MP-PIC Modeling of the Slot Problem

Governing Equations:

Fluid:

Continuity:

$$\frac{\partial}{\partial t}(\rho_f \theta_f) + \nabla \cdot (\rho_f \theta_f \mathbf{u}_f) = 0 \qquad (1)$$

Momentum:

$$\frac{\partial}{\partial t}(\rho_f \theta_f \mathbf{u}_f) + \nabla \cdot (\rho_f \theta_f \mathbf{u}_f \mathbf{u}_f) = -\nabla p + \nabla \cdot [\theta_f \mu_f \nabla \mathbf{u}_f] \qquad (2)$$

$$+\nabla \cdot [\theta_f \mu_f (\nabla \mathbf{u}_f)^T] - \nabla \left\{ \frac{2}{3} \theta_f \mu_f \nabla \cdot \mathbf{u}_f \right\} - \mathbf{F} + \rho_f \theta_f \mathbf{g} \qquad (3)$$

where:

ρ_f	$\longrightarrow$	density of the fluid phase
θ_f	$\longrightarrow$	volume fraction of the fluid phase
$\mathbf{u}_f$	$\longrightarrow$	velocity of the fluid phase
p	$\longrightarrow$	pressure
μ_f	$\longrightarrow$	viscosity of the fluid phase
$\mathbf{g}$	$\longrightarrow$	acceleration due to gravity
$\mathbf{F}$	$\longrightarrow$	rate of momentum exchange per unit volume between the fluid and particle phases

Equation of Motion of the Particles

$$\frac{d\mathbf{u}_p}{dt} = D_p(\mathbf{u}_f - \mathbf{u}_p) - \frac{1}{\rho_p}\nabla p + \mathbf{g} - \frac{1}{\theta_p \rho_p}\nabla \tau \qquad (4)$$

where:

$\mathbf{u}_p \longrightarrow$ velocity of the particle
$\mathbf{u}_f \longrightarrow$ velocity of the fluid at particle location
$\rho_p \longrightarrow$ density of the particle
$\theta_p \longrightarrow$ volume fraction of the particle phase
$\tau \longrightarrow$ interparticle stress
$D_p \longrightarrow$ drag coefficient

D_p is given by:

$$D_p = C_d \frac{3}{8} \frac{\rho_f}{\rho_p} \frac{|\mathbf{u}_f - \mathbf{u}_p|}{r} \tag{5}$$

$$\text{where} \quad C_d = \frac{24}{Re} \left(\theta_f^{-2.65} + \frac{1}{6} Re^{2/3} \theta_f^{-1.78} \right) \tag{6}$$

Re is the Reynolds number defined as

$$Re = \frac{2\rho_f |\mathbf{u}_f - \mathbf{u}_p| r}{\mu_f} \tag{7}$$

where:
$r \longrightarrow$ radius of the particle

Interparticle stress τ is given by

$$\tau = \frac{P_s \theta_p^\beta}{\theta_{cp} - \theta_p} \tag{8}$$

where
$P_s \longrightarrow$ is a constant with units of pressure
$\beta \longrightarrow$ constant
$\theta_{cp} \longrightarrow$ is the particle volume fraction at close packing.

In this approach particles are grouped into computational parcels each containing N_p particles with identical density, radius and velocity at a given location, $\boldsymbol{x}_p$.

Particle properties are then mapped to the Eulerian grid for the numerical solution using a bilinear interpolation function.

The particle volume fraction, θ_p, on the Eulerian grid for a given distribution of particles in the domain is given by

$$\theta_{pij} = \frac{1}{V_{ij}} \sum_p N_p V_p S(\boldsymbol{x}_p)_{ij} \tag{9}$$

where:

$\theta_{pij} \longrightarrow$ particle volume fraction at Eulerian grid point ij

$V_{ij} \longrightarrow$ volume of the Eulerian cell

$V_p \longrightarrow$ particle volume

$S(\boldsymbol{x}_p)_{ij} \longrightarrow$ interpolation function

Similarly the fluid-particle momentum exchange is given by

$$\mathbf{F}_{ij} = \frac{1}{V_{ij}} \sum_p S_{ij}[\boldsymbol{x}_p] \left\{ D_p(\mathbf{u}_f - \mathbf{u}_p) - \frac{1}{\rho_p} \nabla p \right\} N_p m_p \tag{10}$$

where:

$m_p \longrightarrow$ mass of the particle

Equation for particle position:

Particle position is updated using the following equation:

$$\frac{d\boldsymbol{x}_p}{dt} = \mathbf{u}_p \tag{11}$$

For more details please see: Andrews, M.T. & O'Rourke, P.J., 1996, "The multiphase particle-in-cell (MP-PIC) method for dense particle flow," Int. J. Multiphase Flow, vol 22, pp. 379–402.

We solved the system of equations using control-volume method with staggered grids for velocity and pressure.

Results

2D:

In Figure 1 we have presented the result for a 2D simulation of the slot problem.

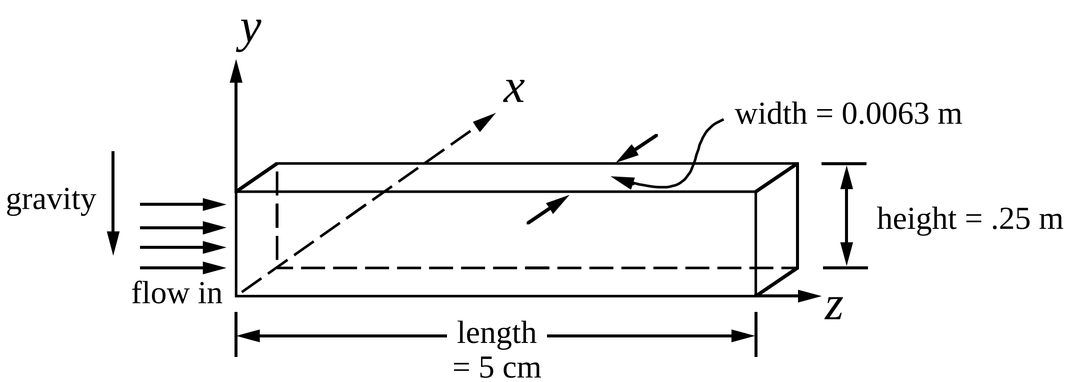
We have a channel of length, x = 5m
channel height, y = 0.25m
Gravity acts in negative y direction
Fluid particle mixture enters the channel at $x = 0$
Inlet velocity of the mixture $= 0.062 m/s$
Fluid density, $\rho_f = 1000$ kg/m^3
Particle density, $\rho_p = 1400$ kg/m^3
Particle radius, $r = 0.03$ cm
Fluid viscosity, $\mu_f = 0.025$ N-s/m^2

29 new parcels enter the computational domain at $x = 0$ at each time-step.
$dt = 0.14$s.
Parcels are equally spaced along the channel height. Each parcel has 4063 particles.
Thus $N_p = 4063$.
Concentration of the particles flowing in = 0.0491

3D:

In Figures 2a - 2c we have presented the results for a 3D simulation of the slot problem.

The channel is shown below.



Gravity acts in negative y direction
Fluid particle mixture enters at $z = 0$
Inlet velocity = 0.062 m/s
ρ_f = 1000 kg/m^3
ρ_p = 1400 kg/m^3
r = 0.03 cm
μ_f = 0.025 N-s/m^2

145 new parcels enter the computational domain at $z = 0$ at each time step.
dt = 0.14s.
The parcels are positioned at equally spaced 29 locations along the height of the channel and equally spaced 5 locations along the width of the channel.
Each parcel has 3 particles.
Concentration of the particles flowing in = 0.029. At the instant presented in Figures 2a-2c, time, t = 12.6s.
Total number of parcels = 13050

Figure 2a shows the parcels inserted in the channel at $z = 0$. Parcels are inserted at 29 locations along the height of the channel. As they move forward in the z direction with the flow they fall due to gravity acting in the negative y direction.

Figure 2b shows the parcels in the y-x planes which is perpendicular to the direction of flow. Parcels are inserted at 5 locations along the width of the channel. This is indicated by 5 vertical lines of higher parcel concentration. As the parcels fall in the negative y direction they disperse from their original x-location in the channel.

Figure 2c shows the parcels in the z-x planes.

Figure 1: View of parcel positions for 2 dimensional simulation. Time = 18.2s, Number of parcels = 3770. Gravity acts in negative y direction. Flow comes in at x = 0.

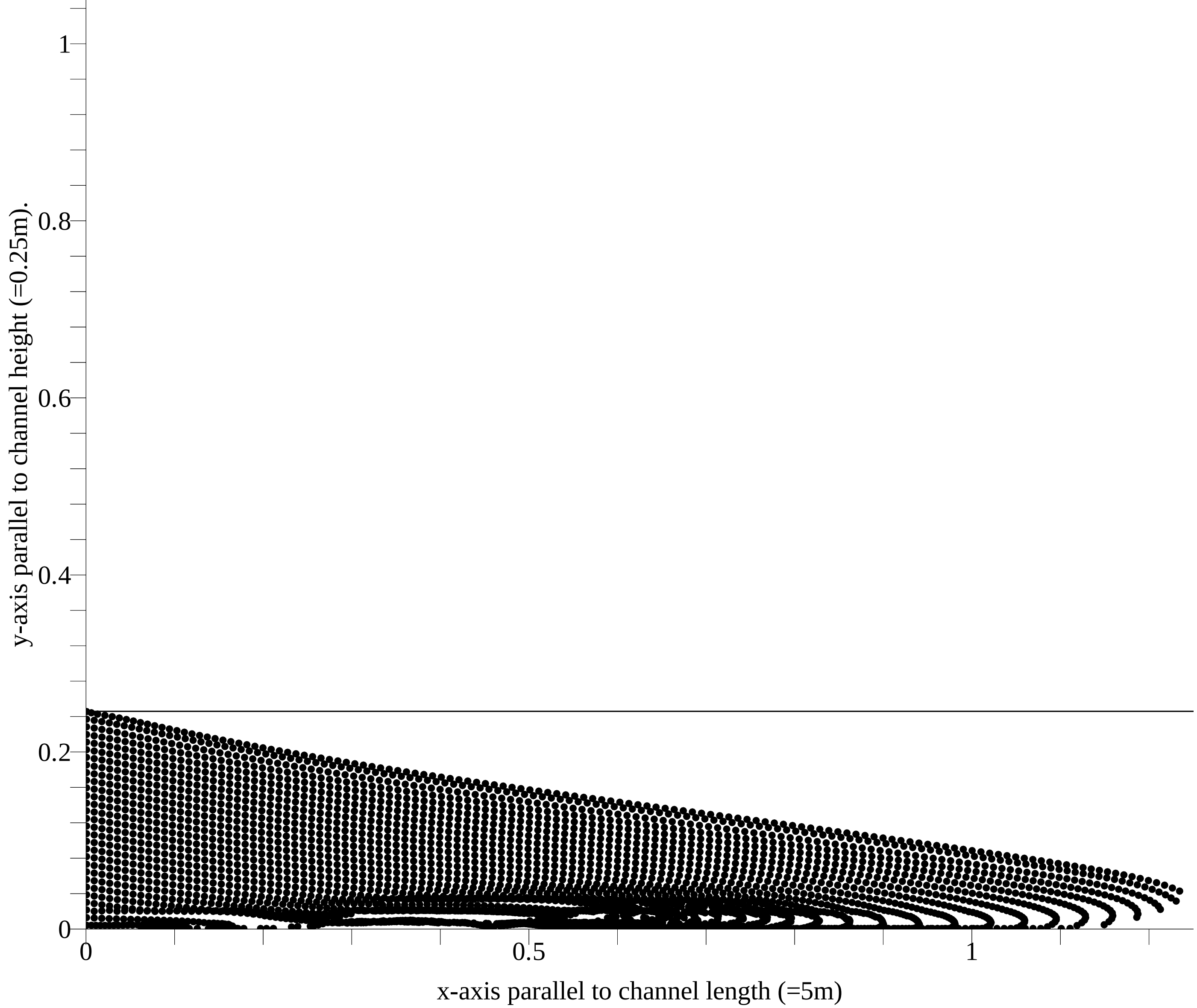
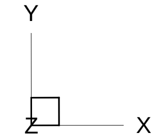


Figure 2b: View in the y-x planes for the 3 dimensional simulation.
Gravity acts in negative y direction.
(Dimensions are not to scale)



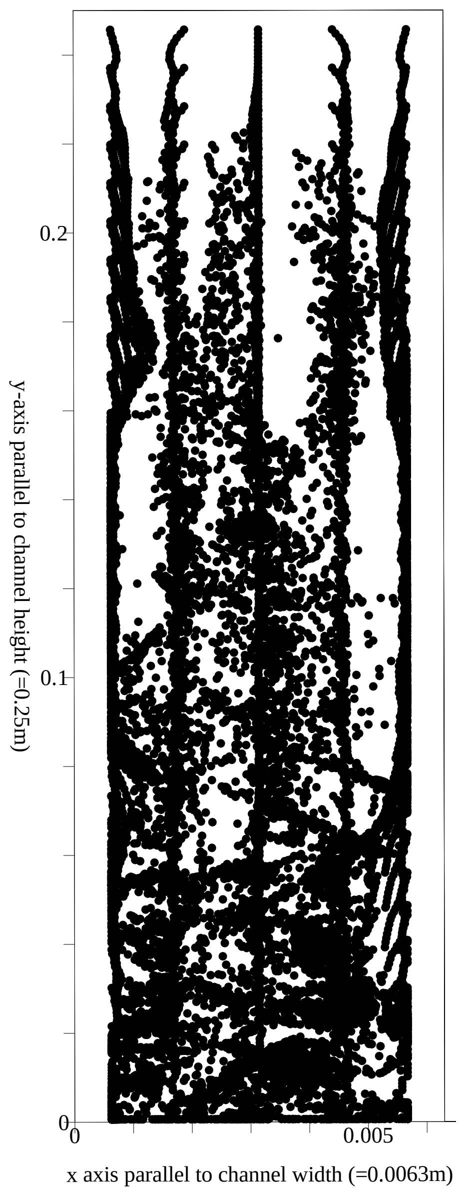


Figure 2c: View in the z-x planes for the 3 dimensional simulation.
Flow comes in at z=0 plane.
(Dimensions are not to scale)

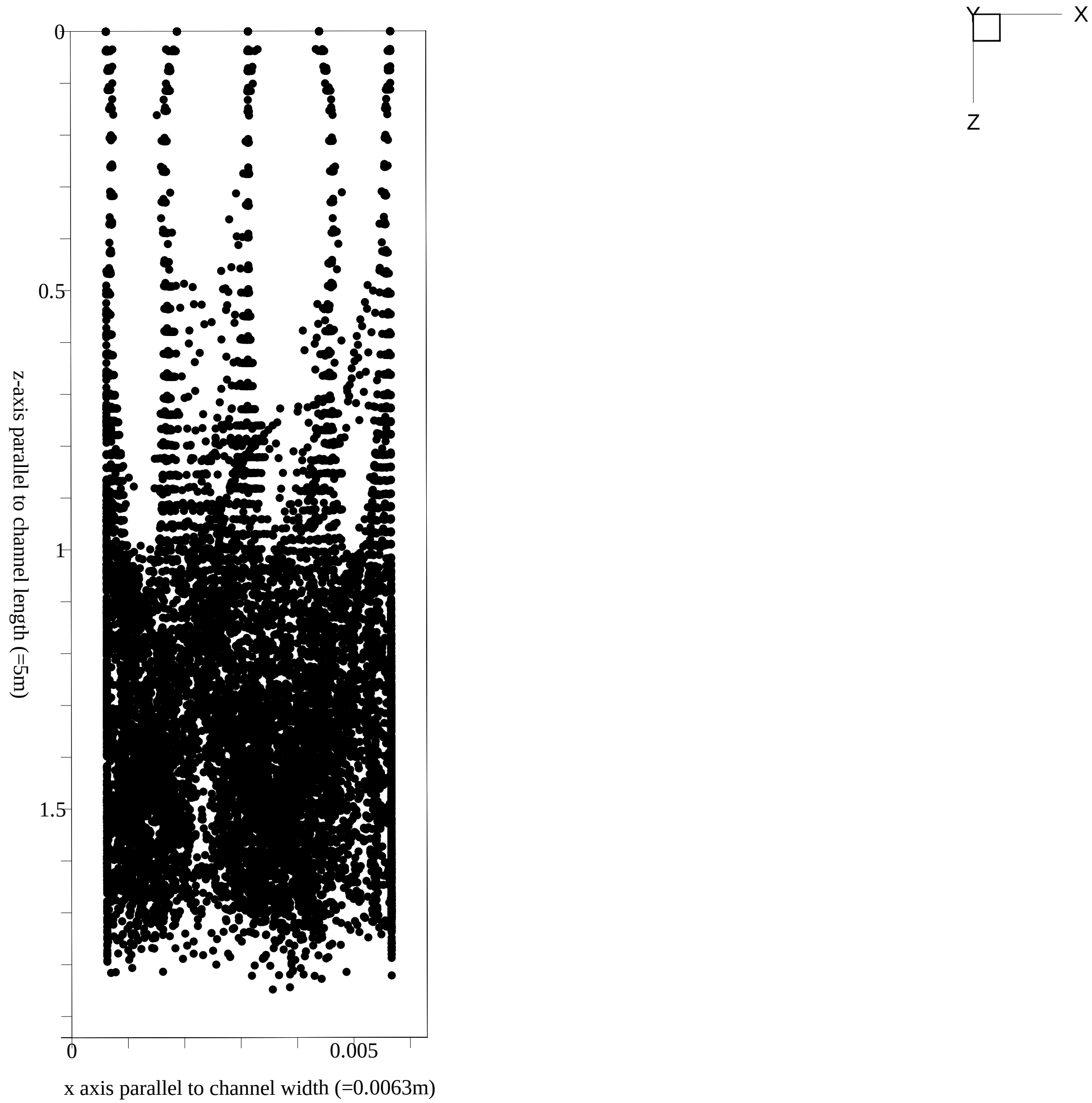


Figure 2a: View in the y-z planes for the 3 dimensional simulation.
Flow comes in at z=0 plane. Gravity acts in negative y direction. (Dimensions are not to scale)