Summary of research activities of D. D. Joseph at Intevep
July 26—Aug 6, 2000

(a) **Foamy Oils in Porous Media:** work with Arjan Kamp and Clara Mata

- We have now finished the revisions of the paper, “Foamy Oil in Porous Media” by Joseph, Kamp, Bai [2000], 43 pages. This paper is posted on the web site at *http://www.aem.umn.edu/people/faculty/joseph/*. The paper will be sent to an archival journal.
- A short version of this paper has been prepared for publication as an SPE paper and for the proceedings of an international conference held at the *Institute of Mathematics and Its Applications* held last year.
- The paper is the first purely analytical model dealing with the increase in primary recovery and rate of production in foamy oils. It also introduces a new approach to problems of description of two-phase flow in porous material replacing the concept of relative with the concept of effective viscosity. It compares well with sand pack data but has not been tested against field data.
- It is recommended that the experiments on bubbling of air through foamy oil be abandoned as unproductive. It seems like the nucleation of gas is quite different than the bubbling of air introduced externally.

(b) **Foam flow experiments of Travis Smieja.** He did experiments in a slit column 1" × ¼" × 43" and took pressure drop vs. volume flow data and visual data with regards to flow type. He got a flow type chart which distinguishes churn flow, bubbly flow and slug (or plug) flow. He analyzed slug (dry) foam (water plus 0.06% by weight) as a lubricated flow, which it is, and he applied the ideal lubrication theory backing out the thickness of the lubrication layer. The layer thickness is not monotonic in the flow rate; it first increased and then decreases. The same behavior has been observed for lubricated flow of Orimulsions. The material from the Smieja experiments was presented to Alida Aponpe and Farkis Kakadjian, and a revised version of Smieja’s masters thesis has been sent to Majella Rivero.

Studies of "Self-lubricated transport of concentrated oil in water emulsions" by Mata, Gurfinkel, Núñez and Joseph. A paper is being prepared by Clara Mata & Mariano Gurfinkel in which techniques are put forward to quantify the extent to which Orimulsion self-lubricates in small and large pipes.

I presented a report on "Lubricated transport of viscous oils." In this I discuss core-annular flow in which oil and water are injected continuously, self-lubricated flows of oil-in-water, and water-in-oil emulsions; all three are different. I argued that the success of these lubricated technologies rests more on the "buildup of fouling" than on fouling itself. This is the reason for the success of Syncrude's water-in-oil pipeline. The natural has colloidal clay, which sticks to the oil preventing the oil from sticking to itself. The oil does foul the wall but the clay covering protects the oil on the wall from the buildup of fouling. In this sense the fouled wall protected from further fouling is a better wall than the naked and unprotected wall.

I suggested the remedy to fouling be sought in terms of treating the water to protect against the buildup of fouling rather than treating the wall. The idea would be to find a cheap surfactant that could protect the oil from sticking to itself, but at the same time would not emulsify the heavy oil. I discussed this with Gustavo Núñez and Hercilio Rivas. Hercilio was interested and has agreed to try to seek such a remedy for VZ heavy oils with high asphatine content.

(c) **Gas Liquid Flows.** I have begun to interact with gas-liquid team of Jose Luis Teallero. This interaction has many facets.

- **Pipeline**. I looked at the experimental facility and at the movies of experiments. I thought that the slugs seen at low speeds might not originate from instability of stratified flow but they could possibly be generated by the way the gas is introduced into the pipeline at the elbow T junction. It seems not to be known if the slug starts at the entrance or if it persists all the way to the exit. *I think that the motion of slugs should be traced over the whole length of the pipeline by equipping the line with pressure transducers at the specified intervals.*[1]

The movies were very interesting. I believe they show that at low speeds stratified flow is stable. The distance between slugs was estimated at 55 meters and the whole line is about 80 meters. The distance between slugs cannot be a long bubble. If the slugs would decay at these low speeds we could say that stratified flow was stable. Maybe the pipeline is not long enough to test the stability of slugs observed at low speeds. *It is of course possible that slugs and stratified flow are both stable.* In fact multiple solutions of nonlinear problems are common; this would mess up the flow diagrams.

[1] Lin, P.Y. & Hanratty, T.J. 1987. Detection of slug flow from pressure measurements, *Int. J. Multiphase Flow*, **13**, 13-21.

- **Separators**. I revised the possibility of using a rotating drum to separate gas and oil downhole using the fact that gas would rapidly separate under centripetal accelerations thousands of times larger than gravity. This is the way the pharmaceuticals are separated. Maybe there are practical problems to doing this downhole but the idea is very clear and the technology, good motors controlled remotely and robust frames with good bearings, is rather low-tech.
- **Kelvin-Helmholz instability.** This is the classical instability of superposed streams moving at different velocities. This instability is *always* involved in discussions of transition of stratified flow to slugs. *The instability has been treated three ways:*

  (a) **Classical-inviscid instability with surface tension.** You can find this in stability books by Chandrasekhar or by Drazin and Reid. They confine their attention to the case of an infinite domain; two fluids with different properties occupy all of space above and below $z = 0$. On page 409 of "Theoretical Hydrodynamics" Milne Thompson gives a KH formula for flow between parallel plates; this formula enters the literature on gas e-liquid flow through the citation by Dobson and Wallis [1973].

  (b) **Viscous potential flow (Joseph & Funada).** In viscous potential flow the viscosity enters into the problem only through the viscous part of the normal stress. The continuity of the shear stress and the tangential component of velocity is not enforced. Viscous potential flow reduces to the classical cases when the viscosity is put to zero.

  (c) **Averaged equations (two-fluid model)** like those used by Ramshaw & Trapp, Lin & Hanratty, Andritson & Hanratty and Barnes & Taitel. Up to now Intevep has been using Barnea & Taitel. These equations are greatly different than viscous potential flow and even differ strongly from the classical case in the limit in which they should be the same.

  - Viscous normal stresses are not considered in averaged equations
  - Viscous stresses are not considered in the classical and potential flow cases.

- The powers of the wave number k and gravity and surface tension terms of the averaged equations is one greater than in classical theory. There are other differences. These differences are being studied by Clara Mata. We are looking to write a paper “Comparison of viscous and surface tension effects in various approaches to Kelvin-Helmholz instabilities“ by C Mata, J Trallero & D.D. Joseph.

- Comparison of dispersion relations for KH instability from viscous potential flow and two-fluid model (VKH).

Viscous potential flow VPFKH (Joseph & Funada 2000).

- Joseph & Funada use disturbances proportional to

$$e^{i\sigma t}\, e^{ikx}. \tag{1}$$

- VKH (Barnea-Taitel 1993) use disturbances

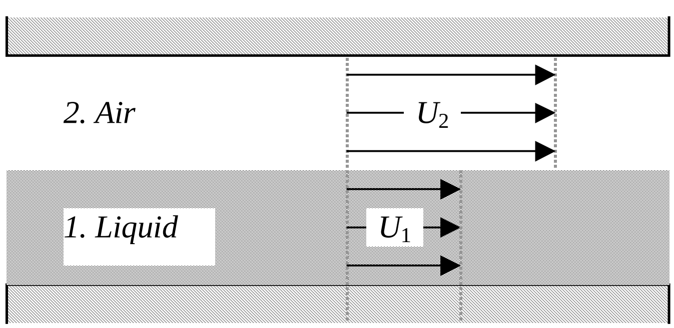
$$e^{i\omega t}\ e^{-i\tilde{k}x}. \tag{2}$$

Hence

$$\sigma = i\omega, \quad \sigma_r = -\omega_i, \quad \sigma_i = \omega_r, \quad k = -\tilde{k} \tag{3}$$

Dispersion relation from VPFKH written with $\sigma = i\omega$

$$\begin{aligned}&(\rho_2 \coth kh_2 + \rho_1 \coth kh_1)\omega^2 + 2\omega\left\{\begin{matrix} k(\rho_2 U_2 \coth kh_2 + \rho_1 U_1 \coth kh_1) \\ -ik^2(\mu_2 \coth kh_2 + \mu_1 \coth kh_1) \end{matrix}\right\} \\ &+ k^2(\rho_2 U_2^2 \coth kh_2 + \rho_1 U_1^2 \coth kh_1) \\ &\qquad - 2ik^3(\mu_2 U_2 \coth kh_2 + \mu_1 U_1 \coth kh_1) - (\rho_1 - \rho_2)gk - \gamma k^3 = 0\end{aligned} \tag{4}$$



Reduction of equation (4) for an infinite domain

$$
\begin{aligned}
&h_1 \to \infty,\ h_2 \to \infty,\ \coth kh_1 \to 1,\ \coth kh_2 \to 1 \\
&(\rho_2+\rho_1)\omega^2 + 2\omega\{(\rho_2 U_2+\rho_1 U_1)k - ik^2(\mu_2+\mu_1)\} \\
&\qquad + k^2\left(\rho_2 U_2^2+\rho_1 U_1^2\right) \\
&\qquad - 2ik^3(\mu_1 U_1+\mu_1 U_1) \\
&\qquad - (\rho_1-\rho_2)gk - \gamma k^3 = 0
\end{aligned}
\tag{5}
$$

Dispersion relation [12] for VKH in Barnea-Taitel [1993] with k replaced by $-k$

$$
\begin{aligned}
&\omega^2 + 2(ak+bi)\omega + ck^2 - dk^4 + eki = 0 \\
&a = \frac{1}{\rho}\left(\frac{\rho_L U_L}{R_L}+\frac{\rho_G U_G}{R_G}\right) \\
&b = \frac{1}{2\rho}\left[\left(\frac{\partial F}{\partial U_{LS}}\right)_{U_{GS},R_L} - \left(\frac{\partial F}{\partial U_{GS}}\right)_{U_{LS},R_L}\right] \\
&c = \frac{1}{\rho}\left\{\frac{\rho_L U_L^2}{R_L}+\frac{\rho_G U_G^2}{R_G} - (\rho_L-\rho_G)g\cos\beta\frac{A}{A'_L}\right\} \\
&d = \frac{\sigma A}{\rho\cdot A'_L}, \qquad e = -\frac{1}{\rho}\left(\frac{\partial F}{\partial R_L}\right)_{U_{LS},U_{GS}} \qquad \rho = \frac{\rho_L}{R_L}+\frac{\rho_G}{R_G}
\end{aligned}
\tag{6}
$$

Inviscid flow (from VPFKH (4))

$$
\begin{aligned}
&(\rho_2\coth kh_2+\rho_1\coth kh_1)\omega^2 \\
&\quad + 2\omega k(\rho_2 U_2\coth kh_2+\rho_1 U_1\coth kh_1) \\
&\quad + k^2\left(\rho_2 U_2^2\coth kh_2+\rho_1 U_1^2\coth kh_1\right) \\
&\qquad\qquad - (\rho_1-\rho_2)gk - \gamma k^3 = 0
\end{aligned}
\tag{7}
$$

Inviscid flow (from VKH (6))

$$
\begin{aligned}
&\omega^2 + \frac{2\omega k}{\rho}\left(\frac{\rho_L U_L}{R_L}+\frac{\rho_G U_G}{R_G}\right) \\
&\quad + \frac{k^2}{\rho}\left\{\frac{\rho_L U_L^2}{R_L}+\frac{\rho_G U_G^2}{R_G} - (\rho_L-\rho_G)g\cos\beta\frac{A}{A'_L}\right\} \\
&\qquad\qquad - \frac{\gamma A}{\rho A'_L}k^4 = 0
\end{aligned}
\tag{8}
$$

- You can see big differences in (7) and (8). VKH is the same for every geometry with A/A'_L the same. You *cannot* predict the effects of geometry.

- Compare (4) VPKH and (6) VKH. The terms with $\mu_1 + \mu_2$ in (4) come from the viscous part of the normal stress and do not appear in (6).
- The terms with F in (6) come from shear and do not appear in (4).
- The terms with a and c multiplying k and k^2 in (6) are like corresponding terms in (4) but with similar physical effects expressed differently.
- The power of k multiplying $g(\rho_L-\rho_G)$ in (6) is k^2, and the power multiplying this term in (4) is k. The power multiplying surface tension in (6) is k^4 and it is k^3 in (4).

- **Long waves, short waves, growth rates and wave speeds, neutral curves, stability limits, most dangerous wave.**

Here k is a wave number. $\lambda = 2\pi/k$ is a wavelength which is long when $k \rightarrow 0$ and short when $k \rightarrow \infty$. Some authors restrict their attention to long waves. This should not be done. All disturbances, waves of all length, can occur and if they are unstable they will grow. We should look for the wavelength, which maximizes the growth rate σ_r for disturbances proportional to

$$e^{\sigma t} = e^{\sigma_r t} e^{i\sigma_i t} \tag{9}$$

Clearly, if $\sigma_r > 0$ the disturbance will grow.

The growth rate and associated wave speed $e = -\sigma_i / k$ are functions of the wave number. The growth rate

$$\sigma_r(k_c) = \max_{k \geq 0} \sigma_r(k) \tag{10}$$

is special because it gives the length of the wave $\lambda_c = 2\pi/k_c$ which grows most rapidly. This is the length of the unstable wave and the wave speed we expect to see in experiments.

A neutral curve is the locus of points in parameter space on which

$$\sigma_r\,(k, U_1, U, \rho_1, \rho_2, \gamma, g, \mu_1, \mu_2; k) = 0 \tag{11}$$

In the case of KH instability this relation equation (11) $\sigma_r = 0$ can be expressed as

$$(U_G-U_L)^2 = f\,(h_1, h_2, \rho_1, \rho_2, \gamma, g, \mu_1, \mu_2; k) \tag{12}$$

This defines the limit of stability.

If we can find a single value of k, any value, say $k = \tilde{k}$, for which

$$(U_G-U_L)^2 = f\,(\tilde{k}\,) \tag{13}$$

the flow is unstable, at $\tilde{k}$ certainly $\sigma_r(\tilde{k}) > 0$.

▪ Kelvin-Helmholz instability and slugs.

Many authors look at the transition from stratified flow to slugs. It is natural to study the stability of stratified flow and this leads to Kelvin-Helmholz instability. The relation of KH instability to slugs is not clear. The best guess is that slugs form out of small amplitude and unstable KH waves when the conditions are right by a nonlinear mechanism not well understood. Moreover, all of the authors in the list below appear to believe that if a flow is stable to KH waves then it will not form slugs. All the authors find that stratified flow is stable when

$$(U_G\text{-}U_L)^2 < f\,(h_1, h_2, \rho_1, \rho_2, g, \gamma, \mu_1, \mu_2) \tag{14}$$

with different f's for different authors which are listed below.

▪ Instability of KH type from the literature listed chronologically

Wallis, G.B. & Dobson, J.E. 1973. The onset of slugging in horizontal stratified air-water flow. *Int. J. Multiphase Flow* **1**, 173-193.

For long waves and when surface tension is negligible (not in his or any other experiments) they get

$$f = (\rho_L - \rho_G)\, g\, (h_G/\rho_G + h_r/\rho_L) \tag{15}$$

They did not go a good job of analysis. If you require that the wave speed c in their quadratic equation [5] be real, then you can get

$$f = \frac{\rho_2 \coth(kh_2) + \rho_1 \coth(kh_1)}{\rho_1\rho_2 \coth(kh_2)\coth(kh_1)}\left[(\rho_1 - \rho_2)g + \gamma k^3\right] \tag{16}$$

without putting $g = 0$ or $k \to 0$.

Kordyban, E.S. & Ranov, T. 1970. Mechanism of slug formation in horizontal two-phase flow. *J. Basic Engng* **92**, 857-864.

$$f = \frac{\dfrac{\rho_L}{\rho_G}\,\dfrac{g}{k}}{\left\{\coth(kh_G - 0.9) + 0.45\coth^2(kh_G - 0.9)\right\}} \tag{17}$$

Taitel, Y.& Dukler, A.E. 1976. A model for predicting flow regime transitions in horizontal and near horizontal gas-liquid flow. *AIChE Jl* **22**, 47-55.

They neglect U_L (they should not) and get (their equation (20) and (22))

$$f = C_1^2 g \frac{\rho_L + \rho_G}{\rho_G} h_G \tag{18}$$

Mishima, K. & Ishii, M. 1980. Theoretical prediction of onset of horizontal slug flow. Trans. *ASME Jl Fluids Engng* **102**, 441- 445.

$$f = \frac{0.236(\rho_L - \rho_G) g h_G}{\rho_G} \tag{19}$$

Lin, P.Y. & Hanratty, T.J. 1986. Prediction of the initiation of slugs with linear stability theory, *Int. J. Multiphase Flow* **12**, 79-98.

Their criterion are given by [38] and [51]. These criteria are derived from many approximations of the effect of viscosity. $(U_G - U_L)^2$ is not there explicitly.

Andritsos, N. & Hanratty, T.J. 1987. Interfacial instabilities for horizontal gas-liquid flow in pipelines. *Int. J. Multiphase Flow* **13**, 583-603.

They say that the inviscid KH theory for viscous flow is a good first approximation (as in Wallis & Dobson).

Barnea, D. & Taitel, Y. 1993. Kelvin-Helmholz Stability Criteria for Stratified Flow: Viscous versus Non-viscous (Inviscid) Approaches, *Int. J. of Multiphase Flow*, vol. 19, No. 4.

Their f is given in [18] and it is the same as the f under Rivero, Laya and Ocando.

Rivero M., Laya A. & Ocando D., June 1995. Experimental Study on the Stratified-Slug Transition For Gas-Viscous Liquids Flow In Horizontal Pipelines, *Multiphase 95—Where are we on the "S" curve?* Proc of BHR Group 7th Int. Conference, Cannes France.

$$\left.\begin{aligned}
&f = K^2\left[(\rho_L R_G + \rho_G R_L)\frac{(\rho_L - \rho_G)}{\rho_L \rho_G} g \cos\beta \frac{A}{dA_L/dh_L}\right], \\
&K^2 = 1 \quad \text{for inviscid flow} \\
&K^2 = 1 - \frac{(C_Y - C_L)^2}{\left\{\frac{\rho_L - \rho_G}{\rho} g \cos\beta \frac{A}{dA_L/dh_L}\right\}} \quad \text{for viscous flow}
\end{aligned}\right\} \tag{20}$$

- **KH instability and flow charts**

The instability criterion

$$|U_G - U_L|^2 > f \quad \text{for some } k$$

can be written as

$$|U_L - U_G|^2 > f \quad \text{for the same } k$$

is perfectly symmetric with respect to liquid and gas. *It's the velocity difference rather than the individual velocities that matter.* The mathematical reason for this is Galilean invariance, which says that the governing equations are the same in any coordinate system moving with a constant velocity from a given one.

If U_G and U_L are steady it should make no difference whether you move with the gas and watch the liquid go by or move with the liquid and watch the gas go by.

The stability criterion $|U_G - U_L|^2 \leq \rho$ can be plotted on the flow chart.

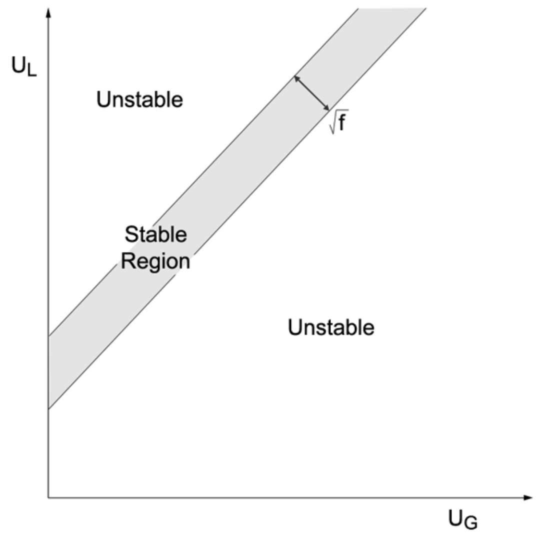


Figure 1. *Flow chart for KH instability.*

The flow chart in figure 1 should be compared with any of the flow charts from experiments given in the literature; for example, figure 2 from Taitel & Dukler is representative. *The experiments do not agree with* $|U_G - U_L|^2 < f$.

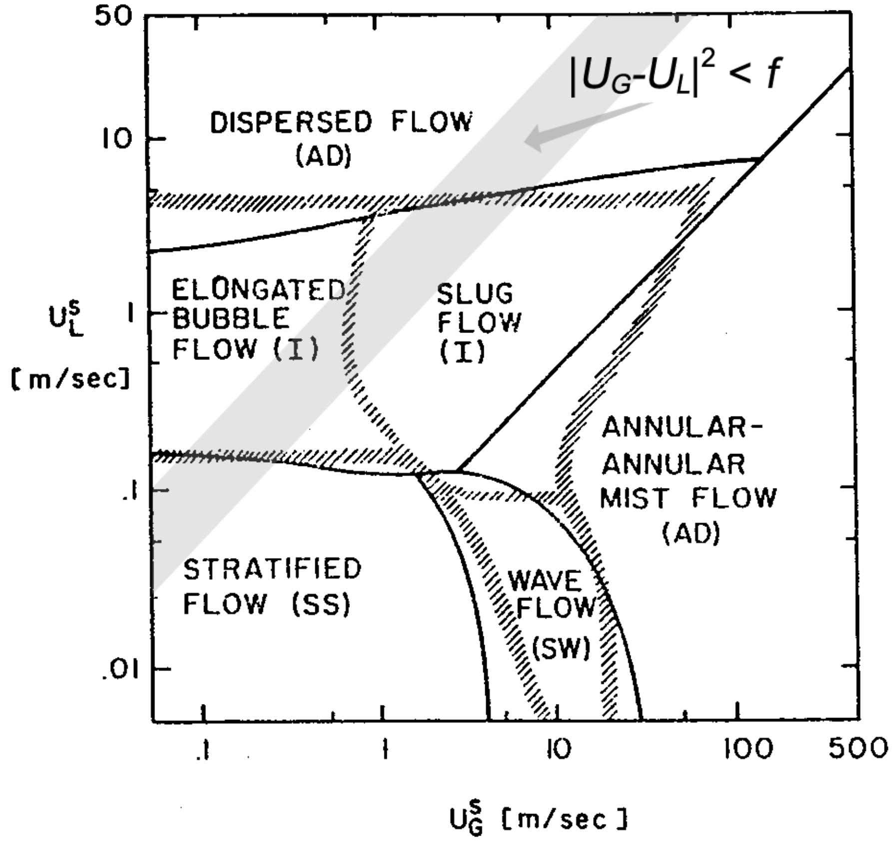


Figure 2. Comparison of theory and experiment. Water-air, 25°C, 1 atm, 2.5 cm. diameter, horizontal. —— theory; ///// Mandhane et al, 1974. Regime descriptions as in Mandhane.

- **Annular flow**

Armando's movies were very interesting. It seemed to me that the slugs that appeared at low gas speed, or better, when $|U_G - U_L|^2$ is small do not arise as an instability of stratified flow. I would say that stratified flow is stable there. Possibly the slugs are also stable at these low speeds.

I saw what I would call KH waves at $|U_G - U_L|^2 \approx 16$, just prior to the transition to annular flow. We should measure the length of these.

Turbulence is required for the appearance of annular flow. When you increase $|U_G - U_L|$ the shear stress must get large. The shear stress in laminar flow would satisfy

$$\mu_{air} \frac{dU_{air}}{dy} = \mu_L \frac{dU_L}{dy} \tag{21}$$

Since $\mu_{air} << \mu_L$, the velocity gradient in the air would be too large. We can balance this with turbulent shear stress

$$\tau_{air} = \mu_L \frac{dU_L}{dy} \tag{22}$$

in annular flow.

Annular flow is a wavy flow with "tiger waves" just like those seen in self-lubricated flow of oil in water emulsion (figure 2 and 3).

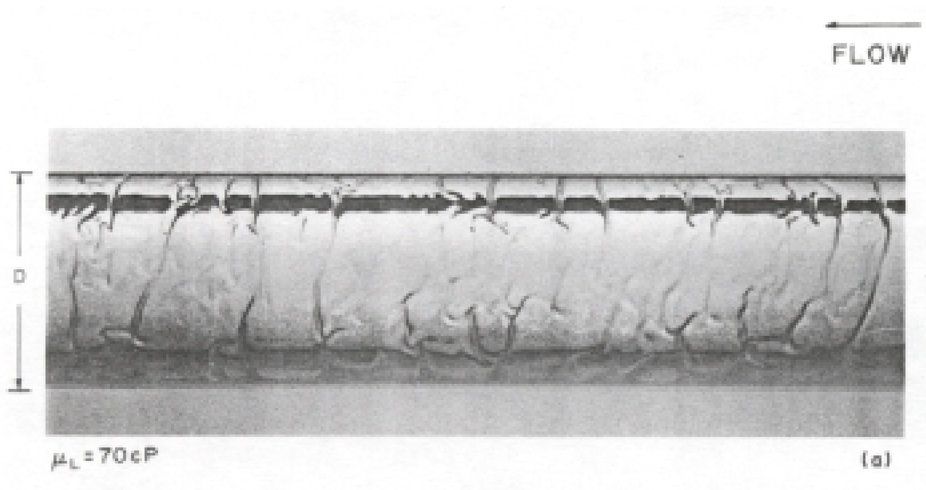


Figure 2. (Andritsos & Hanratty 1987, D = 2.52 cm) Tiger waves in turbulent air flow.

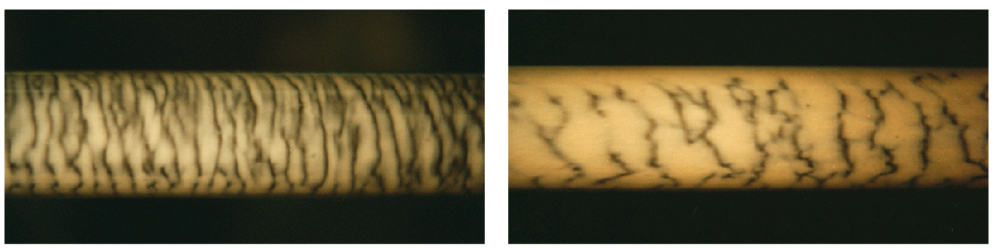


Figure 3. (Joseph, Bai, Mata, Grant 1999). Tiger waves of bitumen froth in water with colloidal clay.

Andritsos and Hanratty 1987 did experiments that were conducted with air and liquid flowing in horizontal pipelines 2.52 and 9.53cm diameter. The liquid viscosity was varied from 1 cp to 80 cp. I have not yet processed the data given there, but it seems to follow trends predicted by viscous potential flow. The wavelengths for low viscosity liquids are between 2-4 cm and for high viscosity liquids between 1-2 cm.

Tiger waves shown in figure 2 and 3 can be KH waves, in figure 2 KH waves are on a liquid driven by air; in figure 3 the waves are on water driven by oil.

- **Transition to annular flow**

Joseph, Bannwart and Liu [1996][2] proposed a criterion for transition to horizontal annular flow. They noted that in core annular of heavy oil in water the viscosity of the oil in the core was large. Noting then in annular gas-liquid flow the gas core is *always* turbulent we looked for a stabilization due to the eddy viscosity of turbulence. We tested

[2] D.D. Joseph, A.C. Bannwart and Y.J. Liu 1996, Stability of Annular Flow and Slugging. *Int. J. Multiphase Flow* **22**(6), 1247-1254.

the idea that stabilization occurred when the eddy viscosity of the gas gets larger than the molecular viscosity of the liquid. This *ad hoc* proposition gave rise to an excellent agreement between the criterion and measured values (see figure 4).

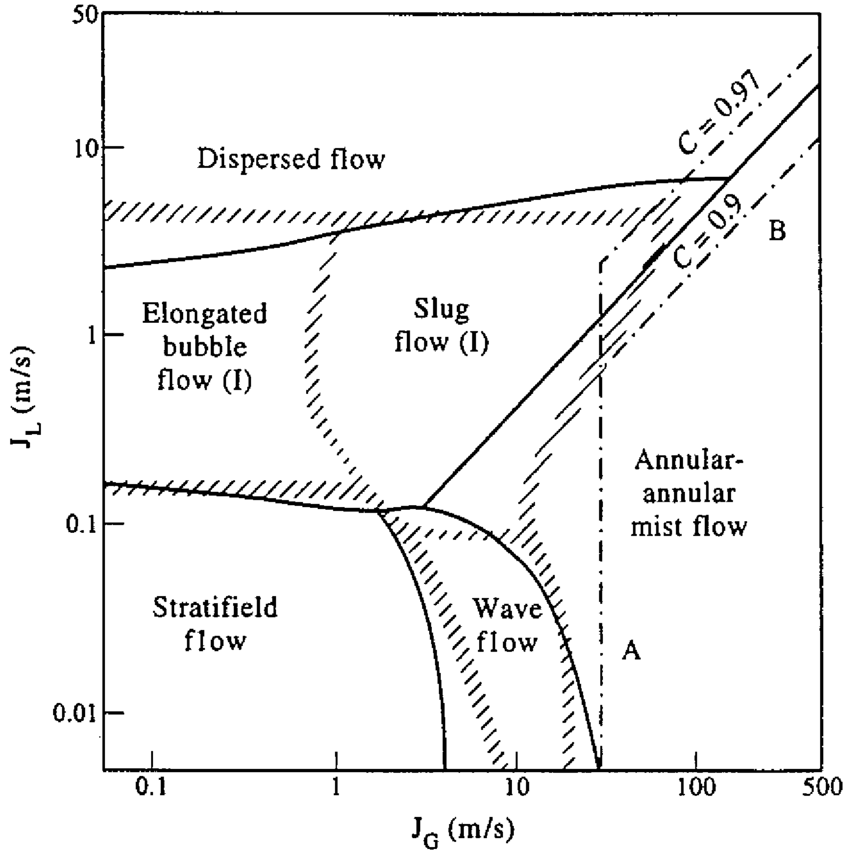


Figure 4. (Joseph, Bannwart and Liu 1996). The vertical dashed-dot line gives the value of the gas velocity 30 m/sec representing the transition to annular flow.

Subsequent comparison of the *ad hoc* criterion with data did not give rise to agreement; if the criterion were right the stabilization of annular flow would require a much higher gas velocity when the liquid viscosity is large. In fact, the stabilization does not appear to depend on the liquid viscosity.

Now I am going to propose another transition criterion, which is based strongly in turbulence theory and appears to be in good agreement with observations. At high gas velocities at which annular flow is observed it is necessary to express the continuity of the shear stress using the Blasius expression for the shear stress τ_o turbulent gas

$$\tau_o = 0.03325\left[\rho^3 \mu U^7 / R\right]^{1/4} \tag{23}$$

where

$$
\left.\begin{aligned}
\rho &= 0.001225 \frac{\text{gm}}{\text{cm}^3} \\
\mu &= 0.000178 \frac{\text{gn}}{\text{cm sec}}
\end{aligned}\right\} \tag{24}
$$

are the density and viscosity of air, R is the pipe radius and U is the superficial gas velocity. Of course for other gases, or air at high pressure, we use other values of ρ and μ.

Our new criterion is again *ad hoc*. We explore the idea that the transition to annular flow occurs when τ_o reaches a critical value τ_{oc}. The shear stress in the liquid film is passive taking on the value that matches the critical value τ_{oc}. The gas velocity at criticality is

$$
U = \left(\frac{\tau_{oc}}{0.03325} \right)^{4/7} \left(\frac{R}{\rho^3 \mu} \right)^{1/7} \tag{25}
$$

This criterion is completely independent of liquid viscosity and depends only weakly on the pipe radius.

We may compute τ_{oc} from the Taitel-Dukler experiments using the data in the Joseph, Bannwart, Liu paper where the transition in a 2.5cm diameter pipe is observed at 3000 cm/sec. Evaluating τ_o to this given data we compute

$$
\left.\begin{aligned}
\tau_{oc} &= 34.40 \frac{dyne}{\text{cm}^2}, \\
U &= 3204 \; R^{1/7}
\end{aligned}\right\} \tag{26}
$$

I have not much data on the dependence of the transition to annular flow on the radius of the pipe. Lin and Hanratty [1987][3] did experiments in two pipe R_1 = 2.76cm and R_2 = 1.26cm

$$
\left(\frac{R_2}{R_1} \right)^{1/7} = 0.82612 \tag{27}
$$

This scaling does appear to agree with the experiments of Lin and Hanraty.

The formula (25) and (26) are consistent with the observations of all experimenters. The make-or-break prediction of (25) is the dependence on R, ρ, and μ. We have already cited agreements with limited data of Lin and Hanratty [1987]. The dependence on ρ and μ can be tested by experiments at very high pressures, which are presently being carried out in Tulsa. We do not expect large variations of μ with pressure p but the change in ρ will be more or less linear and p.

[3] P.Y. Lin, and T.J. Hanratty 1987. Effect of pipe diameter on the interfacial configurations for air-water flow in horizontal pipes. *Int. J. Multiphase Flow* **13**, 549-563.

Preliminary looks at this high pressure data does appear to show a sharp decrease in the transition gas velocity proportional to $\rho^{-3/7}$. In processing experiments it is necessary to take the pressure into account since it may produce a sensible change of density.

▪ Taylor bubble

A Taylor bubble is a long gas bubble rising in a tube. There are two perplexing observations: (1) the bubble rise velocity is independent of the length of the bubble of the bubble volume and (2) the bubble rise velocity is larger in an annulus than in a tube and it increases with the inner radius of the inner cylinder when the tube diameter is fixed.

The papers I will use in this discussion are

R.M. Davies and G.I. Taylor 1950, The mechanism of large bubbles rising through liquids in tubes, *Proceedings of the Royal Society*, **200A**, 375-390.

G.K. Batchelor 1967, *An Introduction to Fluid Dynamics*, Cambridge University Press, section 6.11.

E.F. Caetano, O. Shoham and J.P. Brill 1992, Upward vertical two-phase flow through an annulus—part 1: Single-phase friction factor, Taylor bubble rise velocity, and flow pattern prediction, *J. Energy Resources Technology,* **114**, 1-13.

C.A. Talvy, L. Shemer and D. Barnea 2000, On the interaction between two consecutive elongated bubbles in a vertical pipe. *Int. J. Multiphase Flow* **26**, 1905-1923.

Davies and Taylor [1950] showed that the rise velocity of a long bubble rising in a vertical pipe is

$$U = k\sqrt{gD} = \sqrt{2}\,k\sqrt{gR} \tag{28}$$

where k is about 0.35.

▪ Taylor derivation of the rise velocity

Bernoulli equation on the surface of the bubble:

$$p + \frac{1}{2}\rho q_s^2 = \frac{1}{2}\rho U^2 + \rho q y$$
$$y = R - r(\theta)\cos\theta$$

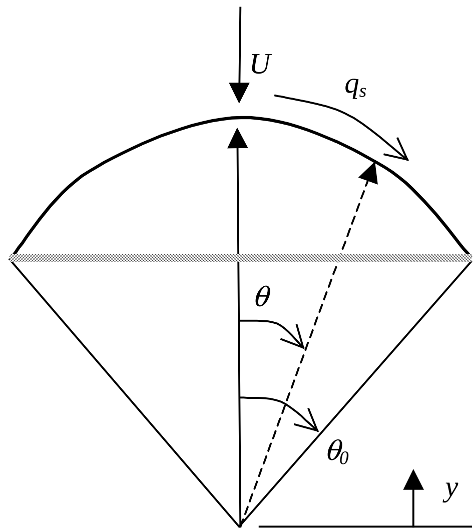


Figure 5. Lenticular bubble.

On the surface of the lenticular bubble p is a constant (equal to the gas pressure inside or it differs by $2\gamma/R$ where γ is surface tension from the pressure inside). Hence, since q_s and y are not constant on the spherical cap $p = \frac{1}{2}\rho U^2$ there,

$$\frac{1}{2}q_s^2 = g\left(R - r(\theta)\cos\theta\right)$$

Now for potential flow over a sphere of radius R

$$q_s = \frac{3}{2}U\sin\theta$$
$$\frac{1}{2}\frac{9}{4}U^2\sin^2\theta = g\left[R - r(\theta)\cos\theta\right] \qquad (29)$$

If I take $r(\theta) = R$, then at small θ, $\sin\theta = \theta$, $\cos\theta = 1 - \theta^2/2$ and

$$\frac{9}{4}U^2 = gR \quad \text{This is approximately correct} \qquad (30)$$

The strange thing about (30) is that velocity of the rising bubble is determined without a drag law and θ_0 doesn't come in. We want to see how buoyancy balances drag and we haven't a clue. Batchelor says

> The remarkable feature of [equations like (30)] and its various extensions is that the speed of movement of the bubble is derived in terms of the bubble shape, without any need for consideration of the mechanism of the retarding force which balances the effect of the buoyancy force on a bubble in steady motion. That retarding force is evidently independent of Reynolds number, and the rate of

dissipation of mechanical energy is independent of viscosity, implying that stresses due to turbulent transfer of momentum are controlling the flow pattern in the wake of the bubble. [4]

- **Why doesn't the rise velocity depend on the length of the Taylor bubble?**

Suppose that hemispherical bubble is rising under buoyancy. The lift is balanced by the drag

$$\frac{2}{3}\pi R^3\left(\rho_l - \rho_g\right)g = D\,.$$

Suppose that

$$D = cR^2U^2\rho_l\pi$$

where πR^2 is projected area. You get this by integrating the pressure difference $P_T(x) - P_B(x)$ over the boundary of the hemisphere (T = top, B = bottom). Presumably $P_B(x)$ is determined by turbulence. Noting that $\rho_l >> \rho_g$ we get

$$2/3\ Rg = CU^2$$

The liquid at the wall drains under gravity without changing the pressure. The equation is

$$\mu\frac{\partial^2 U}{\partial x^2} = \rho_l g \qquad \left(\text{no } \frac{dp}{dx}\right)$$

Then the cylindrical part of the long bubble is effectively not displacing liquid (doesn't change pressure). The buoyancy is still the volume of the hemisphere poking into the liquid at the top. The equation of motion *buoyancy = drag* doesn't change.

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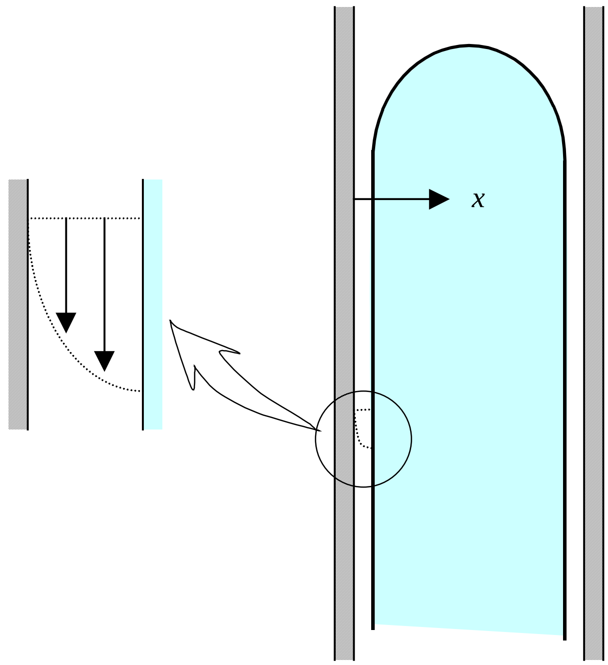
[4] G.K. Batchelor 1967, *An Introduction to Fluid Dynamics*, Cambridge University Press, page 477.



- **Why does the bubble in the annular space rise faster?**
  I don't know. Maybe the annular insert increases the back pressure P_B.