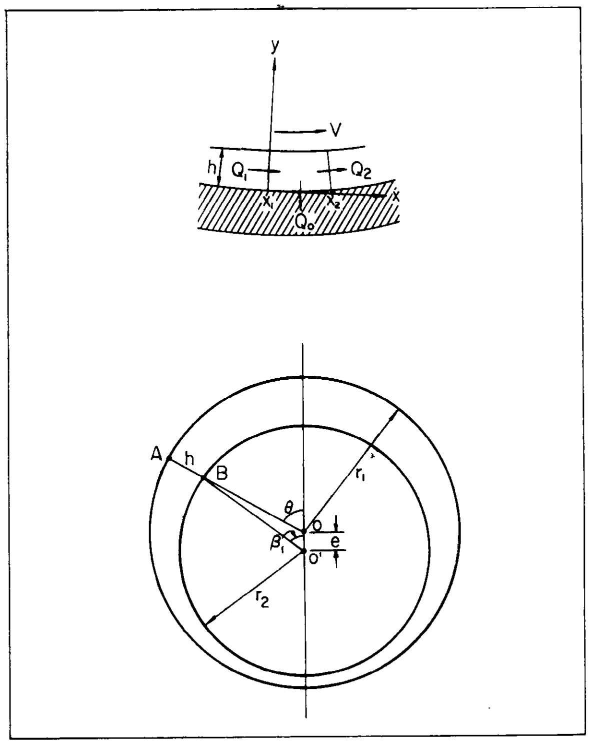

Fig. 2 Geometry of fluid film

$$Q_2 - Q_1 + Q_0 = 0 \tag{4}$$

where Q_0 is given by the Darcy law

$$Q_0 = -\frac{k}{\mu} \left.\frac{\partial \hat{p}}{\partial r}\right)_{r_1} \Delta x \tag{5}$$

From equations (4) and (5), it follows that

$$\frac{dQ}{dx} - \frac{k}{\mu} \left.\frac{\partial \hat{p}}{\partial r}\right)_{r_1} = 0 \tag{6}$$

where

$$Q = \int_0^{h(x)} u dy = -\frac{1}{12\mu}\frac{dp}{dx}h^3 + \frac{V}{2}h \tag{7}$$

Now let $x = r_1\theta$ and combine (6) and (7) to obtain

$$\frac{d}{d\theta}\left(h^3\frac{dp}{d\theta}\right) - 6\mu V r_1 \frac{dh}{d\theta} + 12kr_1^2\left.\frac{\partial \hat{p}}{\partial r}\right)_{r_1} = 0 \tag{8}$$

This is a Reynolds' equation with an additional term to accommodate the mass exchange between the film and bearing. The usual approximation $h(\theta) = c(1 + \epsilon\cos\theta)$ for small clearances is valid in the present context. The term $dh/d\theta = -\epsilon c \sin\theta$ is a driving term for (8).

It is convenient to introduce the following dimensionless variables:

$$\begin{aligned}\eta &= h/e = 1 + \epsilon\cos\theta\\ K &= 12kr_1/c^3\\ \beta &= r_0/r_1\\ R &= r/r_1\\ P &= pc^2/6\mu r_1 V\\ \hat{P} &= \hat{p}c^2/6\mu r_1 V\end{aligned} \tag{9}$$

Using the variables defined in (9), equation (8) becomes

$$\frac{d}{d\theta}\left(\eta^3\frac{dP}{d\theta}\right) - \frac{d\eta}{d\theta} + K\left.\frac{\partial \hat{P}}{\partial R}\right)_1 \tag{10}$$

The quantity $\phi = \frac{kr_1}{c^3}$ $(K = 12\phi)$ is a design factor. It follows that

$$P = \frac{pc^2}{6\mu r_1 V} = F(\theta;\ \phi, \epsilon, \beta)$$

and

$$\hat{P} = \frac{\hat{p}c^2}{6\mu r_1 V} = \hat{F}(r, \theta;\ \phi, \epsilon, \beta)$$

The parameters (ϕ, ϵ, β) characterize the solution.

Solutions

In this section, we develop a formally exact solution and the approximations which make the solution computationally useful. We first note that equation (8) and the ring boundary conditions imply that p and $\hat{p}$ are antisymmetric. Harmonic and antisymmetric $\hat{p}$ with a vanishing normal derivative at $r = r_0$ is given by

$$\hat{p}\,(r, \theta) = \sum_{n=1}^{\infty} B_n\left(\frac{r_1}{r}\right)^n\left[1 + \left(\frac{r}{r_0}\right)^{2n}\right]\sin n\theta \tag{11}$$

The continuity of pressure implies $\hat{p}(r_1, \theta) = p(\theta)$. This reduces our problem to finding B_n compatible with equation (8). We first substitute harmonic

$$\hat{p}(r_1, \theta) = \sum_{n=1} B_n C_n \sin(n\theta) \tag{12}$$

where

$$C_n = 1 + \beta^{-2n},\ \beta = r_0/r_1,\ D = \frac{6\mu r_1 V}{c^2}$$

into

$$\frac{d}{d\theta}\left(\eta^3\frac{d\hat{p}}{d\theta}(r_1, \theta)\right) - D\frac{d\eta}{d\theta} + Kr_1\left.\frac{\partial \hat{p}}{\partial r}\right)_{r_1} = 0 \tag{13}$$

to find a seven-term recursion formula for the coefficients B_n:

$$[(A_0 + A_2)C_1 + Kd_1]B_1 + 2\,(A_1 + A_3)C_2B_2 + 3A_2C_3B_3 + 4A_3C_4B_4 = D\epsilon \tag{14}$$

$$(A_1 + A_3)C_1B_1 + (2C_2A_0 + Kd_2)B_2 + 3C_3A_1B_3 + 4C_4A_2B_4 + 5C_5A_3B_5 = 0 \tag{15}$$

$$A_2C_1B_1 + 2A_1C_2B_2 + (3C_3A_0 + Kd_3)B_3 + 4A_1C_4B_4 + 5A_2C_5B_5 + 6A_3C_6B_6 = 0 \tag{16}$$

$$\begin{aligned}(m-3)A_3C_{m-3}B_{m-3} &+ (m-2)A_2C_{m-2}B_{m-2} + (m-1)A_1C_{m-1}B_{m-1} + (mC_mA_0 + Kd_m)B_m\\ &+ (m+1)A_1C_{m+1}B_{m+1} + (m+2)A_2C_{m+2}B_{m+2}\\ &+ (m+3)A_3d_{m+3}B_{m+3} = 0 \qquad (m = 4, 5, 6, \ldots)\end{aligned} \tag{17}$$

where

$$\begin{aligned}C_m &= 1 + \beta^{-2m},\ d_m = 1 - \beta^{-2m},\ (m = 1, 2, \ldots)\\ A_0 &= 1 + \tfrac{3}{2}\epsilon^2,\ A_1 = \tfrac{3}{2}\epsilon(1 + \tfrac{1}{4}\epsilon^2)\\ A_2 &= \tfrac{3}{4}\epsilon^2,\ A_3 = \tfrac{1}{8}\epsilon^3\end{aligned} \tag{18}$$

To obtain the coefficients B_n, it is necessary to solve an infinite number of the linear equations. The infinite system is approximated with a truncated system. The coefficients B_n $(n > 10)$ are set to zero, and the 10 linear equations are solved for the 10 unknown B_n $(n \leqslant 10)$. It develops, as will be seen in the sequel, that the true solution is very adequately represented with n less than 10. For $\epsilon \leqslant 0.5$, three terms reasonably represent the true solution. For ϕ very large, only one term is enough. The result for $n = 3$, $B_n = 0$ $(n > 3)$ is as follows:

$$\begin{aligned}\Delta = K^3d_1d_2d_3 &+ K^2\left[C_1d_2d_3\left(1 + \frac{9}{4}\epsilon^2\right) + 2C_2d_1d_3\left(1 + \frac{3}{2}\epsilon^2\right) + 3C_3d_1d_2\left(1 + \frac{3}{2}\epsilon^2\right)\right]\\ &+ K\left[6d_1C_2C_3\left(1 + \frac{3}{4}\epsilon^2 + \frac{9}{8}\epsilon^4\right) + 3d_2C_1C_3\left(1 + \frac{5}{4}\epsilon^2 + \frac{45}{16}\epsilon^4\right) + 2d_3C_1C_2\right.\\ &\times\left.\left(1 + 6\epsilon^2 + \frac{39}{8}\epsilon^4\right)\right] + 6C_1C_2C_3\left(1 + \frac{3}{4}\epsilon^2 + \frac{3}{4}\epsilon^4\right)\end{aligned} \tag{19}$$

$$B_1 = \frac{D\epsilon}{\Delta}\left[K^2d_2d_3 + K\left(1 + \frac{3}{2}\epsilon^2\right)(3d_2C_3 + 2C_2d_3) + 6C_2C_3\left(1 + \frac{3}{4}\epsilon^2 + \frac{9}{8}\epsilon^4\right)\right] \tag{20}$$

$$B_2 = -\frac{D\epsilon^2}{2\Delta}\left[KC_1d_3(3 + \epsilon^2) + 3C_1C_3\left(3 + \frac{13}{4}\epsilon^2 - \frac{15}{16}\epsilon^4\right)\right] \tag{21}$$

$$B_3 = \frac{3D\epsilon^3}{4\Delta}\left[KC_1d_2 + \frac{1}{2}C_1C_2(8 + \epsilon^2 + \epsilon^4)\right] \tag{22}$$

This solution and the true solution are indistinguishable for $\epsilon \leqslant 0.5$, Fig. 12.

It follows that

$$B_n = 6\mu r_1 V/c^2 f_n\,(kr_1/c^3, {}_0r/r_1, \epsilon) \tag{23}$$

Since

$$P = \sum_{n=1}^{\infty} f_n\,(kr_1/c^3, r_0/r_1, \epsilon)\sin n\theta \tag{24}$$

our solution is independent of c except as changes in c induce changes in $\phi = kr_1/c^3$. The true pressure scales with $\mu r_1 V/c^2$, except as changes in pressure are induced by permeability (through ϕ), and these changes which depend on c^{-3} become increasingly important as the clearance is reduced.

Force Resultants

Torque. Much interest in lubrication theory focuses on the resultant force and torque. In this section, we calculate these resultants.

The shear stress is

$$\frac{\tau_0}{\mu} = \left.\frac{\partial u}{\partial y}\right)_{y=0} = -\frac{h}{2\mu}\frac{dp}{dx} + \frac{V}{h} \tag{25}$$

$$\frac{\tau_1}{\mu} = \left.\frac{\partial u}{\partial y}\right)_{y=h} = \frac{h}{2\mu}\frac{dp}{dx} + \frac{V}{h} \tag{26}$$

where

$$\frac{dp}{dx} = \frac{1}{r_1}\frac{dp}{d\theta} = \frac{1}{r_1}\sum_{n=1}^{\infty} B_n C_n n \cos n\theta$$

The torques acting on the journal and bearing are M_1 and M_0:

$$M_i = r_1^2 \int_0^{2\pi} \tau_i d\theta; \qquad i = 0,1 \tag{27}$$

$$\frac{M_1}{\mu\pi r_1^2} = \frac{2V}{c(1-\epsilon^2)^{1/2}} + \frac{c\epsilon C_1 B_1}{2\mu r_1} \tag{28}$$

$$\frac{M_0}{\mu\pi r_1^2} = \frac{2V}{c(1-\epsilon^2)^{1/2}} - \frac{c\epsilon C_1 B_1}{2\mu r_1} \tag{29}$$

$$M_1 - M_0 = \pi c\epsilon C_1 B_1 r_1 \tag{30}$$

It is seen that the permeability and the eccentricity are the sources of the difference of the torques upon the bearing and journal. The difference vanishes as the permeability increases and the eccentricity diminishes.

Force. $X_{0,T}$, $X_{1,T}$ and $Y_{0,T}$, $Y_{1,T}$ are the x, y-components of forces acting on the bearing and journal, respectively:

$$X_{1,T} = \int_0^{2\pi} (p \sin\theta + \tau_1 \cos\theta)\, r_1 d\theta \tag{31}$$

$$X_{0,T} = \int_0^{2\pi} (p \sin\theta + \tau_0 \cos\theta) r_1 d\theta \tag{32}$$

$$\frac{X_{1,T}}{\pi r_1} = B_1C_1\left(1 + \frac{c}{2r_1}\right) + B_2C_2\frac{c\epsilon}{2r_1} + \frac{\mu V}{c} \cdot \frac{2}{\epsilon}\left(1 - \frac{1}{(1-\epsilon^2)^{1/2}}\right) \tag{33}$$

$$\frac{X_{0,T}}{\pi r_1} = B_1C_1\left(1 - \frac{c}{2r_1}\right) - B_2C_2\frac{c\epsilon}{2r_1} + \frac{\mu V}{c} \cdot \frac{2}{\epsilon}\left(1 - \frac{1}{(1-\epsilon^2)^{1/2}}\right) \tag{34}$$

$$Y_{1,T} = \int_0^{2\pi} (p \cos\theta + \tau_1 \sin\theta) r_1 d\theta \tag{35}$$

$$Y_{0,T} = \int_0^{2\pi} (p \cos\theta + \tau_0 \sin\theta) r_1 d\theta \tag{36}$$

The pressure force and shear-stress resultant, respectively, are

$$\frac{X_{1,p}}{\pi r_1} = \frac{X_{0,p}}{\pi r_1} = B_1C_1 \tag{37}$$

$$\frac{X_{1,s}}{\pi r_1} = B_1C_1\frac{c}{2r_1} + B_2C_2\frac{c\epsilon}{2r_1} + \frac{\mu V}{c}\frac{2}{\epsilon}\left(1 - \frac{1}{(1-\epsilon^2)^{1/2}}\right) \tag{38}$$

$$\frac{X_{0,s}}{\pi r_1} = -B_1C_1\frac{c}{2r_1} - B_2C_2\frac{c\epsilon}{2r_1} + \frac{\mu V}{c}\frac{2}{\epsilon}\left(1 - \frac{1}{(1-\epsilon^2)^{1/2}}\right) \tag{39}$$

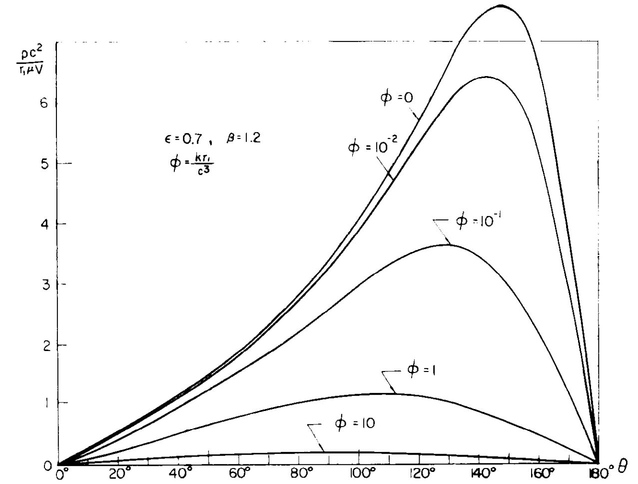

Fig. 3 Pressure distribution in lubricant for fixed eccentricity (ϵ) and radius ratio (β). Pressure is a decreasing function of permeability parameter (ϕ). Pressure maximum decreases and shifts away from position of minimum gap (θ = 180 deg) as permeability parameter increases.

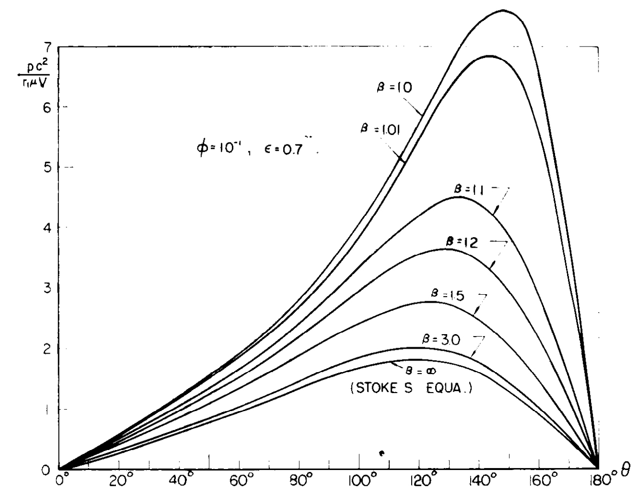

Fig. 4 Pressure distribution in lubricant for fixed values of eccentricity (ϵ) and permeability parameter (ϕ). Pressure maximum decreases and shifts away from position of minimum gap as radius ratio increases. Rate of change of pressure is greatest for a radius ratio of 1 and decreases as radius ratio increases. For very large values of radius ratio ($\beta \to \infty$), this solution tends to the solution of Stokes' equation [4] for the same flow.

For small permeability, the pressure force dominates, and the shear-stress resultant is neglected as small compared with the pressure resultant. But the pressure resultant decreases markedly with permeability, Fig. 6, and the shear stress resultant decreases only slightly and turns negative, Fig. 7. The total force resultant decreases strongly and turns negative as the shear stress resultant becomes dominant, Figs. 9, 10.

For the impermeable case, the shear stress resultant, which is pointed out in [4], changes sign at ϵ = 0.721 and is negative for large values of ϵ. Hence, for large eccentric ratios, the bearing capacity can vanish and even change direction.

From equations (28, 29, 37, 38, 39) and the fact that

$$\frac{B_n c^2}{6\mu r_1 V} = f_n(\phi, \beta, \epsilon) \tag{40}$$

one may infer that

$$\frac{X_p c^2}{\mu r_1 V}, \ \frac{X_s c}{\mu r_1 V}, \ \text{and} \ \frac{Mc}{\mu r_1 V}$$

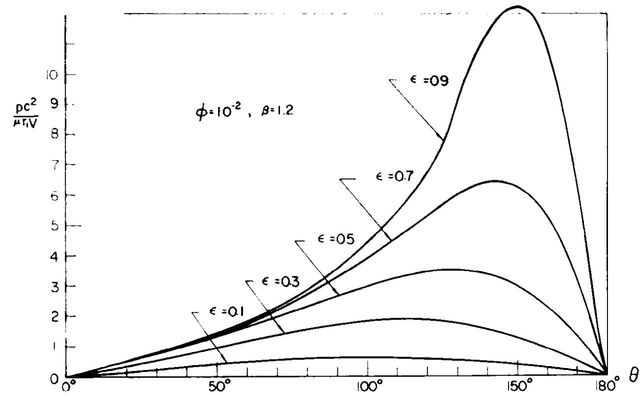

Fig. 5 Pressure distribution in lubricant for fixed values of radius ratio (β) and permeability parameter (ϕ). Pressure magnitude is a decreasing function of eccentricity as in the impermeable case.

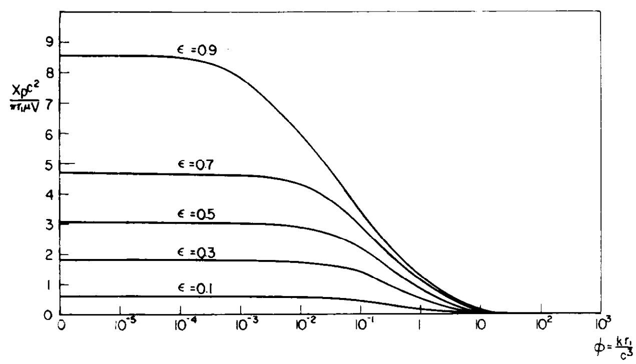

Fig. 6 Resultant pressure force on journal as a function of permeability (ϕ) with eccentricity (ϵ) as a parameter for a fixed radius ratio ($\beta = 1.2$). Pressure is more pronounced for larger values of eccentricity.

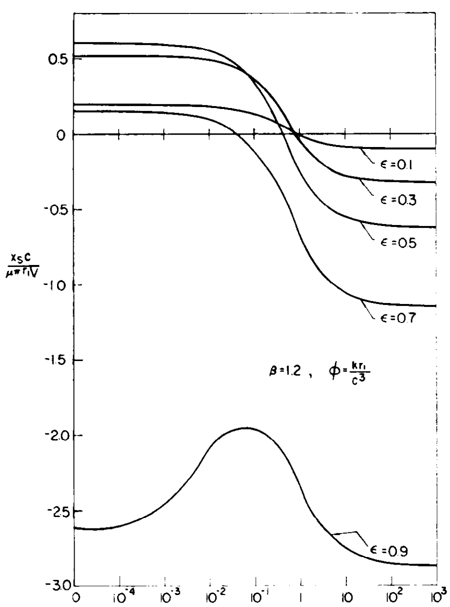

Fig. 7 Resultant shear force on journal as a function of permeability (ϕ) with eccentricty (ϵ) as a parameter for a fixed radius ratio ($\beta = 1.2$). Shear resultant is always negative for sufficiently large values of ϕ or ϵ. For large ϕ, shear resultant may be of same order of magnitude as pressure resultant.

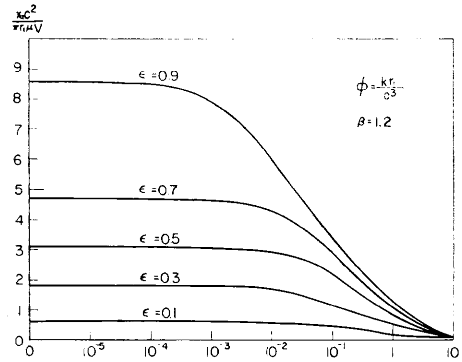

Fig. 8 Resultant total force on journal as a function of permeability (ϕ) with eccentricity (ϵ) as a parameter for a fixed radius ratio ($\beta = 1.2$). Total force is a combination of pressure resultant (Fig. 6), which scales with C^{-2} for small C, and shear stress resultant (Fig. 7), which scales C^{-1} for small C. For small permeabilities, pressure resultant dominates.

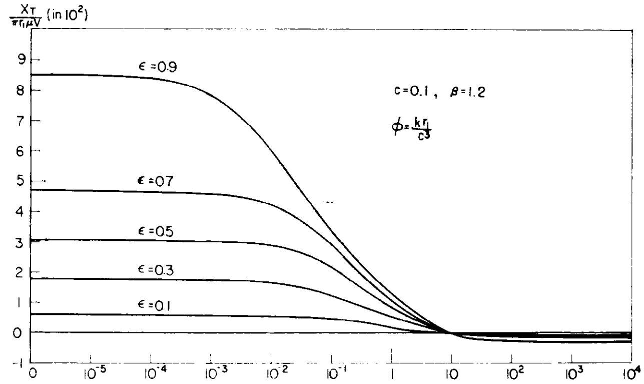

Fig. 9 Resultant total force on journal as a function of permeability (ϕ) with eccentricity (ϵ) as a parameter for fixed values ($\beta = 1.2$, C = 0.1). For large ϕ, force is dominated by shear resultant, which is negative. With $r_1 = 1$ in. and C = 0.001 in., $\phi = 10$ implies $k = 10^{-9}$ sq in., which is a very high permeability and not at all typical of porous bearing.

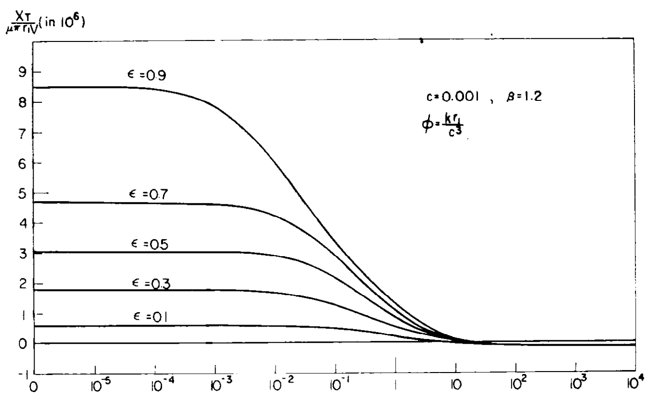

Fig. 10 Resultant total force on journal. This is qualitatively the same as Fig. 9. With $r_1 = 1$ in. and C = 0.001 in., $\phi = 10$ implies $k = 10^{-9}$ sq in., which is typical for porous bearing.

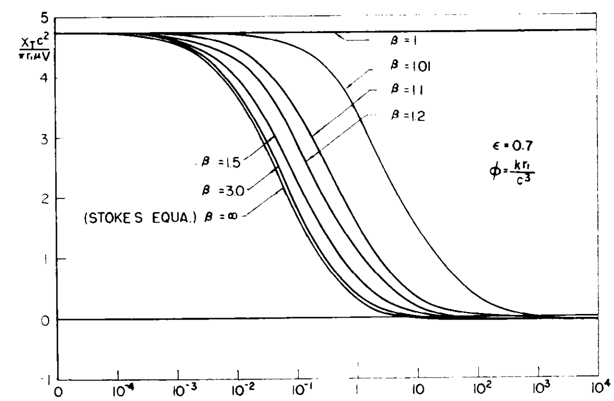

Fig. 11 Resultant total force on journal as a function of permeability (ϕ) with radius ratio (β) as a parameter and a fixed value of eccentricity (ϵ = 0.7). Total force decreases with decreasing rapidity as radius ratio is increased from 1. Value of β = 1 corresponds to an impermeable bearing, and $\beta \rightarrow \infty$ to an infinite porous matrix. In this latter limit, solution may be compared with that given in [4] for same geometry. Solution in [4] starts from two-dimensional equations of Stokes. Reynolds' and Stokes' solutions are nearly identical.

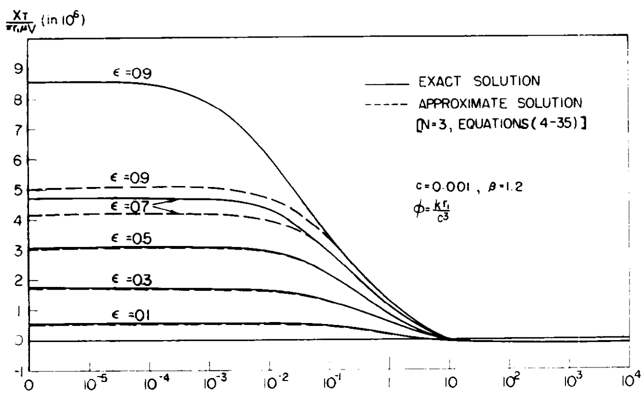

Fig. 12 Resultant total force on journal. The approximate solution, equations (19) to (22), is compared with numerical solution of equations (15) to (18). Approximate solutions are less accurate for larger values of eccentricity.

depend only on the parameters (ϕ, β, ϵ). Since X_P dominates X_T when ϕ is small, and X_S dominates X_T when ϕ is large, it follows that

$$\frac{X_T c^2}{\mu r_1 V} \text{ or } \frac{X_T c}{\mu r_1 V}$$

depend only on (ϕ, β, ϵ) as ϕ is small or large, Fig. 8. In fact, the numerical results indicate that small is approximately $\phi \leqslant 10$ and large is $\phi > 10$.

Discussion on Solutions

The Fourier series which is formally exact will represent the true solution if coefficients B_n can be found which reduce the linear equation to identities. This system is infinite, and its exact convergence characteristics are not easily examined. Suffice it to say that the truncated system does rapidly converge. With $\epsilon \leqslant 0.9$, the solutions obtained by truncating the system with $n > 10$ do not differ from one another within the error (10^{-4}) represented by our graphical representation of the results.

The Fourier series would appear to converge for all values of ϕ and rapidly as ϕ is large. For large values of ϕ ($\geqslant 10$), only one term is enough. As $\phi \rightarrow \infty$ ($k \rightarrow \infty$ or $c \rightarrow 0$), the coefficient $B_1 \sim 0(1/\phi)$, $B_j \sim 0(1/\phi^2)$, $j = 2, 3, \ldots$. The pressure dies like $1/\phi$ in this limit, but the velocities in lubricant and bearing are finite. In general, the Fourier series

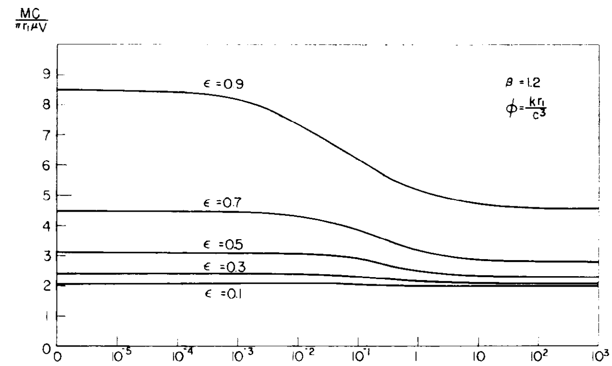

Fig. 13 Resultant moment on journal as a function of permeability (ϕ) with eccentricity as a parameter and for fixed value of radius ratio (β = 1.2). Moment decreases, but not significantly, for large permeability. Torque is largely determined by relative motion of bearing and journal, and this does not change as a result of bearing flow.

converges more rapidly as ϵ is small, with a uniform pressure resulting in the limit $\epsilon \rightarrow 0$, as is consistent with the physics of flow.

As $\epsilon \rightarrow 1$, the series converges slowly; while, if $\phi = 0$, the solution diverges. In this case, the journal and the bearing are in contact, and fluid cannot be carried through the gap.

It is of some interest that Reynolds himself solved the corresponding impermeable problem with a Fourier series [10]. However, the Fourier coefficients A_n were expanded in a power series in ϵ,

$$A_n = \sum_{k-1} C_{k,n}\, \epsilon^k$$

and the interval of convergence of the *power series* is $\epsilon < 0.6$. It is not the Fourier series which diverges, as is sometimes stated in the lubrication literature [13, p. 178], but the power series in ϵ.

Conclusions

Figs. 3–13 represent the effects of the flow in the porous bearing on the pressure distribution and force resultants. The results are obtained from machine calculations with $r_1 = 1$ and $0.1 \leq \epsilon \leq 0.9$. The important parameters are the clearance, eccentricity, permeability, and radius ratio.

The following observations are worth emphasis:

1 The effect of increasing the permeability, other parameters fixed, is to reduce the pressure, shift the maximum (Fig. 3), and to reduce the magnitude of the total-force resultant (Figs. 8, 9, 10) and the pressure-force resultant (Fig. 6). The effects on the torque and shear stress of increases in permeability are less significant (Figs. 7 and 13).

2 The effects of permeability are more pronounced when:

(*a*) The clearance is small. The dependence of the pressure on permeability is of the form

$$\hat{p} \propto (\mu r_1 V/c^2)\, F\, (\phi, \beta, \epsilon)$$

Thus the diminution of c enhances the effect of the permeability parameter even more strongly than the corresponding direct effect on the pressure. The permeability enters the problem through ϕ. The permeability parameter can always be made significant by reducing the clearance (Fig. 3).

(*b*) The eccentric ratio is near unity. An eccentric ratio of 1 implies contact, and our solutions are not valid in this limit. The essential effects are easily read from Figs. 8, 9, 10, and 13. These graphs show that the variation of total force and torque with permeability is much more pronounced as the value of ϵ approaches unity. The range of clearance and eccentricity in which small permeability is important may be read from the graphs.

3 The radius ratio can, of course, have an effect only if the bearing is permeable. In the limit $\beta = r_0/r_1 = 1$, there is no porous matrix, and the impermeable solution applies. In the limit $\beta \to \infty$, the matrix is an infinite porous space with an internal hole in which the journal rotates. There is a solution available for this latter case which was derived from the slow-motion equations of Stokes [4]. Our solution and that of [4] are compared in Figs. 4 and 11. The agreement in all cases is excellent.

The variation of the pressure distribution and force resultant with β is shown in Figs. 4 and 11, respectively. The zero-thickness solution is consistent with Sommerfeld's solution, as was to be expected. It is of importance that the effect of increase in the radius is much more pronounced as the thickness approaches zero. The difference between the pressure distribution in an infinite porous space and for a radius ratio of 1.5 is less than 20 percent. This tends to establish the solutions given in [4] as appropriate to thick bearings ($\beta > 1.5$).

4 The force resultants decrease with increasing permeability. The pressure resultant tends to zero (Fig. 6). This is not true of the shear-stress resultant (Fig. 7), which turns negative for sufficiently large ϵ. The total force, which is the sum of the pressure and shear-stress resultant, turns negative with the shear resultant when the permeability effects have reduced the magnitude of the pressure (Figs. 9 and 10 and cf. [4]).

5 The journal may run at a higher speed for a fixed pressure when the bearing is permeable. If this pressure is the cavitation limit, then a higher speed may be obtained by using a porous bearing. From Fig. 3 ($\epsilon = 0.7$, $\beta = 1.2$), the pressure parameter $pc^2/\mu r_1 V$ is reduced to half of that in the nonpermeable case when $\phi = 10^{-1}$, to 1/5 when $\phi = 1$, to 1/30 when $\phi = 10$. Thus, for a fixed $\hat{p}$, we may increase the journal speed by a factor of 2 when $\phi = 10^{-1}$, a factor of 5 when $\phi = 1$, and a factor of 30 when $\phi = 10$.

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