New concepts about flow induced cavitation of liquids

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ABSTRACT

The problem of the inception of cavitation is formulated in terms of a comparison of the breaking strength or cavitation threshold at each point of a liquid sample with the principal stresses there. A criterion of maximum tension is proposed which unifies the theory of cavitation, the theory of maximum tensile strength of liquid filaments and the theory of fracture of amorphous solids. It is argued that the liquid ruptures in tension at nucleation sites; the cavity then fills with gas and the liquid flows. Liquids at atmospheric pressure which cannot withstand tension will cavitate when and where tensile stresses due to motion exceed one atmosphere. A cavity will open in the direction of the maximum tensile stress which is 45° from the plane of shearing in pure shear of a Newtonian fluid. An analysis of capillary collapse based on viscous potential flow leads to the total collapse of a capillary filament in a finite time; before this the filament enters into tension and presumably would break under tension. For water the critical radius is about 1.5 microns. Consideration is given to the idea that cavitation of water flowing over blunt bodies at values of the pressure above the vapor pressure is due to flow induced tensions. We propose that outgassing is the vaporization of the more volatile component of a two phase partially miscible mixture.

Conventional Cavitation

A fluid will cavitate when the local pressure falls below the cavitation pressure

- The cavitation pressure is the vapor pressure in a pure liquid
- Natural liquids have nucleation sites defined by impurities and may cavitate at higher pressures

What is Pressure?

- In an incompressible Newtonian fluid "pressure" is the mean normal stress. A fluid cannot average its stresses, even though you can. The fluid knows its state of stress at a point.
- In non-Newtonian fluids the pressure is an unknown flow variable, usually not even the mean normal stress, and the definition of it is determined by the constitutive equation. This "pressure" has no intrinsic significance. The fluid doesnt recognize such a "pressure" and knows its state of stress.

Nonconventional Cavitation Based on Principal Stresses

Look at the state of stress at each point in the fluid in principal axis coordinates. Identify the largest of the stresses. Suppose a static fluid cavitates at zero pressure. It will cavitate in flow wherever the maximum tensile stress is positive.

If it cavitates statically when the pressure falls below the vapor pressure, it will cavitate in flow even when the maximum tensile stress is only slightly negative.

Suppose you do an experiment in your lab where the ambient pressure is

$$\textbf{one atmosphere} = 10^5 \textbf{ Pa}$$

Then if you get tensile stresses due to flow larger than this, the fluid will cavitate.

Stress, Principal Axes, Deviator

- Stress in two dimensions

$$\begin{bmatrix} T_{11} & T_{12} \\ T_{12} & T_{22} \end{bmatrix}$$

  "pressure" cannot be recognized in a liquid; it sees a state of stress.

- Principal coordinates (figure 1)
- Mean normal stress and deviator

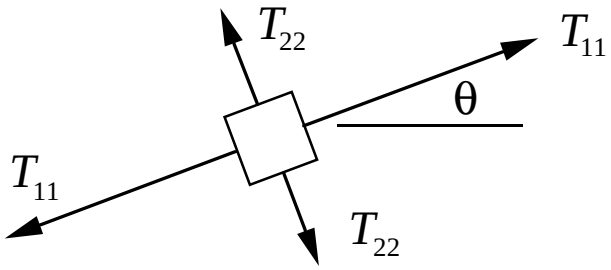
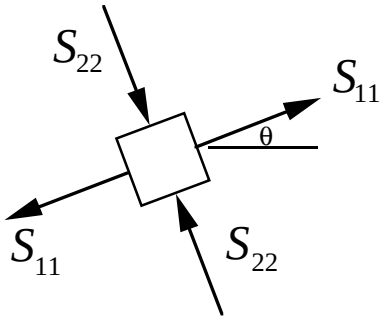


Figure 1: The direction of maximum tension. A cavitation must open in the direction θ; then it can rotate away.

$\mathbf{T} = -p\mathbf{1} + \mathbf{S}$ $\quad$ $\mathbf{S}$ is the extra stress

$p = -\dfrac{T_{11} + T_{22}}{2}$ $\quad$ the fluid cannot average stresses

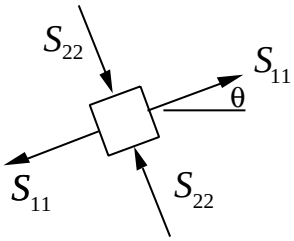
$\mathbf{S} = \begin{bmatrix} S_{11} & 0 \\ 0 & S_{22} \end{bmatrix}, \quad S_{11} + S_{22} = 0$



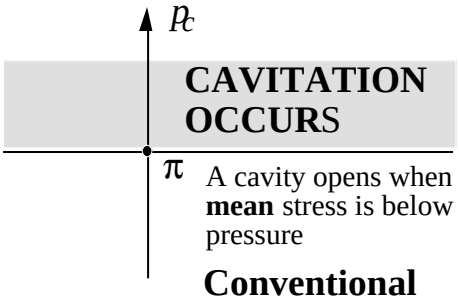
The extra stress is good because it has positive and negative components.

Cavitation Criteria

2D: $S_{22} = -S_{11}$



principal axes



A cavity opens when the **mean** stress is below vapor pressure

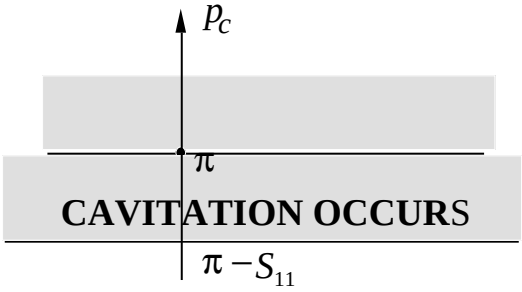
Conventional

$$-\pi = \tfrac{1}{3}(T_{11} + T_{22} + T_{33})$$

p_c is the "vapor pressure", the nucleation threshold.

Maximum Tension:

A cavity opens when **one** of the principal stresses is below vapor pressure



Minimum Tension:

A cavity opens when **all** of the principal stresses are below vapor pressure

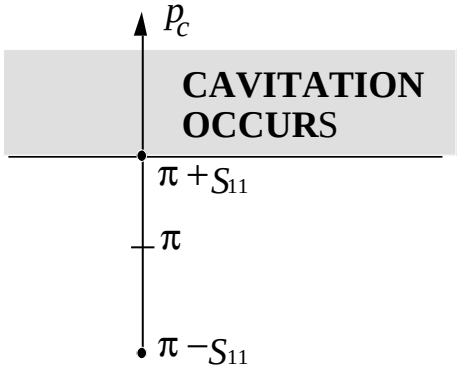


Figure 2: Three criteria for cavitation could be proposed, but the one based on maximum tension is the only one consistent with fracture of solids and solid-like liquids.

Maximum Tension

All *books on cavitation have sections on "tensile strength of liquids." A stress tensor is* ***never*** *introduced.*

"If a cavity is to be created in a homogeneous liquid, the liquid must be ruptured, and the stress required to do this is not measured by the vapor pressure but is the tensile strength of the liquid at that temperature." (Knapp et al. 1970)

FIRST **THE FLUID RUPTURES**
THEN **VAPOR FILLS THE CAVITY**

Cavitation in Shear

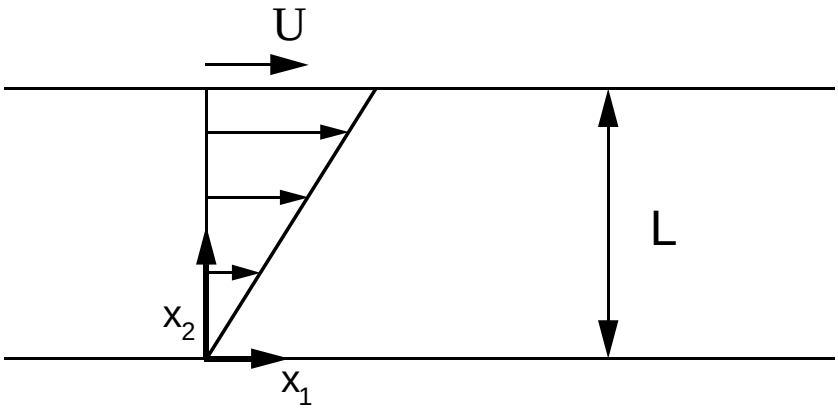


Figure 3: Simple shear between walls.

The stress in this flow is given by

$$\begin{bmatrix} T_{11} & T_{12} & 0 \\ T_{12} & T_{22} & 0 \\ 0 & 0 & T_{33} \end{bmatrix} = -\pi \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \eta \begin{bmatrix} 0 & \frac{U}{L} & 0 \\ \frac{U}{L} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

where $\pi = \frac{1}{3}(T_{11} + T_{22} + T_{33})$ is determined by the "pressurization" of the apparatus. The angle which diagnonalizes $\mathbf{T}$ is given by

$$\theta = 45^\circ$$

(In the break-up of viscous drop experiments in plane shear flow done by G.T. Taylor [1934], the drops first extend at 45° from the direction of shearing.)

In principal coordinates, we have

$$\begin{bmatrix} T_{11}+\pi & 0 & 0 \\ & T_{22}+\pi & 0 \\ 0 & 0 & T_{33}+\pi \end{bmatrix} = \eta\frac{U}{L}\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

where

$$T_{11}+\pi = S_{11} = \eta\frac{U}{L}$$

is a tension.

This tension is of the order of one atmosphere of pressure if

$$\eta\frac{U}{L} = 10^6\,\frac{\text{dynes}}{\text{cm}^2} = 10^5\text{Pa}$$

If $\eta = 1000$ poise, $U = 10$ cm/sec and $L = 10^{-1}$cm, we may achieve such a stress. *A shear stress of this magnitude is enough to put the liquid into tension.*

The production of cavitation in pure shear appears to have been realized recently (1997)

Abstract, "Fracture" phenomena in shearing flows of viscous liquids, L.A. Archer, D. Ternet and R. Larson:

> In startup of steady shearing flow of two viscous unentangled liquids, namely low-molecular-weight polystyrene and α-D-glucose,
>
> The shear stress catastrophically collapses if the shear rate is raised above a value corresponding to a critical initial shear stress of around 0.1 – 0.3 Mpa. The time-dependence of the shear stress during this process is similar for the two liquids, but visualization of samples in situ and after quenching reveals significant differences. **For α-D-glucose, the stress collapse evidently results from debonding of the sample from the rheometer tool, while in polystyrene, bubbles open up within the sample; as occurs in cavitation.** Some similarities are pointed out between these phenomena and that of "lubrication failure" reported in the tribology literature.

We have adhesive and cohesive fracture, 0.1–0.3 Mpa = 1–3 atm. This is enough to put the sample into tension 45° from the direction of shearing.

Breaking Strength of Polymer Strands

The strand breaks at the thinnest cross section of the strand when the tensile stress

$$\sigma = \frac{FV}{A_0} \approx 10^6 \text{Pa} = 10 \text{ atmospheres}$$

for many kinds of polymeric liquids. They say that the breaking stress is a material constant.

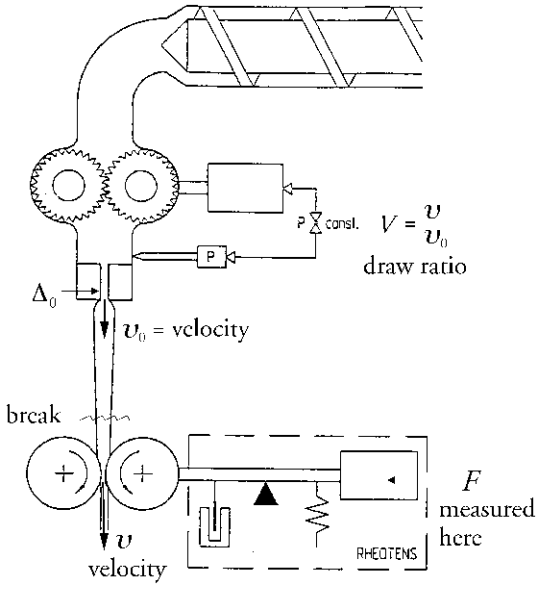


Figure 4: (Wagner, Schulze and Gottfert [1996]) Drawdown apparatus.

Breaking Time & Flow Time Vacuum Cavities

Experiments of Israelachvili and coworkers (Chen & Israelachvili [1991], Kuhl et al. [1994]) on ultrathin (nanometer) films show that cavities open in tension at a threshold value of the extensional stress $2\eta\overset{\circ}{S}$

$$\left(\begin{array}{c} \text{which I estimate as} \\ 3.6 \times 10^5 \text{Pa} < 2\eta\overset{\circ}{S} < 3.6 \times 10^6 \text{Pa} \end{array} \right)$$

and that the formation of cavities is analogous to the fracture of solids except *after* fracture, vapor flows into the cavity "...When a cavity initially forms and grows explosively, it is essentially a VACUUM CAVITY since dissolved solute molecules or gases have not had time to enter the rapidly growing cavity."

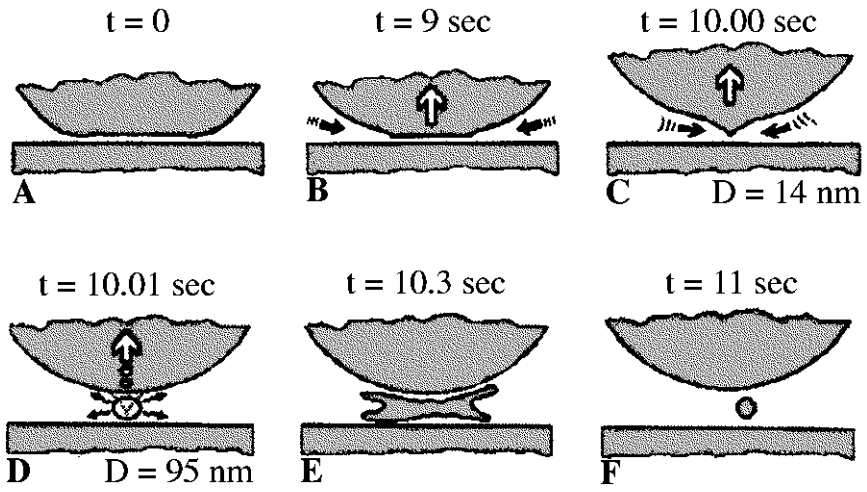


Figure 5: Surfaces separating at high speed, $v = v_c$

Kuhl et al. 1994 describe the experiment shown in figure 5 as

> "If the speed of separation is increased, the surfaces become increasingly more pointed just before they rapidly move apart. Then, above some critical speed ν_c (here about 100 μm/s) a completely new separation mechanism takes over, as shown in Figure 5. Instead of separating smoothly, the liquid 'fractures' or 'cracks' open like a solid.

> It is known that when subjected to very high shear rates, liquids begin to behave mechanically like solids, for example, fracturing like a brittle solid. In our experiments, the point and time at which this 'fracture' occurred was just as the surfaces were about to separate from their most highly pointed configuration (Fig. 5C) - for had the separation velocity been any smaller than ν_c they would have separated smoothly without fracturing. We consider that in the present case, the 'fracturing' or 'cracking' of the liquid between the surfaces must be considered synonymous with the "nucleation" or "inception" of a vapor cavity."

Capillary Collapse and Rupture

It might be thought that flow stress induced cavitation is restricted to rather viscous liquids, where high stress levels can be achieved. However such high stresses can be reached even in water, as the following analysis of Lundgren & Joseph [1997] shows:

> "The breakup of a liquid capillary filament is analyzed as a viscous potential flow near a stagnation point on the centerline of the filament towards which the surface collapses under the action of surface tension forces. We find that the neck is of parabolic shape and its radius collapses to zero in a finite time. During the collapse the tensile stress due to viscosity increases in value until at a certain finite radius, which is about 1.5 microns for water in air, the stress in the throat passes into tension, presumably inducing cavitation there."

Potential flows satisfy the Navier-Stokes equations, though they slip at solid boundaries, the viscosity of the fluid never has to be and should not be, put to zero (see Joseph and Liao [1994], Liao and Joseph [1994] for a complete discussion). The flow at the stagnation point of a collapsing capillary can be treated as a viscous potential flow.

$$[u_z, u_r] = a(t)[z, -\frac{r}{2}] \quad \text{Stagnation flow}$$

$$\varphi = \frac{1}{2}az^2 - \frac{1}{4}ar^2 \qquad \text{Potential flow}$$

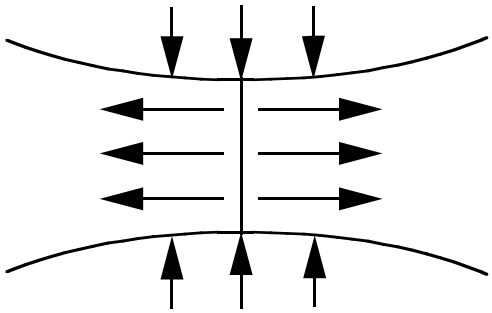


Figure 6: Stagnation point flow

Bernoulli Equations:

$$\frac{\partial \varphi}{\partial t} + \frac{1}{2}(u_r^2 + u_z^2) + \frac{p}{\rho} = \frac{p_0}{\rho}$$
$$\frac{p - p_0}{\rho} = -\left(\frac{1}{2}\dot{a} + \frac{1}{2}a^2\right) z^2 + \left(\frac{1}{4}\dot{a} - \frac{1}{8}a^2\right) r^2$$

$$\begin{aligned} T_{zz} &= -p + 2\mu \frac{\partial u_z}{\partial z} = -p + 2\mu a, \\ T_{rr} &= -p + 2\mu \frac{\partial u_r}{\partial r} = -p - \mu a \end{aligned}$$ Stresses

On $\boldsymbol{r = R(z,t) = R_0(t) + R_2(t)z^2 + O(z^4)}$

$$\begin{aligned} &-T_{nn} - p_a = \sigma\kappa \\ &T_{nn} = n_r^2 T_{rr} + n_z^2 T_{zz} \\ &\kappa = -\frac{\frac{\partial^2 R}{\partial z^2}}{\left(1+\left(\frac{\partial R}{\partial z}\right)^2\right)^{\frac{3}{2}}} + \frac{1}{R\left(1+\left(\frac{\partial R}{\partial z}\right)^2\right)^{\frac{1}{2}}} \\ &\quad = \frac{1}{R_0} - 2R_2 + O(Z^2) \end{aligned}$$ Normal Stress balance

It's zero at $z = 0$ because $\partial u_z / \partial r = 0$ — shear stress

$u_r = \frac{\partial R}{\partial t} + u_z \frac{\partial R}{\partial z}$ $-\frac{1}{2}aR = \frac{\partial R}{\partial t} + az\frac{\partial R}{\partial Z}$ $-\frac{1}{2}aR_0 = \overset{\circ}{R}_0$ Solve for a $-\frac{5}{2}aR_2 = \overset{\circ}{R}_2$ $R_2 = CR_0^5$	Kinematic Condition The parabola flattens as it collapses

$$a = -\frac{2\overset{\circ}{R}_0}{R_0}$$

To lowest order in z^2

$$\frac{p_a - \sigma\kappa}{\rho} = \frac{T_{nn}}{\rho} = \frac{T_{rr}}{\rho} = -\frac{p}{a} - va$$

$$\frac{p_a}{\rho} + \frac{\sigma}{\rho}\left(\frac{1}{R_0} - 2R_2\right) = -\frac{p_0}{\rho} - \left(\frac{1}{4}\overset{\circ}{a} - \frac{1}{8}a^2\right)R_0^2 - va$$

$$R_2 = CR_0^5 \qquad a = 2\overset{\circ}{R}_0/R_0$$

"Rayleigh Plesset" type of equation:

$$\frac{p_0 - p_a}{\rho} - \frac{1}{2}R_0\ddot{R}_0 - \frac{2v\dot{R}_0}{R_0} = \frac{\sigma}{\rho}\left(\frac{1}{R_0} - 2CR_0^5\right).$$

$$\overset{\circ}{R}_0 = \frac{1}{2}\frac{\sigma}{\rho v}$$ Small R_0, balances viscosity against inertia

to leading order in $t_* - t$	$R_0 = \frac{\sigma}{2\rho v}(t_* - t)$ $a = \frac{2}{t_* - t}$ $\frac{p}{\rho} = \frac{p_0}{\rho} - \frac{3z^2}{(t_* - t)^2}$	R_0 collapses to zero in finite time

AXIAL STRESS (to leading order)

$$\frac{T_{zz}}{\rho} = -\frac{2p_0 - p_a}{\rho} + \frac{4v}{t_* - t} = -\frac{2p_0 - p_a}{\rho} + \frac{2\sigma}{\rho R_0(t)}$$

T_{zz} **turns positive for small** R_0

$R_{ocr} = \frac{2\sigma}{2p_0 - p_a}$

Estimating $p_0 = p_a = 10^6$ dynes/cm^2
$\sigma = 75$ dyne/cm

$R_0 = 1.5$microns

At the collapse condition
the Reynolds number is
about 55 based on $\overset{\circ}{R}_0$

The capillary thread cavitates before it collapses to zero.

Cavitation within the laminar separation region on blunt bodies and in the nozzles of spray atomizers

Cavitation on the hemispherical nose of blunt bodies has been seen to commence within the reattachment region of the laminar separation. The terminus of the laminar region is apparently the location of the most intense cavitation and it is the location where cavitation is seen to disappear as the pressure is raised (Parkin & Kermeen [1953], Kermeen et al. [1958], Akrakeri [1973]).

The remarkable feature of this type of cavitation is that *the local pressure exceeds vapor pressure yet the cavitation still occurs*. Let us consider whether this remarkable appearance of cavitation under conditions in which cavitation should not occur can possibly be explained as a flow induced cavitation. Surely the state of stress in the boundary layer of a laminar separartion region is very complicated. It is probable that the maximum stress occurs right at the boundary solid consistent

with the observation of nucleation at the surface of the solid. In the absence of better knowledge we may estimate the stress as

$$\mu \frac{U}{\delta}$$

where $\mu = 10^{-2}$ poise for water, $U \approx 1800$ cm/sec (typical for the experiments) and δ at a value for the displacement thickness

$$\delta = 1.72(\frac{\nu x}{U})^{\frac{1}{2}}$$

Taking $x = 10$cm, we get $\delta \approx 10^{-2}$ and a stress of the order

$$1800 \frac{\text{dynes}}{\text{cm}^2}.$$

This is about one-tenth of the vapor pressure and hence could produce cavitation under these unusual conditions.

Separation regions at the entrance of nozzles of atomizers are conspicuous sites for cavitation especially in Diesel injectors. Chaves et al. [1995] note that

> ... Above an injection pressure threshold that depends on the nozzle geometry and chamber pressure, cavitation appears at the sharp inlet corner of the nozzle. With increasing injection pressure the cavitation reaches the nozzle exit (supercavitation).

Similar results establishing the presence of cavitation at or near the corner of the sharp inlet corner of the nozzle have been presented by Bergwerk [1959], Reitz and Braco [1982], Lichtarourcy [1972] and Souteriou et el. [1993].

It has to be considered that the appearance of cavitation as the pressure is raised above a threshold is very strange; you might expect cavitation as the pressure is dropped. I am suggesting that this anomalous cavitation is associated with the very high flow induced stresses which develop at and near the nozzle inlet. The speeds of the oil in these nozzles can reach hundreds of meters per second; the viscosity of the oils can be 100 times that of oil, leading to very large values of $\mu U/\delta$. In fact the stresses at a an idealized sharp corner are singular and are therefore a favored site for flow stress induced cavitation.

Outgassing as the cavitation of two phase miscible mixture

You could ask what would come out as you draw down the pressure of a mixture of glycerin and water. The logical answer is the constituent with the highest vapor pressure. The case of air and water is like this with the added caveat that air is only partially soluble in water; the solubility is a thermodynamic quantity. At equilibrium a certain amount of air is condensed in water at a given temperature and pressure. If the pressure is reduced the liquid air vaporizes into gaseous air. Glycerin and water are like this except that they are miscible in all proportions.

In studies of cavitation it is necessary to know which constituent is vaporizing.

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