Homogeneous model of production of heavy oil through horizontal pipelines and wells based on the Navier-Stokes equations in the pipeline or the well and Darcy's law in the reservoir

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▪ Homogeneous model:

The gas and oil can be described as a mixture in which the gas content is represented at every point of the mixture by a volume fraction ϕ of gas.

I do not expect that the homogeneous model will work well if slugs are the flow type and certainly it won't work if the gas is connected in streams as in stratified or annular flow. The homogeneous model should work best in the case that all the gas (or most of it) is dispersed in small bubbles that move with the oil, which may be the case for foamy oils, heavy oil like Zuata, Cerro Negro and other heavy oils in the Orinoco belt, Canada, Albania, etc.

Emilio Guevara told me that in his experience with multiphase pumps it was always much easier to monitor the flow of foamy oil because the gas was dispersed. I tried on my last visit in early March 2001 to find out if the flow type was slugs or homogeneous in horizontal pipes and wells in reservoirs of heavy oil. No one that I asked seemed really to know the answer to this important question.

▪ Horizontal pipe and horizontal well.

There is no flow through the wall of the pipe; all the oil is introduced at the entrance of the pipe. The well has liner with many slots that allow the oil to enter the well by overpressure in the reservoir; probably this can be modeled by the continuity of velocity of the oil and gas normal to the liner wall. It is probable that the continuity of velocity normal to the liner is modeled well by the continuity of the normal component of velocity given by Darcy's law in the reservoir and Navier-Stokes in the pipe.

▪ Flow loops can't duplicate field conditions

In horizontal wells and pipe live oil at saturation is driven into the pipe by a pressure created by the drawdown pressure at the pump. The gas evolves by outgassing of dissolved gas as the pressure drops by gradients in the direction of the pump.

In flow loops the gas is introduced by pumping in gas and oil and outgassing is not an important factor. Of course if a lot of dissolved gas is released by outgassing it might collect into slugs which could be modeled in a flow loop.

▪ Mathematical model of homogeneous flow

(i) Pressure P vs dispersed gas fraction ϕ (all of the gas is dispersed into bobbles which more or less move with the fluid).

Arjan Kamp and I derived the equation of state (called a solubility isotherm) for dispersed as a function of pressure

$$\frac{\beta\phi}{1-\phi}=\frac{\tilde{P}-P}{P} \tag{1}$$

where $\beta(T)$ depends on temperature and $\tilde{P}$ is the saturation or bubble point pressure. When $P \geq \tilde{P}$ there is no dispersed gas, ϕ = 0. We computed β = 3μ for Cerro Negro and values different but near 3.4 for Lloydminster and Lindbergh heavy oil.

Obviously the most severe outgassing occurs at the pump where $P = P_\rho$ is the smallest. Using (1) we may estimate

$$\phi = 1\Big/\left[1+\frac{\beta}{\Delta P/P_\rho}\right]$$

where $\Delta P = \tilde{P} - P_T$. If $\tilde{P}$ = 1000 psi and P_p = 900 psi, the ϕ = 1/30.6. If $\tilde{P}$ = 700 psi, P_p = 400 psi, then $\phi = {}^1/_{2.6}$. These are the worst conditions; in the pipe or well ϕ is much smaller. It certainly appears that in many cases not very much gas will come out except near the pump. This supports the idea that the homogeneous model might hold mostly anywhere away from the pump.

It will be convenient to work with the pressure drop

$$\pi = P - \tilde{P} \leq 0.$$

Then $(1)_1$ may be written as

$$\frac{\beta\phi}{1-\phi}=\frac{-\pi}{\pi+\tilde{P}} \tag*{$(1)_2$}$$

(ii) The viscosity μ (ϕ) and density $\rho(\phi)$ of the mixture depend on ϕ. The viscosity is assumed in the separable form

$$\mu(\phi) = \mu(0)\, f(\phi) = \mu_0 f(\phi) \tag{2}$$

There are may empirical formulas for $f(\phi)$ but we really don't know the correct $f(\phi)$ in the case of foamy oil. On the other hand, the mixture density can be considered to be accurately described by

$$\rho(\phi) = \rho_0(1-\phi) + \rho_g\phi == \rho_0(1-\phi) \tag{3}$$

(iii) The substantial derivative in a fluid and in a porous media

$$\frac{D\phi}{Dt} = \frac{\partial \phi}{\partial t} + \mathbf{u} \cdot \nabla \phi \quad \text{in a pure fluid} \tag{4}$$

$$\frac{D\phi}{Dt} = \alpha \frac{\partial \phi}{\partial t} + \mathbf{q} \cdot \nabla \phi \quad \text{in a porous media with porosity } \alpha \tag{5}$$

(iv) Darcy's Law

q is the seepage velocity of the mixture in the porous media and it satisfies Darcy's law

$$\mathbf{q} = \frac{k}{\mu_0 f(\phi)} \nabla \pi \tag{6}$$

where *k* is the permeability (*v*) continuity equation

$$\frac{D\rho(\phi)}{Dt} = \alpha \frac{\mathrm{d}\rho}{\mathrm{d}\phi} \frac{D\phi}{Dt} = -\rho_0 \frac{D\phi}{Dt} \tag{7}$$

From

$$\frac{D\rho}{Dt} + \rho \operatorname{div} \mathbf{u} = 0 \tag{8}$$

we get

$$-\frac{D\phi}{Dt} + (1-\phi) \operatorname{div} \mathbf{u} = 0 \quad \text{in the fluid} \tag{9}$$

$$-\frac{D\phi}{Dt} + (1-\phi) \operatorname{div} \mathbf{q} = 0 \, \text{porous media} \tag{10}$$

- ## The stress

This doesn't exist in the porous media

$$\mathbf{D}[\mathbf{u}] = \frac{1}{2}\left(\nabla \mathbf{u} + \nabla \mathbf{u}^T\right) \tag{11}$$

In the Navier-Stokes equation we have a term corresponding to the viscosity part of the stress

$$\begin{aligned} &2 \operatorname{div} \mu(\phi) \mathbf{D}[\mathbf{u}] = 2\mu_0 \operatorname{div}\left(f(\phi) \mathbf{D}[\mathbf{u}]\right) = \mu_0 \mathbf{S}(\phi, u) \\ &\mathbf{S}(\phi, u) = \operatorname{div}\left[f(\phi)(\nabla \mathbf{u} + \nabla \mathbf{u}^T)\right] \end{aligned} \tag{12}$$

▪ Navier-Stokes equation for dispersed flow

$$\rho_0(1-\phi)\left[\frac{\partial \mathbf{u}}{\partial t} + y \cdot \nabla y\right] = -\nabla\pi + \mu_0 S(\phi, \mathbf{u}) \tag{13}$$

▪ Horizontal well and pipeline: Boundary conditions

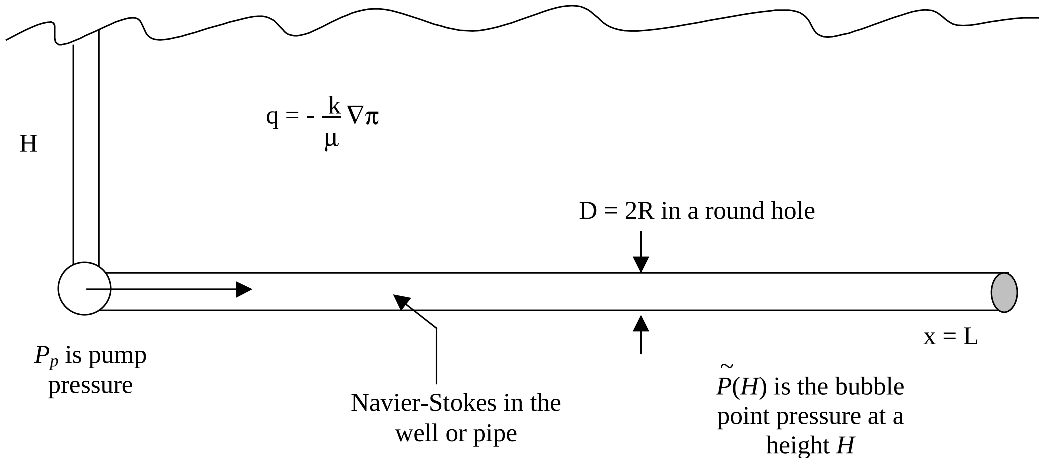

Figure 1.

We are going to assume that there is one bubble point pressure $\tilde{P}$ which doesn't depend on H.

(1.) Horizontal pipe.

$\mathbf{u} = 0$ at $r = R$. The porous media determines the pressure at the end of the pipe at $x = L$. P_p is the lowest pressure in the system. Flow is from reservoir to pipe entrance; then from pipe entrance to pump.

(2.) Horizontal well.

The normal component of velocity is continuous across the liner from the porous media into the well

$$\mathbf{u} \cdot \mathbf{e}_r = \mathbf{q} \cdot \mathbf{e}_r = -\frac{k}{\mu_0 f(\phi)} \frac{\partial \pi}{\partial r} \text{ at } r = R \tag{14}$$

The tangential component on the pipe liner vanishes

$$\mathbf{u} \cdot \mathbf{e}_x = 0 \text{ at } r = R \tag{15}$$

(3.) Pressure at the inlet end x = L of a long pipe.

It can be shown that if the pipe is long the pressure P_L at $x = L$ is very nearly the reservoir pressure

$$P_L \approx \tilde{P} \tag{16}$$

(4.) Reynolds number in a long pipe.

Assume that the oil is homogeneous and in Poiseuille flow in a long pipe. The mean velocity is given by

$$\bar{u} = \frac{R^2}{8\mu}\,\frac{\Delta P}{L}$$

where μ is the viscosity

$$\Delta P = \tilde{P} - P_p \quad .$$

The Reynolds number is

$$\mathrm{Re} = \frac{\bar{u}R}{\nu} = \frac{\rho}{8}\frac{R^3}{\mu^2}\frac{\Delta P}{L}$$

Numerical estimate:

$$\Delta P = \tilde{P} - P_p \cong 210\,\mathrm{psi} = 15\times 10^6\,\mathrm{dynes/cm^2}$$

$$L = 5000\,\mathrm{ft} \approx 1.5\times 10^5\,\mathrm{cm}$$

$$\Delta P / L = 10\,\mathrm{dynes/cm^3}$$

$$R = 1\mathrm{in} \cong 2.5\mathrm{cm}$$

$$\bar{u} = \frac{(2.5)^2}{\mu}\,\frac{5}{4}$$

$$\mathrm{Re} = \frac{(2.5)^3}{\mu^2}\rho\frac{5}{4}$$

The Reynolds number is very small

Re << 1 for heavy oil

(5.) Conclusion.

The Reynolds number for heavy oil $\mu > 100$ poise is very small. Neglect the inertia (left side) in (13).

$$\pi = P - \tilde{P} \quad \text{Pressure difference}$$

$$\pi_p = P_p \text{-} \tilde{P} < 0 \quad \text{prescribed pressure difference which drives the flow}$$

$$\frac{\beta\phi}{1-\phi} = \frac{-\pi}{\pi + \tilde{P}} \quad \text{Solubility isotherm}$$

The porous media does not enter this problem

$$\mu(\phi) = \mu_0 f(\phi) \quad \text{Viscosity}$$

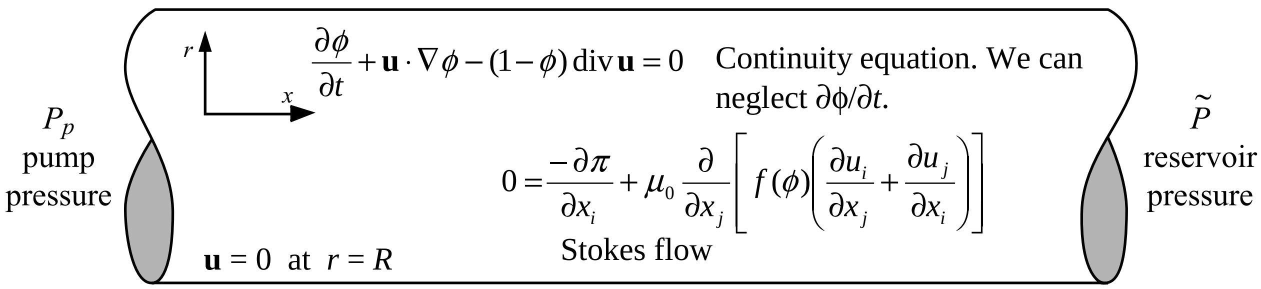

Figure 2. Horizontal pipe (length L*, Radius* R = r*). The unknowns are ϕ and* **u***; π may be eliminated. ϕ and* **u** *depend on* x *or* r.

All the equations in the horizontal pipe hold in the horizontal well. In addition, we need reservoir equations:

$$\mathbf{q} = \frac{-k}{\mu_0 f(\phi)} \nabla\pi \quad \text{Darcy law}$$

$$\alpha\frac{\partial\phi}{\partial t} + \mathbf{q}\cdot\nabla\phi - (1-\phi)\,\mathrm{div}\,\mathbf{q} = 0 \quad \text{Continuity equation}$$

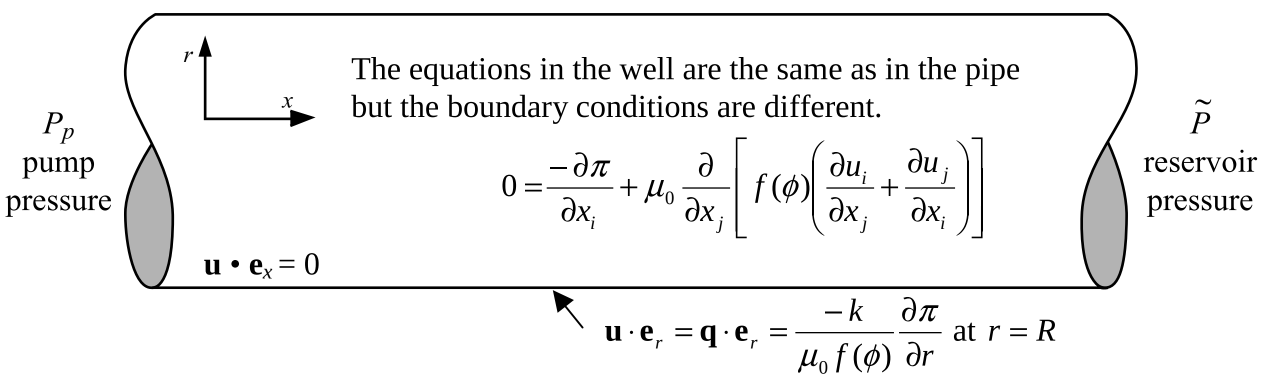

Figure 3. Horizontal well. The porous media is very important. Flow is driven by the reservoir pressure reference π_p toward the pump through the liner.

The equations in figures 2 and 3 should be made dimensionless in order to identify the controlling parameters. Quite frankly, I am not at all clear on how to do this. It is easy to make the equations dimensionless, but choices of scales must be made and I do not yet know the optimal choice.

You could try your hand at getting a good dimensionless formulation.