

Reaction to “Theory of recovery of foamy oil” by Dan D. Joseph

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1. Introduction

This memo is a short reaction to a write-up of a foamy heavy crude model by Dan Joseph (1997a, 1997b). His model differs from a related model by Hammond (1997a, 1997b) by the fact that it is essentially a two-equation model (with as variables pressure and void fraction), so that no equations are written down for the quantity of dissolved gas. It also treats the gas phase as being incompressible. It is not an equilibrium model in the sense that gas fraction different from equilibrium can occur, but that these relax back to the equilibrium state by first order kinetics with a certain time constant. In this sense the model is similar to Hammond’s model (1997b), where relaxation of the dissolved gas fraction occurs, also modelled by a linear kinetics mechanism.

The development of two different models for the same phenomenon in the current foamy heavy crude project might be justified by the fact that the objective of the two models is not the same. The Hammond model tries to give a more accurate description at the price of being more difficult to solve and needing possibly more closure relations. The Joseph model is probably better suited to get a rough understanding of the physics and it is more easy to solve. The model has some limitations as well, which will be discussed throughout this memo and especially in the conclusions.

2. Relation between void fraction and pressure

The first part of Dan’s reasoning is the following. If the quantity of gas that can be dissolved in a oil is a linear function of pressure, which is supported by the measurements of Svrcek & Mehrotra (1982, 1989), then the gas fraction of free gas in the oil should be a linear function of pressure as well. This reasoning supposes that:

- uniformity of gas fraction exists;
- the pressure variations are small enough for the gas to be considered as incompressible.

The second part of his reasoning is to admit that there might be a deviation from equilibrium behaviour (that is to say, the quantity of dissolved gas might sometimes be different from the quantity which can be dissolved under thermo-dynamic equilibrium at a

given pressure) and that this can be accounted for by linear relaxation of void fraction or pressure towards the equilibrium value.

Let us reconsider his derivation. The problem is schematically resumed in figure 2.1.

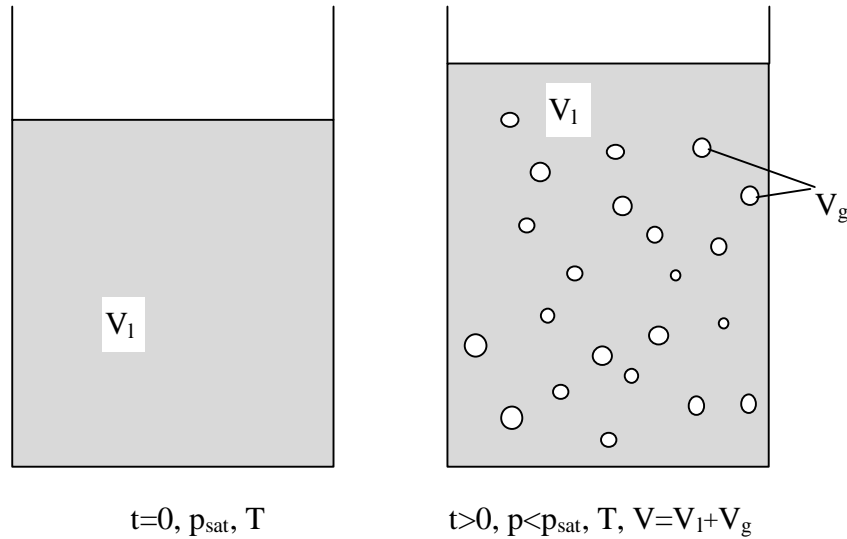


Figure 2.1 Expansion of a live oil by bubble nucleation during de-pressurisation

The pressure in a live oil, which is initially at saturation pressure, is decreased. The quantity of gas which can be dissolved then decreases, so that some of the gas evolves out of the oil and small bubbles are formed. It is supposed that these bubble stay in the oil, at a homogeneous phase fraction ϕ , defined by

$$\phi = \frac{V_g}{V_l + V_g} \quad (2.1)$$

where V_l is the volume of the liquid and V_g the volume of the gas phase which is created. We suppose that the liquid phase can be considered as incompressible and that the liquid phase density ρ_l does not depend very much on the quantity of dissolved gas, which seems to be a reasonable hypothesis (see figure 2.2).

The mass fraction of dissolved gas at a given pressure is approximately a linear function of pressure, as for example shown in figure 2.3

$$\xi = K(T)p \quad (2.2)$$

Dan expresses this in a fictive void fraction

$$\hat{\phi} = \frac{V_{g,r}}{V_{g,r} + V_l} \quad (2.3)$$

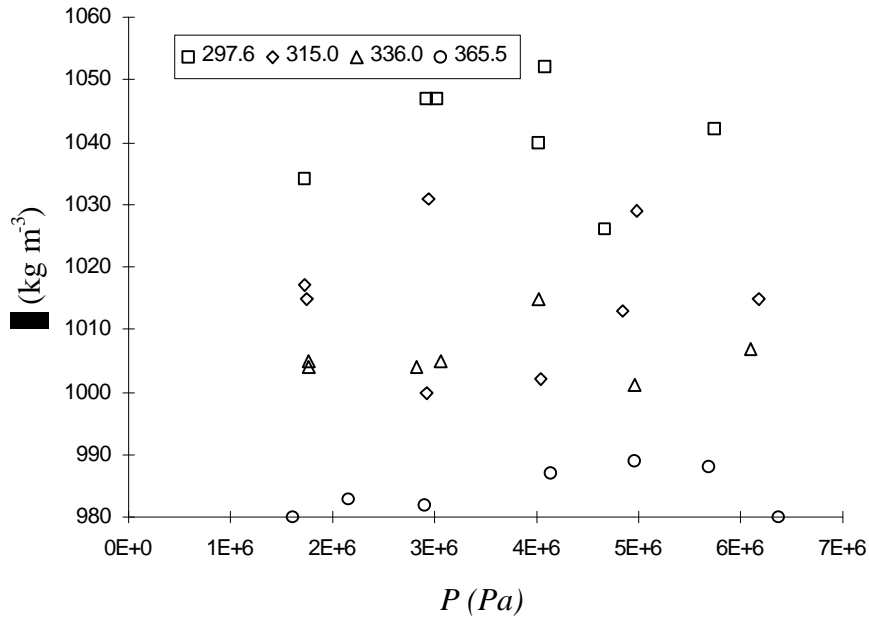


Figure 2.2 Density of Athasca bitumen saturated with CO₂ as a function of pressure for different temperatures (297.6 K, 315.0 K, 336.0 K, 365.5 K.), data from Svrcek & Mehrotra., 1982.

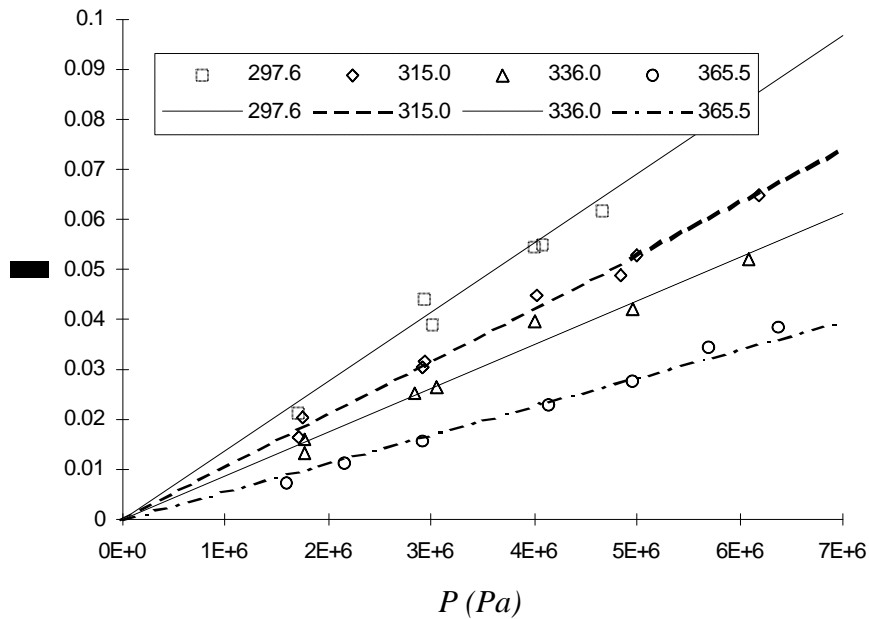


Figure 2.3 Mass fraction of CO₂ in Athasca bitumen as a function of pressure for different temperatures (297.6 K, 315.0 K, 336.0 K, 365.5 K.), data from Svrcek & Mehrotra., 1982; The slopes of the linear regressions are given by $1.39 \times 10^{-8} \text{ Pa}^{-1}$, $1.06 \times 10^{-8} \text{ Pa}^{-1}$, $8.77 \times 10^{-9} \text{ Pa}^{-1}$, $5.66 \times 10^{-9} \text{ Pa}^{-1}$.

where $V_{g,r}$ is the volume of gas which is released if the pressure is dropped to atmospheric pressure and V_l is the volume of the live crude oil.

The mass fraction of dissolved gas can be written as

$$\xi = \frac{N_{g,d} M_g}{\rho_l V_l} \quad (2.4)$$

where $N_{g,d}$ are the number of moles of dissolved gas and M_g is the molar mass. When released to atmospheric pressure p_a and ambient temperature T_a these $N_{g,d}$ moles of gas occupy the volume $V_{g,r}$, which is related to $N_{g,d}$ by

$$N_{g,d} = \frac{p_a V_{g,r}}{RT_a} \quad (2.5)$$

where $R=8.31 \text{ J}\cdot\text{mole}^{-1}\cdot\text{K}^{-1}$ is the gas constant, assuming that an ideal gas law holds. Combination of (2.3), (2.4) and (2.5) then results in

$$\xi = \frac{p_a M_g V_{g,r}}{RT_a \rho_l V_l} = \frac{p_a M_g}{RT_a \rho_l} \frac{\hat{\phi}}{1 - \hat{\phi}} \quad (2.6)$$

and after inserting (2.2), the relation between this fictive void fraction $\hat{\phi}$ and the pressure becomes:

$$\hat{\phi} = \frac{p}{p + \frac{p_a M_g}{KRT_a \rho_l}} \quad (2.7)$$

or inverting it

$$p = \frac{p_a M_g}{KRT_a \rho_l} \frac{\hat{\phi}}{1 - \hat{\phi}} \quad (2.8)$$

For low fictive void fractions $\hat{\phi} \ll 1$

$$\frac{\hat{\phi}}{1 - \hat{\phi}} \approx \hat{\phi}(1 + \hat{\phi}) \approx \hat{\phi} \quad (2.9)$$

and then (2.8) becomes the second equation on page 4 of Dan's memo

$$p = \gamma(T)\hat{\phi} + C \quad (2.10)$$

with the difference that we have supposed that ξ is a linear function of p and that $\xi = 0$ at $p=0$, which seems a reasonable hypothesis according to figure (2.3). γ and K are thus related by

$$\gamma(T) = \frac{p_a M_g}{K(T)RT_a \rho_l} \quad (2.11)$$

Dan's model is essentially incompressible for the gas phase. This can be easily seen by equation (2.6) which states that the fictive volume fraction is proportional to the mass fraction of dissolved gas (or the pressure) if $\hat{\phi} \ll 1$. If $\hat{\phi}$ however increases due to an increasing pressure then the incompressibility hypothesis becomes less and less good, the gas occupying less space due to the increased pressure.

We are however not really looking for a relation between this fictive void fraction and the pressure, but rather for a relationship between the real void fraction, defined

by (2.1) and the pressure. If we assume that no bubbles move out of the system, then the total number of moles of gas in the system remains constant. At the saturation pressure all the gas is dissolved and the number of moles in the system is given by

$$N_{g,d} = \frac{Kp_{sat}V_l\rho_l}{M_g} \quad (2.12)$$

when the pressure is decreased the number of dissolved moles of gas decreases and the number of moles of free gas increases

$$\frac{Kp_{sat}V_l\rho_l}{M_g} = \left(\frac{KV_l\rho_l}{M_g} + \frac{V_g}{RT} \right) p \quad (2.13)$$

This can be re-written as

$$\frac{1 - \phi}{1 - \left(1 - \frac{M_g}{K\rho_l RT} \right) \phi} = \frac{p}{p_{sat}} \equiv p^* \quad (2.14)$$

or as

$$\phi = \frac{1 - p^*}{1 - \left(1 - \frac{M_g}{K\rho_l RT} \right) p^*} \quad (2.15)$$

For

$$\left(1 - \frac{M_g}{K\rho_l RT} \right) \phi \ll 1 \quad (2.16a)$$

so for very solubilities or for low void fraction, first order expansion leads to

$$1 - \frac{M_g}{K\rho_l RT} \phi = p^* \quad (2.16b)$$

This can be re-written, using (2.11) in

$$p + \gamma \left(\frac{p_{sat}}{p_a} \frac{T_a}{T} \right) \phi = p_{sat} \quad (2.17)$$

which looks very alike equation (4) in Dan's paper. For notation convenience we define

$$\gamma' = \gamma \left(\frac{p_{sat}}{p_a} \frac{T_a}{T} \right) \quad (2.18)$$

So that

$$p + \gamma' \phi = p_{sat} \quad (2.19)$$

So Dan's result seems all right, but the way he obtained it is not completely correct, and his equation (3) should yield:

$$\gamma(\hat{\phi}_2 - \hat{\phi}_1) = \gamma'(\phi_1 - \phi_2) \quad (2.20)$$

For Zuata oil at room temperature

$$\frac{\gamma'}{p_{sat}} = \frac{M_g}{K\rho_l RT} \approx \frac{16 \cdot 10^{-3}}{3.5 \cdot 10^{-9} \times 10^3 \times 8.31 \times 293} \approx 1.9$$

which shows us that a little bit below the bubble pressure (otherwise there is no nucleation) an increase of 2% in pressure is associated with a rise in void fraction of 0.01.

3. *Non-equilibrium behaviour*

We assume that (2.19) holds in an equilibrium situation and that in a distortion from equilibrium the system relaxes back to the equilibrium according to a first order rate mechanism:

$$\frac{D\phi}{Dt} = -\alpha \frac{\phi - \phi_{eq}}{\tau} \quad (3.1)$$

where ϕ_{eq} is given by a solution of equation (2.19)

$$p + \gamma' \phi_{eq} = p_{sat} \quad (3.2)$$

The porosity α appears in this equations for the case in which the nucleation happens in a porous medium where only a α part of the volume is not occupied by rock or sand. This is related with the definition of the derivative D/Dt , which is a substantial time derivative, indicating that the gas fraction in an volume element changes both by mass transfer and by transport of gas fraction into this element

$$\frac{D\phi}{Dt} = \frac{\partial(\alpha\phi)}{\partial t} + \mathbf{q} \cdot \nabla\phi \quad (3.2a)$$

Substituting (3.2) in (3.1) we obtain

$$\kappa \frac{D\phi}{Dt} = -\gamma' \phi - p + p_{sat} \quad (3.3)$$

where

$$\kappa = \frac{\tau\gamma'}{\alpha} \quad (3.4)$$

Equation (3.3) is the first equation on page 5 of Dan's paper.

On of the restrictions of Dan's theory is that bubbles should move all the time with the liquid phase. This can be explained as follows. The void fraction is not a fundamental parameter in the nucleation problem. The rate of nucleation is determined by the difference between the available quantity of dissolved gas and the quantity which thermodynamically can be present at the given temperature and pressure. This rate of nucleation can, in the case that the bubbles move with the liquid phase, be expressed in a void fraction and a pressure, which is essentially what Dan does. But in a case where the bubbles leave the oil zone in which they have been nucleated or in a case where the oil leaves the bubbles behind, this does not seem possible to me. Void fraction can be constantly reduced by transport of bubbles out of the reservoir without this affecting the nucleation rate. So this is one of the things that we have to keep in mind.

In a revision (Joseph, 1997c), Dan writes a relaxation equation in pressure. This equation can be obtained by supposing that close to equilibrium the substantial derivative of pressure and void fraction are related by (2.19):

$$\frac{D\phi}{Dt} = -\frac{1}{\gamma'} \frac{Dp}{Dt} \quad (3.4a)$$

Substituting this in equation (3.4) we obtain

$$\frac{\tau}{\alpha} \frac{Dp}{Dt} = \gamma' \phi + p - p_{sat} \quad (3.4b)$$

In my opinion the relaxation equation should not be given as in (3.3), neither as in (3.4b), but rather as:

$$\frac{\tau'}{\alpha} p \frac{D\phi}{Dt} \approx \gamma' \phi + p - p_{sat} \quad (3.4c)$$

which is a non-linear differential equation and which cannot be transformed in one of the equations proposed by Dan. I will defend this proposition more in detail in section 4.

Density of dissolved gas

I must admit that I have difficulties following Dan's argument about the density of dissolved gas and the density of pure liquid. Very ingeniously, he is defining a volume fraction Ψ of dissolved gas. What is however this volume fraction of dissolved gas? If we imagine the gas molecules sitting individually between oil molecules it is difficult to see what the relative volumes of each phase are. Therefore I will refrain from arguing about equation (5) in Dan's paper.

I think that there are two things you can measure. The first is the compressibility c of a dead oil

$$\frac{\delta\rho_{l,p}}{\rho_{l,p}} = c\delta p \quad (3.5)$$

The second is the compressibility c' of a live oil

$$\frac{\delta\rho_l}{\rho_l} = c'\delta p \quad (3.7)$$

The difference between c and c' comes from the fact that the density of the oil changes when its composition changes. In the Svrcek & Mehrotra (1982) experiments for solubility of CO₂ in Athabasca bitumen the constant c' proves to be close to zero. This is however not a general rule as shown by the CH₄ data in the same paper.

Now it is possible to define a constant d by the definition:

$$\frac{\delta\rho_l}{\rho_l} = c\delta p - d\delta\xi \quad (3.6)$$

as is done by Hammond (1997b) so that with help of (2.2)

$$c - dK = c' \quad (3.7)$$

The constant d does however not have any fundamental value as such because it is impossible to distinguish the effects of compressibility and composition change.

4. Transport equations

4.1. Mixture transport for incompressible flow

The transport of the foamy crude is governed by a system of 3 transport equations (one for the liquid without dissolved gas, one for the dissolved gas and one for the free gas) completed by the Darcy equation for the mixture (Hammond, 1997a)

$$\frac{\partial}{\partial t} \{ \alpha(1-\phi)(1-\xi)\rho_l \} + \nabla \cdot \{ (1-\phi)(1-\xi)\rho_l \mathbf{q} \} = 0 \quad (4.1a)$$

$$\frac{\partial}{\partial t} \{ \alpha(1-\phi)\xi\rho_l \} + \nabla \cdot \{ (1-\phi)\xi\rho_l \mathbf{q} \} = -f^{l \rightarrow g} \quad (4.1b)$$

$$\frac{\partial}{\partial t} \{ \alpha\phi\rho_g \} + \nabla \cdot \{ \phi\rho_g \mathbf{q} \} = f^{l \rightarrow g} \quad (4.1c)$$

$$\mathbf{q} = \alpha \mathbf{u} = -\lambda \nabla p \quad (4.1d)$$

This write-up assumes that the gas bubbles move with the same velocity as the liquid, although an extension of the model including relative velocities is possible by including two Darcy equations and relative permeabilities.

On purpose I have deviated a little bit from Dan's definition of the velocity because I think that (4.1d) more clearly indicates that \mathbf{q} is a ratio between volume flow rate and area, whereas \mathbf{u} is a local velocity in the porous medium. The equations (4.1a)-(4.1d) is a set of six equations of six variables, ϕ , ξ , \mathbf{q} and p , assuming that \mathbf{q} has three components. α is a parameter and closure modelling of λ and $f^{l \rightarrow g}$ should be provided, as well as constitutive equations for ρ_l and ρ_g .

If we choose to add all three, we obtain a mixture conservation equation

$$\frac{\partial}{\partial t} (\alpha\rho) + \nabla \cdot (\rho\mathbf{q}) = \frac{\partial}{\partial t} (\alpha\rho) + \rho\nabla \cdot \mathbf{q} + \mathbf{q} \cdot \nabla\rho = 0 \quad (4.2)$$

with the mixture density defined by

$$\rho = (1-\phi)\rho_l + \phi\rho_g \quad (4.3)$$

Assuming that the gas density is much lower than the oil density: $\rho_g \ll \rho_l$, (4.3) writes

$$\rho \approx (1-\phi)\rho_l \quad (4.3)$$

Substituting this mixture density in equation (4.2) and re-arranging we obtain an equation for the void fraction evolution

$$\frac{\partial}{\partial t} (\alpha\phi) + \mathbf{q} \cdot \nabla\phi = (1-\phi)\nabla \cdot \mathbf{q} \quad (4.4)$$

Inserting the Darcy equation (4.1d) in equations (3.4b) and (4.4) we obtain

$$\alpha \frac{\partial\phi}{\partial t} - \lambda \nabla p \cdot \nabla\phi = (1-\phi)\nabla \cdot (-\lambda \nabla p) \quad (4.5a)$$

$$\alpha \frac{\partial p}{\partial t} - \lambda |\nabla p|^2 = \alpha (\gamma' \phi + p - p_{sat}) / \tau \quad (4.5b)$$

which relates void fraction and pressure drop.

There seems to be an incompatibility between the complete model shown by equations (4.1a)-(4.1c) and the neglect of the gas phase density compared to the liquid phase density. If we add (4.1a) and (4.1c) and we suppose that the liquid is incompressible, then we obtain:

$$\frac{\partial}{\partial t} (\alpha\phi) + \mathbf{q} \cdot \nabla\phi - (1-\phi)\nabla \cdot \mathbf{q} = f^{l \rightarrow g} / \rho_l \quad (4.6)$$

which together with (4.4) would impose $f^{l \rightarrow g} / \rho_l \approx 0$. This however represents the fact that the liquid density is hardly effected by the transfer of mass.

4.2. Other method

It is possible to avoid making hypotheses on what the relaxation equation (3.3) or (3.4b) should look like, altogether eliminate the dissolved gas fraction as a parameter and to keep compressibility in the model. This can simply be done by adding equations (4.1a) and (4.1b) and by substituting constitutive equations in the result and in equation (4.1c). The transport equations are then

$$\frac{\partial}{\partial t} \{ \alpha(1-\phi)\rho_l \} + \nabla \cdot \{ (1-\phi)\rho_l \mathbf{q} \} = -f^{l \rightarrow g} \quad (4.7a)$$

$$\frac{\partial}{\partial t} \{ \alpha\phi\rho_g \} + \nabla \cdot \{ \phi\rho_g \mathbf{q} \} = f^{l \rightarrow g} \quad (4.7b)$$

The constitutive equations for the densities can be written as

$$\rho_l = \rho_{l,sat} \exp[c'(p - p_{sat})] \quad (4.8a)$$

$$\rho_g = \frac{\rho_{g,sat}}{p_{sat}} p \quad (4.8b)$$

and after substitution in (4.7)

$$c'(1-\phi) \left(\alpha \frac{\partial p}{\partial t} + \mathbf{q} \cdot \nabla p \right) + \frac{\partial}{\partial t} \{ \alpha(1-\phi) \} + \nabla \cdot \{ (1-\phi)\mathbf{q} \} = -f^{l \rightarrow g} / \rho_l \quad (4.9a)$$

$$\phi \left(\alpha \frac{\partial p}{\partial t} + \mathbf{q} \cdot \nabla p \right) + p \left[\frac{\partial}{\partial t} \{ \alpha\phi \} + \nabla \cdot \{ \phi\mathbf{q} \} \right] = f^{l \rightarrow g} p_{sat} / \rho_{g,sat} \quad (4.9b)$$

The transfer term $f^{l \rightarrow g}$ does however depend on ξ . A simple way to model it would be a linear kinetics mechanism (Hammond, 1997b)

$$f^{l \rightarrow g} = \alpha(1-\phi)\rho_l \frac{\xi - Kp}{\tau} \quad (4.10)$$

supposing that the pressure is lower than the saturation pressure of the mixture. We could now imagine that if the liquid is super-saturated, i.e. $\xi - Kp > 0$, that the “void fraction is under-saturated”. Or in other words, gas has to be transferred from the liquid into the free gas phase. This is indicated by substituting (2.2) in (2.14)

$$\xi - K\gamma'\phi = Kp_{sat} \quad (4.11)$$

Substituting the ξ value this obtained in equation (4.10) we obtain

$$f^{l \rightarrow g} = \rho_l(1-\phi)\alpha \frac{(1-\phi)K(p_{sat} - p) - \frac{M_g p \phi}{\rho_l RT}}{\left[1 - \left(1 - \frac{M_g}{K\rho_l RT} \right) \phi \right] \tau} \quad (4.12a)$$

For low void fractions, this can be written as

$$f^{l \rightarrow g} = \rho_l(1-\phi)K\alpha \frac{p_{sat} - p - \gamma'\phi}{\tau} \quad (4.12b)$$

This looks a lot like the right-hand side of equation (3.3), which is of course not surprising, since both are based on equation (2.2). Instead of (4.10), which depends on ξ , we now have a transfer term which only depends on void fraction and pressure. Without the assumption of low void fractions, the transfer term should yield

Recalling the definition of the substantial derivative D/Dt

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \mathbf{q} \cdot \nabla \quad (4.13)$$

we can write (4.9) as

$$c'(1-\phi) \frac{Dp}{Dt} - \frac{D\phi}{Dt} = -f^{l \rightarrow g} / \rho_l - (1-\phi) \nabla \cdot \mathbf{q} \quad (4.14a)$$

$$\phi \frac{Dp}{Dt} + p \frac{D\phi}{Dt} = f^{l \rightarrow g} p_{sat} / \rho_{g,sat} - p\phi \nabla \cdot \mathbf{q} \quad (4.14b)$$

There are a few interesting observations that we can make about these equations. Combining the equations in order to obtain two equations with either Dp/Dt or $D\phi/Dt$ in the left-hand side, we obtain

$$\frac{D\phi}{Dt} = \left(1 + \frac{(1-\phi)c'p}{\phi}\right)^{-1} \left\{ \frac{\alpha}{\tau} \left[K(p_{sat} - p) - \frac{p_{sat} M_g}{RT\rho_l} \phi \right] \left(1 + \frac{(1-\phi)c'p}{\phi} \frac{\rho_l}{\rho_{g,sat}}\right) + (1-c'p)(1-\phi) \nabla \cdot (-\lambda \nabla p) \right\} \quad (4.15a)$$

$$\frac{Dp}{Dt} = \left(1 + \frac{(1-\phi)c'p}{\phi}\right)^{-1} \left\{ \frac{\alpha}{\tau} \left[\frac{K(p_{sat} - p)}{\phi} - \frac{p_{sat} M_g}{RT\rho_l} \right] \left(\frac{\rho_l p_{sat}}{\rho_{g,sat} p} - 1 \right) - \frac{p \nabla \cdot (-\lambda \nabla p)}{(1-\phi)\phi} \right\} \quad (4.15b)$$

As a limiting behaviour for $c'p \ll \phi \ll 1$ these simplify to:

$$\frac{D\phi}{Dt} = \frac{\alpha}{\tau} \left[K(p_{sat} - p) - \frac{p_{sat} M_g}{RT\rho_l} \phi \right] + (1-\phi) \nabla \cdot (-\lambda \nabla p) \quad (4.16a)$$

$$\frac{Dp}{Dt} = \frac{\alpha}{\tau} \left[\frac{K(p_{sat} - p)}{\phi} - \frac{p_{sat} M_g}{RT\rho_l} \right] \left(\frac{\rho_l}{\rho_{g,sat}} p_{sat} - p \right) - \frac{p \nabla \cdot (-\lambda \nabla p)}{(1-\phi)\phi} \quad (4.16b)$$

The first of these equations looks very much alike equation (4.5a) which we had earlier obtained, with the difference that a contribution to of the mass transference occurs. The second equations is significantly different from (4.5b) and it does not look particularly nice for low void fraction.

Instead, it seems a better idea of replacing (4.16b) with another equation which follows directly from substituting (4.12b) in (4.14b)

$$\frac{D(p\phi)}{Dt} = \frac{\alpha}{\tau} (1-\phi) \left[K(p_{sat} - p) - \frac{p_{sat} M_g}{RT\rho_l} \phi \right] \frac{\rho_l}{\rho_{g,sat}} p_{sat} - (p\phi) \nabla \cdot (-\lambda \nabla p) \quad (4.17)$$

which of course is nothing else than the mass transport into the compressible gas phase as represented by equation (4.1c).

For low void fractions this equation reduces to

$$p \frac{D\phi}{Dt} \approx \frac{\alpha}{\tau} \left[K(p_{sat} - p) - \frac{p_{sat} M_g}{RT\rho_l} \phi \right] \frac{\rho_l}{\rho_{g,sat}} p_{sat} \quad (4.17)$$

which makes think that neither (3.3), nor (3.4b) is the right equation, but that the relaxation occurs according to

$$\tau' p \frac{D\phi}{Dt} = -\gamma'\phi - p + p_{sat} \quad (4.18)$$

which would then not result in a nice telegraph equations such a obtained by Dan.

We can conclude from equations (4.16b) and (4.17), that the transport equations (4.1) cannot be reduced to Dan's equation (3.4b) or (4.5b).

5. Interpretation of the equations

Under the low void fraction and low compressibility assumptions, the model is described by

$$\alpha \frac{\partial \phi}{\partial t} - \lambda \nabla p \cdot \nabla \phi = \frac{\alpha}{\tau} \left[K \left(\frac{p_{sat}}{p} - 1 \right) - \frac{p_{sat} M_g}{p R T \rho_l} \phi \right] \frac{\rho_l}{\rho_{g,sat}} p_{sat} \quad (5.1a)$$

$$\alpha \frac{\partial \phi}{\partial t} - \lambda \nabla p \cdot \nabla \phi = \frac{\alpha}{\tau} \left[K(p_{sat} - p) - \frac{p_{sat} M_g}{R T \rho_l} \phi \right] + \nabla \cdot (-\lambda \nabla p) \quad (5.1b)$$

The mobility λ will in generally depend on ϕ and thus on the spatial co-ordinate because of pore-blocking by bubbles and because of reduction of dissolved gas concentration associated with the increase in void fraction. If the dissolved gas concentration decreases then the viscosity of the live oil increases, which creates a decrease in mobility.

A combination of equations (5.1a) and (5.1b) gives

$$\nabla \cdot (\lambda \nabla p) = -\frac{\alpha}{\tau} \left[K(p_{sat} - p) - \frac{p_{sat} M_g}{R T \rho_l} \phi \right] \left(\frac{\rho_l p_{sat}}{\rho_{g,sat} p} - 1 \right) \quad (5.2)$$

which is a diffusion equation for the pressure. If the pressure goes not too much below the saturation pressure then $\rho_l p_{sat} / (\rho_g p) \gg 1$ so that the first term on the right-hand side of (5.1b) can be neglected

$$\alpha \frac{\partial \phi}{\partial t} - \lambda \nabla p \cdot \nabla \phi = \nabla \cdot (-\lambda \nabla p) \quad (5.3)$$

Assume that we start at $p = p_{sat}$ and $\phi = 0$. If in a small volume the pressure is then slightly decreased then the right-hand side of equation (5.1a) becomes positive. The left-hand side tells us then that the void fraction in the cell has to increase (nucleation of bubble !).

In one-dimensional space co-ordinates the system is described by

$$\alpha \frac{\partial \phi}{\partial t} - \lambda \frac{\partial p}{\partial x} \frac{\partial \phi}{\partial x} = \frac{\alpha}{\tau} \left[K \left(\frac{p_{sat}}{p} - 1 \right) - \frac{p_{sat} M_g}{p R T \rho_l} \phi \right] \frac{\rho_l}{\rho_{g,sat}} p_{sat} \quad (5.4a)$$

$$\alpha \frac{\partial \phi}{\partial t} - \lambda \frac{\partial p}{\partial x} \frac{\partial \phi}{\partial x} = \frac{\partial}{\partial x} \left(-\lambda \frac{\partial p}{\partial x} \right) \quad (5.4b)$$

These can be further simplified by supposing that the mobility is constant and by dropping inertia

$$\alpha \frac{\partial \phi}{\partial t} = \frac{\alpha}{\tau} \left[K \left(\frac{p_{sat}}{p} - 1 \right) - \frac{p_{sat} M_g}{p R T \rho_l} \phi \right] \frac{\rho_l}{\rho_{g,sat}} p_{sat} \quad (5.5a)$$

$$\alpha \frac{\partial \phi}{\partial t} = -\lambda \frac{\partial^2 p}{\partial x^2} \quad (5.5b)$$

It should be pretty straight forward to solve (5.5) numerically, but analytical solution seems pretty tough to me. Maybe it is possible supposing that a very small pressure variation already creates a significant increase in void fraction, in which case the equation become de-coupled.

6. Conclusions

I am not sure that all this is right, but I am sure that Dan or Paul Hammond will correct me if I am wrong. There might still be some typing or fundamental errors in the memo which I have not found yet.

Concerning the equilibrium behaviour it seems possible to base our initial arguments on the dissolved gas fraction, eliminating it afterwards, because this directly gives access to a more correct pressure-void fraction relation (equation 2.14) which then for low void fractions reduces to the relation found by Dan (equation 2.19). This method avoids as well the introduction of the variable $\hat{\phi}$ which by some people is confused with a void fraction, whereas it is a fictive void fraction of gas in solution. There seems to be some erroneous arguments in Dan's memo (see equation 2.20) although he ends up with more or less the right equilibrium equation.

The corrected relaxation equation should in my opinion be given by (4.18), which is almost linear when the pressure variations are small, but which is different from the equations given by Dan. In fact, the product of pressure and void fraction $p\phi$ relaxes back to equilibrium by interface mass transfer, instead of only the pressure or only the void fraction. The present write-up permits as well to keep liquid compressibility in the model, although I think that this is not essential. It has been shown that the criteria for neglect of compressibility yields $c'p \ll \phi$. If the void fraction is not small, then the model equations are resumed by (4.15a) and (4.17), although for both of these equations the mass transfer term is based on the small void fraction assumption. This can be solved by using equation (4.12b) instead of (4.12a).

The main limitation of the model is that it seems impossible to include drift between the gas and liquid phase. Even if this can be done by including an additional Darcy equation for the velocity of the gas, it becomes impossible to relate the quantity of dissolved gas to the void fraction. Furthermore, Dan's model is only valid for low void fractions and still lower products of pressure and compressibility, although this shortcomings can be removed if it turns out to be necessary, as shown in the preceding sections. Another limitation of the model is that the modelling of mass transfer by nucleation is more or less fixed and more difficult to modify than in Paul's model. The advantage of the model is that it is more easy to solve. For these reasons I think that the model might be very useful to do parameter studies which might give an idea about how production can be increased or optimised. It is probably less well suited for inclusion in a reservoir simulation code.

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7. References

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