CRITICAL VELOCITY U_c

Under certain conditions particles cannot be sucked through or blown through a hole.

When $U < U_c$, particles go through
When $U > U_c$, particles don't go through

$U_c(\phi, \frac{D_p}{D_T}, E)$

ϕ volume fraction of particles

$\frac{D_p}{D_T} = \frac{\text{particle diameter}}{\text{tube diameter}}$

$E = \frac{\lambda\eta}{\rho D_p^2}$ elasticity number

$E = 0$ (newtonian) particles always go through

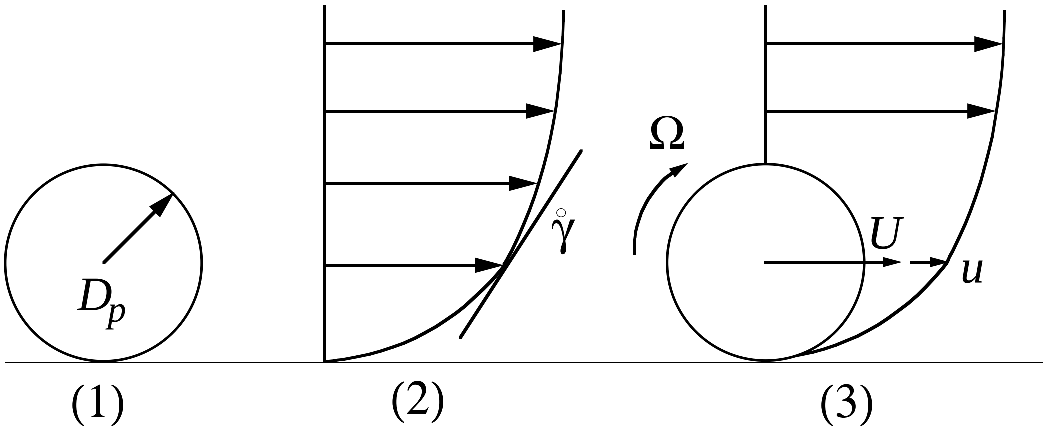
Dependence of U_C on ϕ, D_p/D_T, E

- When the volume fraction of solids is large, the particles will go through
- Smaller particles enter the perforation more easily
- The more viscoelastic the fluid the greater is the screening effect in which particles don't go through

LIFT OFF OF PARTICLES ON A PLANE WALL IN A SHEAR FLOW

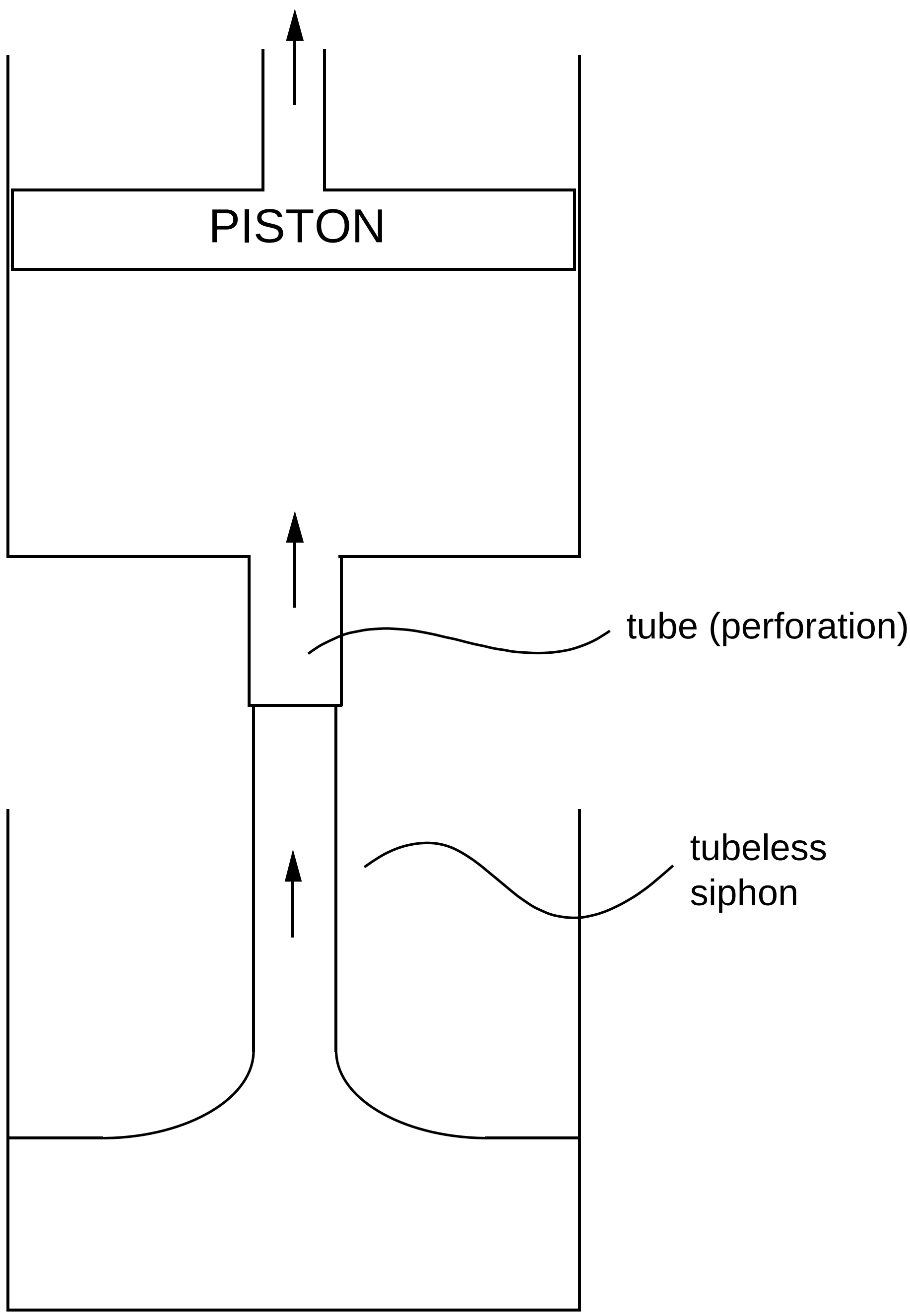
- Description of the problem
- Analysis
- Simulation

CRITICAL CONDITION FOR LIFT OFF

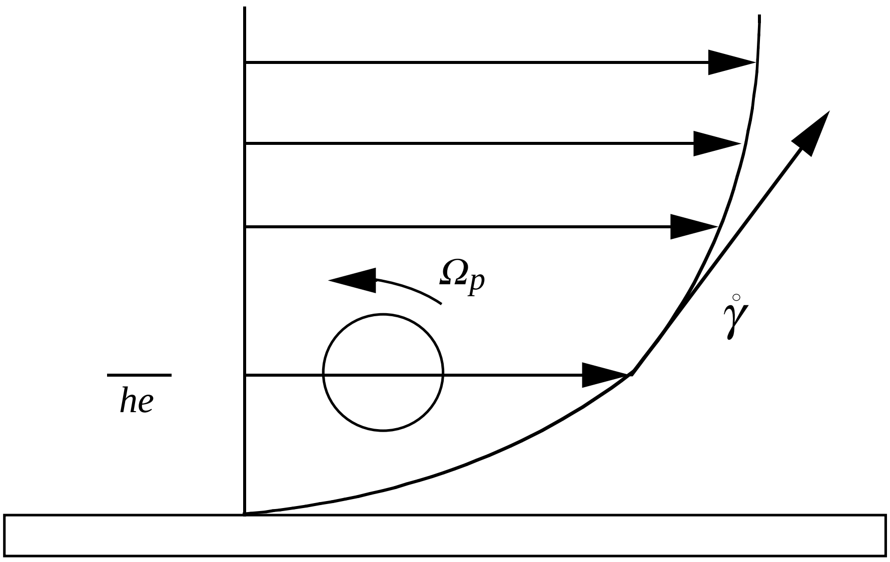


1. Particle of density ρ_p rests on a wall
2. Pressure flow with gradient dp/dx is started
3. The particle slips with velocity U and rotates with angular velocity Ω. The slip velocity is $u - U$
4. If $\frac{dp}{dx}$ is large enough, the particle lifts off. (If the slip velocity is large enough, the particle lifts off)

PARTICLE TRANSPORT THROUGH PERFORATIONS



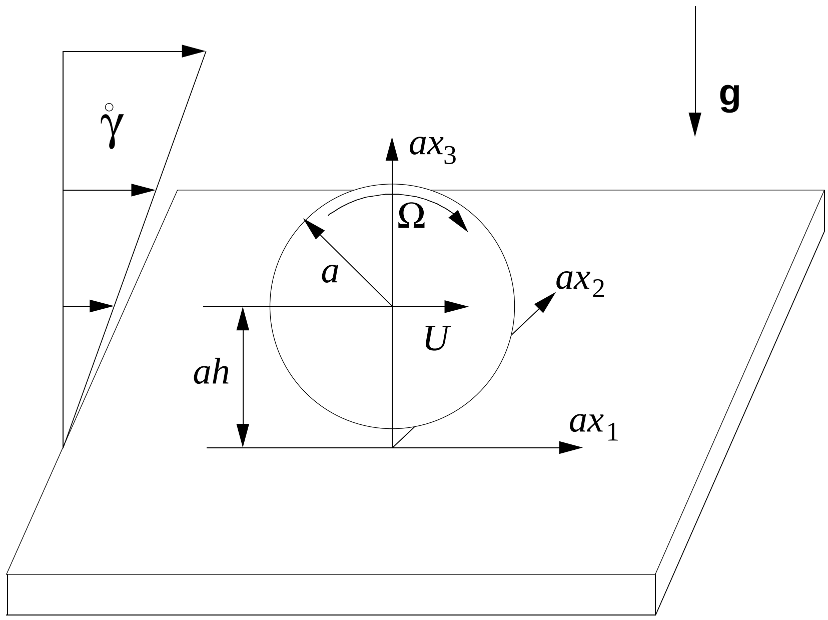
Particle at equilibrium height he



The particle rises to a height in which lift balances weight. Then the slip velocity $u - U$ and angular slip velocity $\Omega_f - \Omega_p = \frac{\overset{\circ}{\gamma}}{2} - \Omega_p$ are positive and at equilibrium values.

If the speed is increased, the particle rises, reducing lift until the lift equals the weight.

PERTURBATION ANALYSIS OF LIFT OFF



Sphere moving along a planar wall. The non-dimensional minimum gap between the sphere and the wall is $\varepsilon = h - 1$

$\mathbf{u} = -U\mathbf{e}_1$ sliding velocity

$\mathbf{u} = \Omega a \mathbf{e}_2 \wedge (\mathbf{x} - h\mathbf{e}_3)$ on the sphere

$\mathbf{u} = (\mathring{\gamma} a x_3 - U)\mathbf{e}_1$ for $|\mathbf{x}| \to \infty$

Lift and weight

$\mathbf{L} = L\mathbf{e}_3$ lift

$\mathbf{W} = -W\mathbf{e}_3$ weight

$$W = \Delta\rho g \frac{4}{3}\pi a^3, \Delta\rho = \rho_p - \rho_f$$

The sphere lifts off when $L/W \geq 1$

Perturbation for slow steady flow

$$\nabla^2 \mathbf{u} = -\nabla p + Re(\mathbf{u} \cdot \nabla)\mathbf{u} + De\,\mathrm{div}\left\{\mathbf{B} + \frac{\alpha_2}{\alpha_1}\mathbf{A}^2\right\}$$

$$Re = \frac{\rho V a}{\eta} \qquad \text{Reynolds}$$

$$De = \frac{-\alpha_1 V}{a\eta} \qquad -\alpha_1 > 0 \quad \text{Deborah}$$

V is a scale velocity U or $\overset{\circ}{\gamma} a$ or Ωa

$\mathbf{u} = \mathbf{u}_0 + Re\mathbf{u}^{(1)} + De\mathbf{u}^{(2)} +$ higher order

$\mathbf{u}_0$ is Stokes flow

$$\mathbf{u}_0 = \mathbf{u}_{01} U + \mathbf{u}_{02} \overset{\circ}{\gamma} a + \mathbf{u}_{03} \Omega a$$

LIFT INTEGRALS

$$L = ReL^{(1)} + DeL^{(2)}$$

Use the reciprocal theorem to calculate integrals

$$L^{(1)} = \int_{\text{fluid}} \boldsymbol{v} \cdot \left\{ \mathbf{u}^{(0)} \cdot \nabla \mathbf{u}^{(0)} \right\} dV$$

$\boldsymbol{v}, \mathbf{u}_0$ are Stokes flows

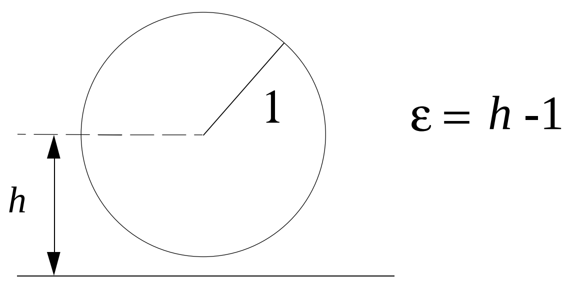
Use Giesekus theorem to simplify; if $\alpha_1 + \alpha_2 = 0$ the 2nd order correction modifies the pressure but not the velocity, as in Tanners theorem.

$$L^{(2)} = -\int_{\text{sphere}} \left\{ \mathbf{u}^{(0)} \cdot \nabla p^{(0)} + \frac{1}{4} \mathbf{A}^2[\mathbf{u}_0] \right\} \mathbf{n} \cdot \mathbf{e}_3 \, dS$$

INERTIAL LIFT was calculated by Krishnan & Leighton [1955] following Leighton and Acrivos [1985, $U = \Omega = 0$]

$$\frac{L^{(1)}}{W} = \frac{3\rho}{4\pi\Delta\rho}\left\{ \frac{1.755U^2}{a} + 0.546\Omega^2 a - 2.038Ul - 9.044U\overset{\circ}{\gamma} + 1.212\Omega\overset{\circ}{\gamma}a + 9.257\overset{\circ}{\gamma}^2 a \right\}$$

THE STOKES FLOW SOLUTION IS SINGULAR IN THE GAP $h - 1 \to 0$



O'Neill & Stewartson [1967] and Cooley & O'Neill solved the problam by matched asymptotic methods. The inner or gap region is singular. This singularity is integrable and doesn't effect the inertial lift. The singularity does enter into three of the viscoelastic lift integrals.

LIFT / WEIGHT RATIOS

$$\frac{L}{W} = \frac{L^{(1)} + L^{(2)}}{W}, \frac{L}{W} \begin{cases} < 1 \text{ the particle does not lift off} \\ > 1 \text{ lift off} \end{cases}$$

$$\frac{L^{(1)}}{W} = \frac{3\rho a}{4\pi\Delta\rho g}\left[1.75\frac{U^2}{a^2} + 0.54\Omega^2 - 2.04\frac{U}{a}\Omega - 9.04\frac{U}{a}\mathring{\gamma} + 1.21\Omega\mathring{\gamma} + 9.25\mathring{\gamma}^2\right]$$

If $\frac{U}{a}$ is $O(1)$, then $\frac{L^{(1)}}{W} = O(a)$ and small particles don't lift

$$\frac{L^{(2)}}{W} = \frac{6(-\alpha_1)}{5\Delta\rho g}\frac{1}{a\varepsilon}\left[\frac{U^2}{a^2} + \Omega^2 - \frac{1}{2}\frac{U}{a}\Omega\right]$$

If $\frac{U}{a}$ is $O(1)$, then $\frac{L^{(2)}}{W} = O\left(\frac{1}{a\varepsilon}\right)$ is very large even if a is not small and the particle will lift no matter what.

LIFT OFF VELOCITY U_{LO}

The critical condition for lift off is $\frac{L}{W} = 1$
Assume that the slip velocity term is the most important

$$\frac{L^{(1)}}{W} = \frac{5.25U^2}{4\pi g \Delta\rho a} = 1,$$
$$\frac{L^{(2)}}{W} = \frac{6\eta\lambda_1 U^2}{5g\Delta\rho a^3\varepsilon} = 1$$

$-\alpha = \rho\lambda$ for Maxwell models

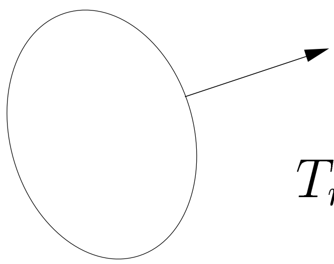
$U_{LO} = \sqrt{4\pi g\Delta\rho a/5.25}$ inertia

$U_{LO} = \sqrt{5g\Delta\rho a^3\varepsilon/6\eta\lambda_1}$ viscoelastic

$\varepsilon = 0$ means that the particle is on the wall. If $\varepsilon = 0$, then $U_{LO} = 0$ and the particle will lift for any U however small.

INTUITIVE EXPLANATION OF VISCOELASTIC LIFT PROPORTIONAL TO $1/\varepsilon$

The normal stress on a body due to viscoelasticity in slow steady 2D flow of a viscoelastic fluid is always *compressive*; it can also be called viscoelastic “pressure”.



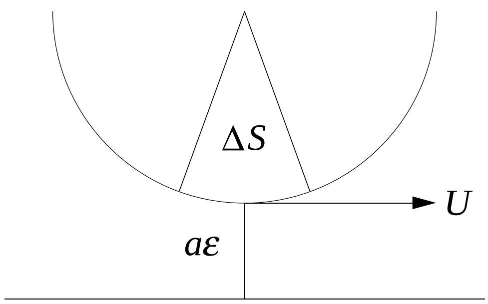
$$T_{nn} = \frac{-\Psi_1(0)}{4}\overset{\circ}{\gamma}^2$$

where

$\Psi_1(0) > 0$ is the coefficient of the first normal stress difference,
$\overset{\circ}{\gamma}$ is the shear rate at the wall

The viscoelastic pressure is large where the inertial pressure is small and vice versa.

$$L = \int |T_{nn}|ds \approx \frac{\Psi_1(0)}{4}\overset{\circ}{\gamma}^2 \Delta S$$



$$\Delta S = K\varepsilon$$

$$\overset{\circ}{\gamma} = U/a\varepsilon$$

$$L = \frac{\Psi_1(0)K}{4a^2}\frac{\varepsilon}{\varepsilon^2}$$