Memo: From Dan Joseph
To: Ignacio Layrisse, Simon Suarez, Gustavo Núñez
Review of research at Intevep, March 14-24, 1997

In this memo I will review my interactions with Intevep people during my visit to Los Teques, March 14-24, 1997. The memo is organized by topics.

FOAM CONTROL USING A FLUIDIZED BED. Josés thesis established that you could suppress the formation of foam and reduce the gas-holdup in a foaming bubble reactor by fluidizing hydrophilic particles in a bubbly mixture. Clara has just shown you can do substantially more suppression with hydrophobic than with hydrophilic particles and we believe that the degree of hydrophobicity matters. There appears to be considerable interest in these results in oil companies. Shell in Houston is trying to get us money from the 30 million dollar pot of GPRI (Global Petroleum Research Institute) money that a consortium of oil companies has put together. A first try at a research proposal for GPRI on foam control using a fluidized bed is given here in Appendix 1. Exxon has invited me to give their engineers a lecture on foaming and there have been other expressions of interest. Since we own the patents and have the know-how it is possible that we can find good business opportunities. We should keep our eyes open about applications involving foam.

It is probable that hydrophilic particles suppress foam in the reactor because they increase liquid and solid hold-up markedly. Hydrophobic particles appear to break, and not only to suppress, foam; they may have a greater application.

IN-SITU FOAMING. In many applications you would like to block a high permeability crack so as to get a fluid preferentially into a lower permeability (less damaged) crack. One way to do this is by *in situ* foaming; the gas and liquid surfactant are injected through separate conduits and foamed downhole.

José found a criterion for foaming. If gas and liquid are injected continuously with superficial velocities U_g and U_l, respectively, then there is a critical condition

$$U_l = aU_g + b \tag{1}$$

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for foam formation. If you fix U_l the reactor will not foam for low U_g and will foam when U_g is increased by the value given by (1). The more gas you put in, the greater is the amount of foam. You can stop foaming by fixing U_g and increasing U_l above the value given by (1). In general, to get a surfactant to foam you have to shake it up; even detergent in water won't foam unless you stir it.

In our video you could see that we can make foam in a fixed bed according to (1). The fixed bed is formed by spheres too heavy to fluidize. Probably you could foam a sand pack in this way.

Experiment 1. Try to determine a critical condition like (1) for foaming a sand pack

First we inject water plus surfactant; then we increase the gas flow until foam appears. This is an option for foam production which could find applications for proppant transport and possibly secondary recovery.

The advantage of *in situ* foaming is that it is easier to inject gas and water separately and produce foam in place than to mix and transport the foam to place. A technology for *in situ* foaming requires separate liquid and gas lines.

ACIDIZING IN SITU. We want to block the more permeable undamaged cracks with foam so that we acidize the damaged cracks. Here is the plan. Flood the reservoir with surfactant and water. Then inject gas fast enough to foam; the most conductive cracks will foam first. Then the acid will flow into the less conductive (damaged) cracks. Maybe this is a better way to foam an undamaged crack.

EXPERIMENT ON THE SELECTIVE FOAMING OF CRACKS. Here is one kind of experiment we can do; it is described in the caption of Figure 1.

IN SITU FOAMING FOR DOWNHOLE CLEANING IN HORIZONTAL DRILLING. It should be easier to pump liquid and gas to the bit separately than as a foam. We should use the technology that is used now as much as possible. As I see it, the main problem is how to pump in the gas. One idea is to use an annular drill string (figure 2), or a drill string with an auxiliary gas line. I don't know what would be best but different ideas could be proposed and tested at Intevep with cheap experiments before going to the fields.
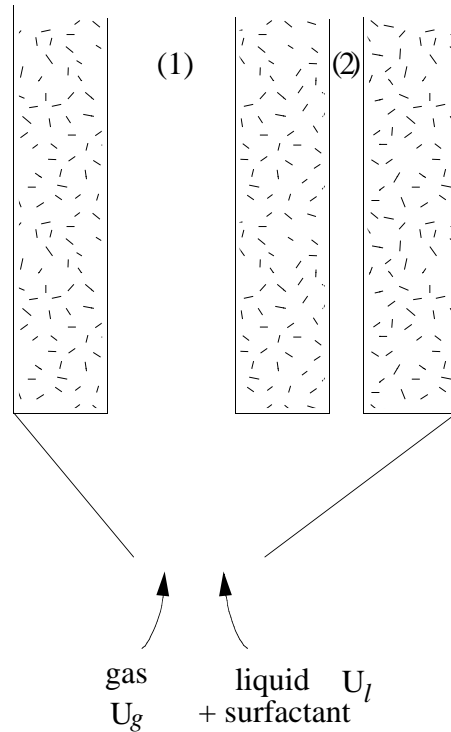
Figure 1: Selective foaming. Channels (1) and (2) have different flow resistance and holdup of gas and liquid. These can be manipulated by design. According to equation (1) we can expect that as we increase U_g at a fixed U_l one of the channels will foam first. In this way we can determine properties for in situ foaming.

MWD (Measure while drilling). Foams and aerated muds are aggressive and mechanical MWD tools used in more benign muds don't work. I have the idea that the previous technology could be used also as an MWD tool. To pump in water, you need a certain pressure head and a different head is required to push in the gas. The pressures required could be monitored at the well head and maybe we could develop a little theory and a short program to get a continuous record of the pressure downhole. I don't know enough about this on the practical side to evaluate this myself but I am sure that Mayella can have some thought about whether this is a good idea. I need myself to go into the fields and see how they do things there.
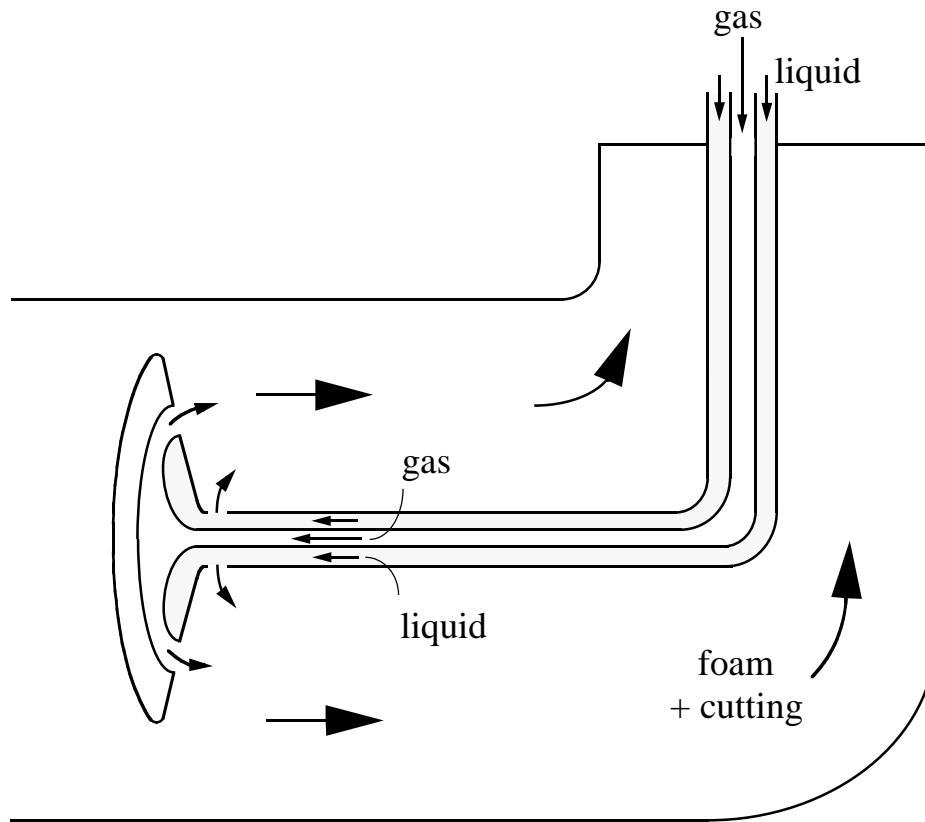
Figure 2: In situ creation of foam downhole using an annular drill string in which the liquid and gas are injected separately. This is one idea, maybe not the best, for separate injection of gas downhole.

RECYCLING OF FOAM USING A SHAKER BOTTLE. When drilling with foams, the used foams are dumped into a pit and the sunshine breaks up the foam; you get surfactant plus water which can be recycled. Some new surfactant must be added but there is at present no way to know how much. It's probable that the operators in the field put in too much new surfactant, as a margin of safety. To know exactly how much surfactant to add we can use a shaker bottle. This bottle is very low tech and anybody can understand and use it with confidence.

José and I wrote a paper called "Foaminess measurements using a shaker

bottle." We gave a little theory in our paper and showed that it gave reliable measurements of the foaminess

$$\epsilon_f = \frac{V_{\text{foam}} - V_{\text{liquid}}}{V_{\text{foam}}}$$
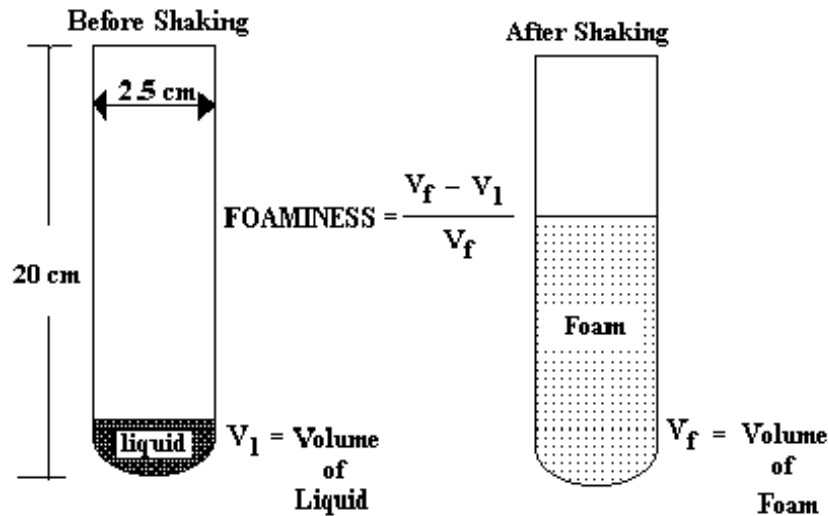
(see figure 3)

Figure 3: Cartoon of the shaking bottle to measure foaminess. This bottle could be graduated so that the amount of surfactant necessary to add to recycle the old foam could be read like a thermometer.

DOUGLAS: BENCHTOP-SLIT DRILLING HOLE APPARATUS. Following our earlier discussions Douglas Ocando is building a slit device in which we can measure everything. Douglas should add some cartoons and drawings to this part of the memo: The apparatus allows for continuous injection of liquids, can also use a separate line for gases and for continuous injection of solids as we have in actual drilling operations. We can test different kinds of muds. The apparatus has two degrees of freedom around an axis through the center and around the bottom as in figure 4.

CUTTINGS TRANSPORT. Since we inject particles and drilling fluid at controlled rates we can see how effectively the mud removes cuttings. We
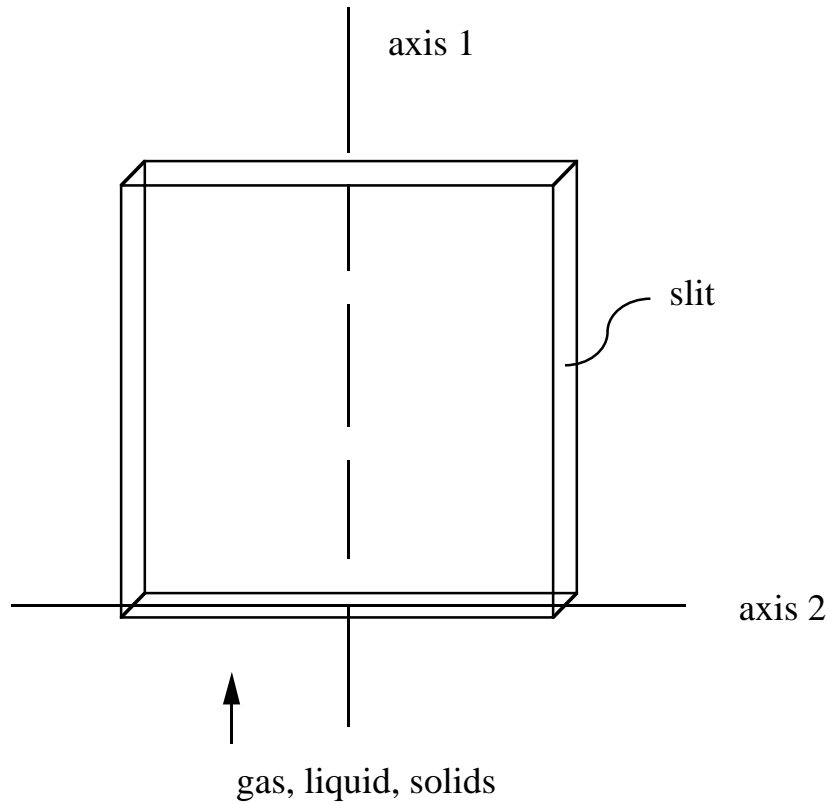
Figure 4: The slit can be tilted around two axis, vertical to horizontal, flat face perpendicular to parallel to gravity.

can see how particles migrate in flow of different muds. We can see how effectively foam removes particles of different size, weights and shapes. A prime variable to measure for all this is the *cuttings hold up*. The device can also be used to study proppant transport into hydraulic fracture.

BARITE SAG. We can study this well known problem in wells deviated around 45° from gravity when you have solids (cuttings) in suspension. The barite sag is like a Boycott effect (enhanced settling) with the caveat that particles are injected continuously and not in batch. In the Ocando apparatus, we can inject the particles continuously as in real drilling. The channel can be tilted from the vertical and then rotated around the central axis. So we can look at the slit in every orientation relative to gravity. This is a

definite plus.

Here is a thought that may not have been expressed before. I think that there should be a great difference between foam and other drilling fluids with respect to barite sag. Foams do not circulate like other fluids. The fluid currents set up by settling particles in ordinary liquid drilling muds will not occur in foams. This difference can be imagined by analogy between the bubbly mixture and the foam in the video of the reactor José studied in his thesis; the bubbly mixture, even with particles, undergoes violent vortical and turbulent motion while the foam above is very passive even when large gas bubbles rise through it. I think that there are no studies of Barite sag in foam drilling muds and that ours should be the first. Of course, I don't know this literature well.

MODELS FOR FOAMS, MISTS AND AERATED MUDS. Mayela, Saad Saleh, Douglas, Raad and I met to discuss modeling issues. The first part of our meeting was a discussion of MUDLITE which is a computer code prepared by Maurer engineering for predicting the rheological behavior and cuttings transport properties of foams and mist. Maurere is a reputable service company but we agreed that the code is not documented in a way we can understand, it has definite undesirable features that we can identify and I am not satisfied with the documentation of validization of the code. The two people at Maurer who wrote the paper "Foam computer model helps in analysis of underbalanced drilling" do not seem qualified for modeling. Medley has a BS in civil engineering from Texas A&M and a lot of practical experience; Liu has a Masters degree in Mechanics and practical experience. I think that this kind of education is not right for people who develop models. You have to have a very good understanding of fundamentals to do models and I think that I would be happier with the code if it were developed by *professionals.*

There are a bunch of equations in the aforementioned paper, and in the documentation they give to customers, but there are no derivations. Neither do they give reference to a place where derivations can be found. As nearly as I can tell, MUDLITE is taken from FOAMUP which is a 1972 Chevron program, for which documentation is also not given. The chief difference between FOAMUP and MUDLITE seems to be that FOAMUP is run on a mainframe while MUDLITE runs on a PC. They made some other changes, but they get good agreement between MUDLITE and FOAMUP because the codes aren't very different. This comparison is ridiculous. The same remark goes for their validating MUDLITE with data from Chevron. Apparently, this data was used to build FOAMUP, and since MUDLITE is made from

FOAMUP, you are comparing a code with data used to construct the code. Validity is automatic but vacuous.

They did another comparison, comparing predictions and observations for a couple of points. The comparisons don't go far and it is in their interest to make them good. I don't trust MUDLITE.

Another things is that to use MUDLITE you have to input the gas, oil and water transfer from the reservoir. These should not be inputs; these should be outputs.

The very idea of trying to get one model to work for Mists and Foams is insane right from the start.

Mayela and Douglas are going to see if we can get the derivation of the equations used in the MUDLITE model. On the other hand, if they are using this model in the fields we ought to see that they are getting useful information. Maybe they run the code and then do what they want anyway. If they are getting useful results we ought to know. You have to be skeptical about what people tell you because they always try to make a good impression.

FUTURE PLANS FOR MODELING. We are going to give the commercial models a hard look, but try to see the good in them. We are going to try to develop our own model. We have a good team, Mayella, Paul Hammond, me and a Ph.D. from Houston working on this in Japan that Mayela knows. We will meet in Intevep at the end of June and beginning of July (I plan to come for two weeks then).

I think we want a one dimensional model which uses actual data, correlations for closure. My idea is to use a Richardson-Zaki type of model as José did for foaming reactors and bubbling mixtures and as Richardson and Zaki did for fluidized suspensions of solids. Basically, this kind of theory satisfies the continuity equations for the separate phases and models the *slip velocity* with an empirical equation. This modeling of the slip velocity can be regarded as a substitute for the momentum balance. I have prepared some further explanation of the Richardson-Zaki type of theory in an appendix to this memo (to be sent later).

I think that a useful model for drilling should be highly empirical and easy to use. A Richardson-Zaki theory is like this – maybe MUDLITE is too complicated and insufficiently empirical. It's a point we should discuss.

The Richardson-Zaki two phase flow theories are easier when the components are incompressible. In this case, the unknowns are the velocity and volume fraction. When one of the materials is compressible, we have the density as an additional variable. For many kinds of flows, gas can be

treated as incompressible. Gas effects ought to be different in mists, foams and aerated muds.

COMPRESSIBLITY OF FOAMS. We need PVT diagrams for the foams we use. I mentioned this to Mayela and we did not know if this information is available. If it's not available, we ought to create it. The same is being done for foamy oil, which is allied to aerated muds. We should ask people about data on the compressibility of foam and if there is insufficient data we could design some simple experiments when I come down in June.

FOAMY OILS. I met with Marlene Huerta and looked at what her group does. They are in charge I would say of material characterization of foamy oils. Foamy oils are oils which contain dissolved gases and the gases come out when the pressure is reduced below the bubble point. These oils often give rise to foam (films & plateau borders) at the well head. I need more education about foamy oils, but I think that the generation of foam is entirely a secondary issue and the important thing about these oils is the release of gas.

According to Marlene, the interest in these oils stems from the "anomalous" high production rate and the primary recovery factor. I asked her to explain these anomalous results to me in precise way with all details. I did not get to the explanations I wanted. I looked then at the possibility that Marlene's group does not focus on production. It is not supposed to so focus, but the understanding of what to measure, what is really needed in a lab like Marlene's should come from an intimate understanding of just what the problems are at the well head.

In attempting to understand what the "anomalous" production was all about, I asked Marlene if there was someone around who really knew a lot about it. She identified the group of Carlos Otero who is a group leader for integrated reservoir studies for Corpoven. He formerly worked with Marlene in reservoir simulation and appears to have a better understanding of foamy oils than Marlene. He tried to explain the details I needed, though I still lack understanding and need more explanations.

Huerta, Otero, Rico, Jiminez, Mirabel and Roja wrote an interesting SPE paper on "Understanding Foamy Oil Mechanisms for Heavy Oil Reservoirs during Primary Production." I discussed the experiments described in that paper with Marlene.

They did conventional and non-conventional PVT tests, and in the non-conventional test the oil is not agitated. They believe this test is more like what actually occurs in reservoirs. I think that they are probably right; in

any case the difference between the two tests is not so great.

I would like to know the density, temperature and pressure of oil plus trapped gas – designated as psuedo phase in figure 2. These are needed for the model of the horizontal drilling hole, which follows this. Now perhaps the PVT data that Marlene gets can give this. It depends on where she takes the measurements. If the measurements are in the free gas the data for the *psuedo phase* would not follow directly. For example, is the tranducer for pressure in figure 3 in the free gas?

Usually, if gas is released all of a sudden, and the thermal conductivity of the oil is small, the gas expansion will be adiabatic and after a long time it will get to the temperature of the oil. So to know how much the gas expands to get the gas fraction we should get an opinion about the thermodynamics under way.

Figure 5 in their paper shows the viscosity of live oil as a function of pressure. There is a difference in values taken by capillary and rotational viscometers. We should know if this is due to material measured or the instrument. I have urged Marlene to measure the viscosity of 500–1000 cp silicon oil in both instruments. Since silicon oils are well characterized, the viscosity should be the same for both instruments. If we see a difference it will be due to the instrument.

I guess that the viscosity of dead oil should be larger than live oil? The value 9.1 cp in Table 1 is hard to believe.

The viscosity in figure 5 are very interesting. They show that at lower pressures as gas is released, the viscosity increases. This is expected. The viscosity of a dispersion should be greater.

Gas coming out of live oil can be regarded as cavitation, like a phase change. In fact, I think it is a phase change. When the pressure is increased above the bubble point, gas is forced into solution. I think that this means that the gas is "liquified" and that the oil and liquid gas is an ordinary mixture of miscible *liquids*. Naturally the viscosity of solution in which the liquid gas is of low viscosity, would be reduced. I don't know that this idea is correct, but it seems to make sense.

MODEL OF HORIZONTAL DRILLING HOLE IN A RESERVOIR OF FOAMY OIL. (This model could be considered for critical evaluation by the beautiful sisters Araujo)
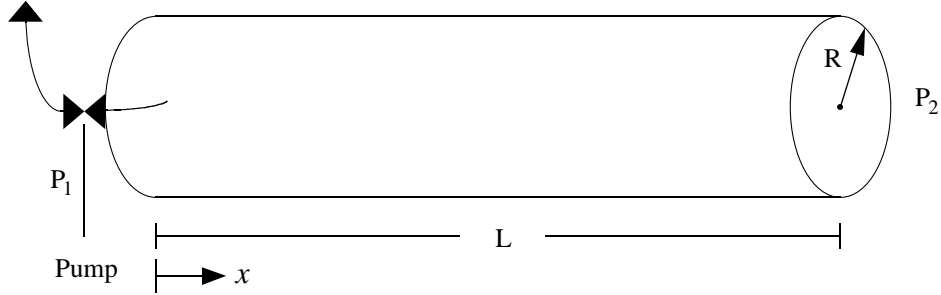

Figure 5: P_2 is the reservoir pressure. $P_2 < P_b$ where P_b is the bubble point. This means that as we lower the pressure, bubbles will come out.

I would assume the reservoir around the pipe is at a pressure P_2. Gradients of the pressure are created by pumping. These gradients move oil through from the pipe inlet and also by seepage from the porous formation through the pipe walls.

Can I assume that the oil has volume of small bubbles of size $r = a$ with a volume fraction $\phi = \frac{V_{\text{gas}}}{V_{\text{gas}} + V_{\text{oil}}}$ of dispersed gas?

The oil is treated as a one-phase fluid but with properties like viscosity and density that depend on ϕ. This assumption should be OK provided that the gas remains in disperse bubbles and does not percolate.

When you pump the oil, you create a pressure gradient in the pipe and in the formation. As the pressure drops gas bubbles grow and more bubbles are liberated. The liberation of the gas is probably responsible for improved production, but I think it is very hard to model.

For the flow in the porous formation I would start with Darcy's law in the form

$$\nabla P = \frac{\eta(\phi)}{\kappa(\phi)} \mathbf{U} \tag{2}$$

where P and $\mathbf{U}$ are the pressure and velocity and the viscosity η and permeability κ depend on the gas fraction.

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Oil without dispersed gas can be considered to be incompressible; $d\rho/dt = 0$, hence div$\mathbf{U} = 0$. If in addition η/κ is independent of φ, P is a harmonic function; $\nabla^2 P = 0$ is a classical equation for flow in a porous media. The situation for foamy oil is quite different since $\eta(\phi)$ and $\kappa(\phi)$ depend on ϕ and so does the density $\rho = \rho(\phi)$; then $d\rho/dt = \rho'(\phi)d\varphi/dt$ and hence

$$\rho'(\phi)\frac{d\phi}{dt} + \rho(\phi) \ \text{div}\mathbf{U} = 0. \tag{3}$$

In the pipe we use the Navier-Stokes equation for velicty $\mathbf{u}$ and pressure p together with a continuity equation identical to (3), with $\mathbf{u}$ replacing

$$\rho'(\phi)\frac{d\phi}{dt} + \rho(\phi) \ \text{div}\mathbf{u} = 0, \tag{4}$$

$$\rho(\phi)\Big(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u}\Big) = -\nabla p + \eta(\phi)\nabla^2\mathbf{u} \tag{5}$$

At the interface $r = R$ of the pipe and porous media we require

$$\left.\begin{array}{ll} p = P & \text{the pressure is continuous} \\ u_r = U_r & \text{the normal (radial) component of} \\ & \text{velocity is continuous} \end{array}\right\} \tag{6}$$

$$\begin{array}{ll} u_x = 0 & \text{the tangential component of velocity} \\ & \text{in the pipe vanishes} \end{array} \tag{7}$$

No conditions are prescribed on U_x. There will be a discontinuity of the tangential velocity at $r = R$. Equations (4) and (5) are 4 equations for the 5 variables $\mathbf{u}, \varphi, p$. To complete this description I need to invent an equation governing the evolution and distribution of the gas fraction ϕ. For example (just a guess)

$$\alpha\Big(\frac{\partial\varphi}{\partial t} + \mathbf{u}\cdot\nabla\phi\Big) = f(\phi, p) \tag{8}$$

where $\mathbf{U}$ and P replace $\mathbf{u}$ and p in the formation.

Equation (8) couples φ to isobars through f. I need to guess a good f. In the static case

$$f(p, \phi) \tag{9}$$

relates the gas fraction to the pressure.

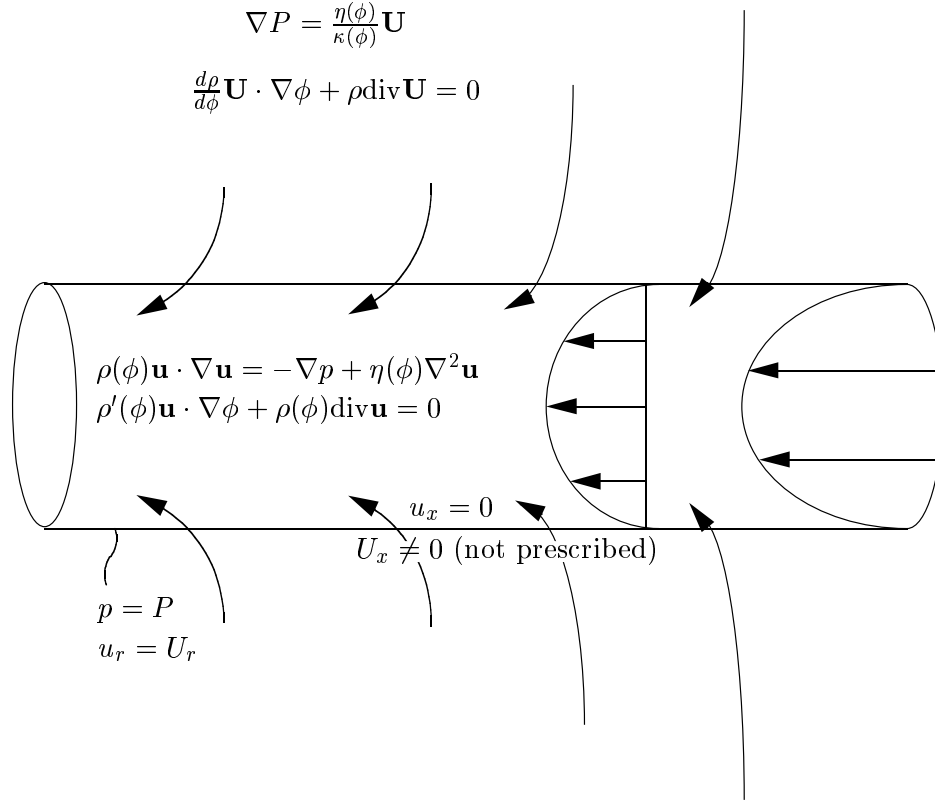Now we have the same number of equations as unknowns, in the pipe and in the formation (see figure 6).

$$\nabla P = \frac{\eta(\phi)}{\kappa(\phi)}\mathbf{U}$$

$$\frac{d\rho}{d\phi}\mathbf{U}\cdot\nabla\phi + \rho\,\mathrm{div}\,\mathbf{U} = 0$$

$$\rho(\phi)\mathbf{u}\cdot\nabla\mathbf{u} = -\nabla p + \eta(\phi)\nabla^2\mathbf{u}$$
$$\rho'(\phi)\mathbf{u}\cdot\nabla\phi + \rho(\phi)\mathrm{div}\,\mathbf{u} = 0$$

$$u_x = 0$$
$$U_x \neq 0 \text{ (not prescribed)}$$

$$p = P$$
$$u_r = U_r$$

Figure 6: Infinite horizontal pipe producing foamy oil. $P = P_2$ @ $r \to \infty$, $\mathbf{U} \to 0$ at $r \to 0$. ϕ satisfies some equation like (8).

To solve a pipe flow it is enough to prescribe the values of $\mathbf{u}$ at $r = R$. Here $u_x = 0$ and the two continuity conditions (6) (count each of them as 1/2) are equivalent to one prescribed value, so we have the right number of boundary conditions and (6) is also 1 condition at $r = R$ in the formation, which is all that the lower order Darcy Law (2) will allow. So far, I can't see a problem with this mode formulation.

To set up a problem we might actually hope to solve, we have to choose a tractable formulation; for example, we might look for steady, spatially periodic flow of period L in an infinitely long $(-\infty < x < \infty)$ pipe driven by constant pressure gradient $c = \frac{p_2 - p_1}{L}$.

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$$p = \frac{p_1(L-x) + p_2 x}{L} + \Pi(x,r) \left.\vphantom{\frac{p_1(L-x)+p_2x}{L}}\right\}$$
$$\Pi(0,r) = \Pi(L,r) \tag{10}$$

where $\mathbf{u}, \Pi, \varphi$ are periodic in x with period L. We could try to solve (2) with

$$\left.\begin{aligned} P(x,r) &= \left[\frac{b_1(L-x) + p_2 x}{L}\right] g(r) + \zeta(x,r) \\ g(R) &= 1,\ g(\infty) = 0, \zeta(0,r) = \zeta(L,r) \\ \zeta(x,\infty) &= P_2 \\ \mathbf{U}(0,r) &= \mathbf{U}(L,r) \end{aligned}\right\} \tag{11}$$

Of course, I don't know if this works, it's just an idea.

To get functions needed for this theory Marlene Huerta's group would need to measure

$$\frac{\eta}{\kappa}(\phi), \eta(\phi), \rho(\phi)$$

and

$$\rho(\phi) \text{ i.e., } f(p, \phi) = 0$$

in equilibrium. Maybe $p(\phi)$ and $\rho(\phi)$ are given by the P, V, T measurements.

GAS ANCHORS. José Robles is building the plexiglass slit apparatus which I suggested in figure 3 of my memo of November 25. This device should be working in the next months. We could ask him to give a cartoon of this device for this memo. The idea of the slit separator is to determine the conditions under which gas bubbles rise out and separate even though they are experiencing downward drag from oil going down the annulus. In the slit device you can see how it all works.

José has decided to build the big separator he originally proposed. He thinks he needs this for data. He modified the design of the big apparatus to simplify it, eliminating the feature that will duplicate the benchtop slit apparatus.

PROGRESSIVE CAVITY PUMP (José Robles). The problem that José now identifies as central is leaking seals. After some discussion we came to the following understanding. The pump cavities are identical except at intake and discharge. We may assume that the volumes of gas and liquid

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is the same in each cavity but not necessarily at the intake and discharge. Therefore, the seals at the intake and the outlet are at greatest risk. The pressure in the cavities is less than at the discharge. The overall problem with progressive cavity pumps is that they wear out, which evidently means that you develop unacceptable leakage between cavities.

The following questions came up:

1. Is there critical pressure difference for leakage across a seal?

2. Is there a preferential breaking of gas or liquid?

3. What is the nature of the deterioration of progressive cavity pumps which are worn out?

We discussed some experiments for answering 1. and 2., but no decision was taken. I would like the information to answer 3.

DRILL STRINGS. I did not have a chance to talk to Miguel Ford. I contacted Paul Paslay, an old friend of mine, who according to Saad is one of the great experts in drill string and bore hole stability. He is in the consulting business and is employed mainly by Shell; but he keeps and wants his independence. He helps me when he can and would be glad to come down here. I think it could be a good idea because Miguel did tell me there were drill strings they need to address. Ford uses the work of Thomas Hill. Paslay likes Hill, thinks he is reputable, but technically weak. I don't think Paslay would ask much money and an expert review of drill string problems might be useful.

LUBRICATION WITH DILUENTS. Ignacio rasied the possibility of lubrication of crudes using diluent, like kerosene, rather than water. The idea is to create a lubricated core flow. I think that this is an excellent idea. Instead of diluting heavy oil with kerosene you pump the kerosene as you would water in a core-annular flow. Of course the kerosene and oil will mix, but maybe the mixing takes place very slowly. We have to do some experiments and some thinking about this. We have to hope that diluent will wet the carbon steel walls more strongly than the crude. Other diluents might be used. We could consider dispersions of kerosene droplets in water as a lubricant.

Even if diluents don't work well enough to lubricate core flow, they are still important for cleanup of fouling. If I wash a carbon steel wall with

kerosene in water, will the kerosene stick to the wall?

LUBRICATED INTERMITTENT GAS LIFT. Ali Hernandez also had this idea to lubricate crude with kerosene. He proposed to do gas lift of heavy crude when the walls are wet with kerosene. We hope that the crude won't stick to the kerosene wet walls. Then we will get a dramatic reduction in the cost of lift.

Ali has designed some versions of benchtop gas lift to test his idea, as I suggested. The main problem we see is how to wet the walls with kerosene. Ali has several good ideas and I am asking him to prepare some cartoons to go with this report.

It would be best if the kerosene was injected as a mist with the gas.

Appendix 1.

Foam Control Using a Fluidized Bed

Daniel D. Joseph, University of Minnesota,
Minneapolis, MN 55455, March, 1997

This proposal is based on research by Guitián and Joseph [1996] which was carried out at the University of Minnesota. A patent application for foam control using a fluidized bed has been filed jointly by Intevep S.A. and the University of Minnesota. The paper "How bubbly mixtures foam and foam control using a fluidized bed" on which this proposal is based is appended to this proposal.

Part 1. Foam creation in a bubbly column

Here we consider projects associated with how bubbly mixtures foam, without particles, corresponding to Part 1 of the paper by Guitián and Joseph in the appendix. We shall use the same figure and equation numbers in the appendix.

1. Scale up
Is the foaming criterion

$$U_g = a + bU_l \tag{1}$$

independent of the foam height H (see figure 1)? The percentage of foam $f = h/H$ in the column depends on U_g for a fixed U_l and $f = 0$ when the U_l is larger than the threshold value given by (2) (see figure 2).

The coefficients a and b depend on the foaminess (surfactant) and foam quality (the distributor) is a factor

- Do a and b depend on H?

2. Foam Quality
In our video tape, we show that bubbling through a fixed bed produces a fine foam

- Determine a and b for fine foam

- Determine gas hold up vs. gas velocity for fixed liquid velocities for fine foam. *Repeat the measurements described in the appended paper for fine foam.*

- In the previous work we used pentanol-SDS mixtures to reproduce the foaming characteristics of a real reactor (figure 6). The pentanol is toxic, now we use butanol-SDS mixtures and the foam quality is visually different. Repeat the measurement described in the appended paper for Butanol-SDS mixture.

- Seek a foaming system for organic, non-aqueous liquids which resemble commercial hydrocarbon reactors more closely.

3. Foam Rheology

We think that the rheology of foams may be initially confined to the determination of viscosity, yield stress, elasticity and normal stress in foam. We need to consider particles as a tool for characterizing the rheology of foams.

Viscosity. We used an ad hoc theory and foaming data to determine a formula (22) for the viscosity of foam shown in figure 7.

- Repeat the viscosity calculation for fine foams and butanol SDS - water foaming mixtures.

- Compare the formulas for viscosity that arise from our ad hoc analysis with literature expressions.

Yield stress. Small particles are trapped in foam; large particles are blocked from entering or if they enter the foam, they will not circulate and they drop out in linked chains as in figure 8.

- Determine the size and weight of particles that can be trapped in foam. Use this information to characterize the yield stress. It may be better to describe this property of foam as its *particle carrying capacity* rather than yield stress.

- How does the shape of particles and the size and weight distribution of a polydispersion effect the particle carrying capacity?

Normal Stresses and elasticity of foams. The orientation of cylinders settling in a viscoelastic fluid are sensitive indicators of the normal stresses developed in flow. In situations, in which cylinders with round ends

fall with their long axis parallel to gravity, cylinders with flat ends will tilt and the angle of tilt is greater when the normal stresses are greater. We propose to use this property to construct a device to determine the magnitude of normal stresses in foam, shown below in cartoon form.
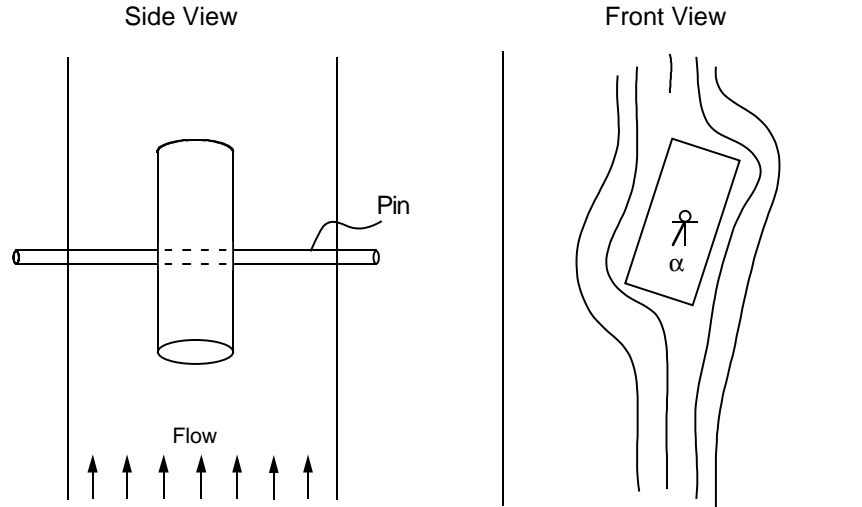

The cylinder center is fixed by a pin through its center and is anchored on the side wall. It is tilted normal stresses in flow of foam and measurements of the tilt angle correlate with the intensity of the normal stresses.

4. Foam suppression with hydrophilic particles

More data of the type collected by Guitián and Joseph needs to be collected. The effects of size, weight and concentration of monodisperse particles in batch should be thoroughly documented. New types of data should be collected.

- Polydisperse particle. The foam should act as a filter removing small and light particles. The effect on the gas hold up and foam height should be determined. We are getting 12 different proppant sands used in well fracturing from Stimlab. These sands are polydisperse and have other differences which may enter into foam suppression.

- Shape effects can enter into properties of fluidized beds used to suppress foam. The bed expansion could be controlled to a degree by the shape of particles. We propose to make cylinders with flat ends from feed stock.

"Square"   "Rectangle"
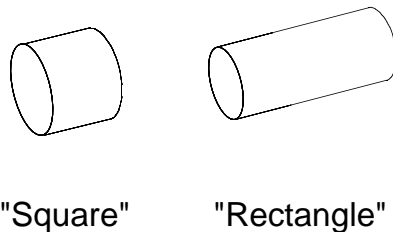
Particles like these can be made cheaply in our shop from different feed-stock size and weight. The drag on these particles is greater than the drag on an equivalent spherical leading to less foam through enhanced bed expansion.

- It is known (see figure 14) that the liquid hold up will increase with size of monodisperse spheres of fixed density and concentration only up to a certain size; for larger sizes the liquid hold up decreases.

- Determine the optimum size of monodisperse particles of fixed density and concentration.

Since

$$1 = \epsilon_l + \epsilon_s + \epsilon_g$$

large spherical particles can increase liquid hold up even when they decrease gas hold up.

There are probably many criteria for optimization of different mechanisms of foam suppression which will emerge.

5. Continuous injection of particles
This is a very practical topic of study since most reactors use continuous rather than batch injection of particles. Injected particles must go out of or accumulate in the reactor.

- Determine size, weight and injection rate of particles for which steady state conditions, without accumulation may be established.

- The limiting factor in particle transport is the foam. We expect to see large hold up of particles in the bubbly mixture with only small amounts of particles held up in the foam. This hold up will depend on the foam but more strongly on the particles.

- Determine hold up properties of foam and bubbly mixtures under steady conditions corresponding to different rates of particles (3). The light and small particles of a polydisperse slurry will be driven out with the foam. The foaming reactor can be used in this way as a particle demixer, like a flotation device, which needs documentation.

6. In situ foaming

The foaming criterion (2) works also in a packed bed. We may create foam in a packed bed by injecting gas and liquid at rates above critical. This creates an opportunity for foam injection. The surfactant or foaming solution is injected, then gas is forced through at a rate fast enough to create foam. The foam is created in situ, instead of injecting foam we create it down-hole.

There are many opportunities for in situ foaming; acidizing to name one. It would be useful to see if foam could be created in fluid filled tightly packed sands.

7. Foam suppression and destruction using hydrophobic particles.

Though no reference to the use of fluidized beds appears in the vast literature on defoaming, there are discussions of foam breaking using hydrophobic particles. Guitián & Joseph studied foam suppression with glass and plastic (hydrophilic) particles. Hydrophobic particles also can be fluidized in the bubbly mixture and perhaps they will attack foam at the interface where foam appears. The reduction or destabilization of foam above a bed of hydrophobic particles fluidized in a bubbly mixture is a new topic worthy of study.

The study of foam breaking or destabilization is different than foam suppression. The suppression of foaming in a reactor means less foam is in the reactor, but the rate of foam production does not necessarily decrease. For many applications, foam suppression is not enough, the foam should be destroyed or destabilized to the greatest extent. For this, we look to the fluidization of hydrophobic particles.

It is certain that not all hydrophobic particles are alike. A classification based on size, weight, concentration and degree of hydrophocity (contact angle) should be established along the lines in the appended paper. Continuous injection of hydrophobic particles should also be considered.

Appendix 2.

Richardson-Zaki Theory: This "theory" is a constant state theory (gradients are not allowed) in which the dispersed and continuous phases are characterized by constant values of the volume fraction and velocity. In solid-liquid or solid-gas systems, the solids are the dispersed phase; in bubbly mixtures and aqueous foams the gas is the dispersed phase and the liquid is the continuous phase. The continuity equations for the disperse and continuous phase give rise to three equations for four unknowns; another equation is required and basically it has to come from the momentum balance. Two phase flow theories elaborate complicated momentum balances. Richardson and Zaki theory short circuits this by postulating a constitutive equation for the slip velocity in terms of the volume fraction. I am going to show this below. The great utility of RZ theory is that the constitutive expression for the slip velocity is obtained from correlations of experimental data. This makes RZ theory highly empirical, but highly systematic. The empiricism is to be highly praised in complicated two sphase systems since foolishness which you often see in two phase flow models is circumvented. The RZ theories are the simplest possible.

Ensemble Averages.
I am going to derive continuity equations for the two phases. The derivation will show how two fluid equations arise by averaging. I am going to write briefly about ensemble averaging; I could get the same basic equations by taking some other kinds of averages like time averages and space averages.

Ensemble averages are obtained by measuring a property of flow at a given time and place for many repeated realizations. You could think of doing the same experiment many times. You fix your attention at some point $\mathbf{x}$ of your apparatus and take the measurement after waiting a time t and you do this N times and let $N \to \infty$.

To be concrete lets consider a bubble reactor where gas is the dispersed and liquid is the continuous. Now define an indicator function

$$H(\mathbf{x}, t) = \begin{cases} 0 \text{ when } \mathbf{x}, t \text{ is in the gas} \\ 1 \text{ when } \mathbf{x}, t \text{ is in the liquid} \end{cases} \tag{1}$$

$$\begin{aligned} \langle H \rangle (\mathbf{x}, t) &\stackrel{\text{def}}{=} \lim_{N \to \infty} \frac{1+0+0+1+1+1+0+1+1+1+1+0+\cdots}{N} \\ &= \varepsilon(\mathbf{x}, t), \qquad 0 \leq \varepsilon \leq 1 \end{aligned} \tag{2}$$

Obviously ε is a measure of how often we see gas at $\mathbf{x}, t$. We assume now that

$$\varepsilon = \frac{\text{vol of gas}}{\text{total volume}} \tag{3}$$

Now we define the liquid fraction

$$\phi(\mathbf{x}, t) = \langle 1 - H \rangle = 1 - \varepsilon(\mathbf{x}, t) = \frac{\text{vol of liquid}}{\text{total volume}} \tag{4}$$

Obviously

$$\varepsilon + \phi = 1 \tag{5}$$

Notice that we have just generated two continuous fields $\varepsilon(\mathbf{x}, t)$ and $\phi(\mathbf{x}, t)$ defined at the same point. This is what is behind the notion of interpenetrating continua.

Now I am going to define the gas and liquid velocity for interpenetrating fields

$$\mathbf{u}_g(\mathbf{x}, t) \stackrel{\text{def}}{=} \frac{\langle H\mathbf{u} \rangle}{\langle H \rangle} = \frac{\langle H\mathbf{u} \rangle}{\varepsilon} \tag{6}$$

The liquid velocity

$$\mathbf{u}_l(\mathbf{x}, t) \stackrel{\text{def}}{=} \frac{\langle (1 - H)\mathbf{u} \rangle}{\langle 1 - H \rangle} = \frac{\langle (1 - H)\mathbf{u} \rangle}{\phi} \tag{7}$$

The "superficial" velocities are

$$\mathbf{U}_g = \varepsilon \mathbf{u}_g = \langle H\mathbf{u} \rangle \tag{8}$$

$$\mathbf{U}_l = \phi \mathbf{u}_l = (1 - \varepsilon)\mathbf{u}_l = \langle (1 - H)\mathbf{u} \rangle \tag{9}$$

Obviously

$$\mathbf{U}_g \leq \mathbf{u}_g, \mathbf{U}_l \leq \mathbf{u}_l. \tag{10}$$

The indicator function doesn't change if $\mathbf{x}, t$ is always on the same material particle, liquid or gas

$$\frac{dH}{dt} = \frac{\partial H}{\partial t} + \mathbf{u} \cdot \nabla H = 0 \tag{11}$$

Obviously

$$\frac{d(1 - H)}{dt} = \frac{\partial(1 - H)}{\partial t} + \mathbf{u} \cdot \nabla(1 - H) = 0 \tag{12}$$

Continuity Equations.
Now, I will derive the continuity equation for the liquid by taking the ensemble average of $1 - H$ using the fact that $\text{div}\mathbf{u} = 0$ in the liquid and

$$\langle \frac{\partial(1-H)}{\partial t} \rangle = \frac{\partial}{\partial t}\langle 1 - H \rangle = \frac{\partial \phi}{\partial t} \tag{13}$$

$$\langle \mathbf{u} \cdot \nabla(1 - H) \rangle = \langle \text{div}(\mathbf{u}(1 - H)) \rangle = \text{div}\langle u(1 - H) \rangle \tag{14}$$

$$= \text{div}\phi\mathbf{u}_l$$

Hence

$$\frac{\partial \phi}{\partial t} + \text{div}\phi\mathbf{u}_l = 0 \tag{15}$$

This derivation doesn't work for the gas because $\text{div}\mathbf{u} \neq 0$; in fact, there

$$\frac{\partial \rho}{\partial t} + \text{div}\mathbf{u}\rho = 0. \tag{16}$$

To get the equation multiply (11) by ρ and take an ensemble average

$$\rho\frac{\partial H}{\partial t} + \rho\mathbf{u} \cdot \nabla H = 0$$

$$\langle \frac{\partial}{\partial t}PH \rangle + \text{div}\langle \rho\mathbf{u}H \rangle - \langle H\{\frac{\partial \rho}{\partial t} + \text{div}\rho\mathbf{u}\} \rangle$$

$$= \frac{\partial}{\partial t}\rho_g\varepsilon + \text{div}\varepsilon\{\rho\mathbf{u}\}_g = 0 \tag{17}$$

where

$$\{\rho\mathbf{u}\}_g \overset{\text{def}}{=} \frac{\langle H\rho\mathbf{u} \rangle}{\varepsilon}. \tag{18}$$

When ρ_g is a constant independent of $\mathbf{x}, t$, (17) reduces to

$$\frac{\partial \varepsilon}{\partial t} + \text{div}\varepsilon\mathbf{u}_g = 0 \tag{19}$$

People who do "mixture theory" like Ishii and Truesdell get

$$\frac{\partial \rho_g\varepsilon}{\partial t} + \text{div}\varepsilon\rho_g\mathbf{u}_g = 0 \tag{20}$$

instead of (17) which may be written as

$$\frac{\partial \varepsilon\rho_g}{\partial t} + \text{div}\varepsilon\rho_g\mathbf{u}_g + \text{div}[\varepsilon\{\rho\mathbf{u}\}_g - \varepsilon\rho_g\mathbf{u}_g] = 0 \tag{21}$$

Hence, the mixture theory guys miss out the last term.

The density changes associated with flow don't amount to much if the Mach number is much less than one. However, in the oil industry the variation of the density due to hydrostatic pressure cannot be neglected. In this case, we could write

$$\rho_g(\mathbf{x}, t) = \tilde{\rho}(z) + r(\mathbf{x}, t) \tag{22}$$

and think that $r(\mathbf{x}, t)$ is very small when the Mach number is much less than one. Then

$$\begin{aligned} \{\rho\mathbf{u}\}_g - \rho_g u_g &= \langle H\rho\mathbf{u}\rangle - \rho_g\langle H\mathbf{u}\rangle \\ &= \langle H\tilde{\rho}\mathbf{u}\rangle - \tilde{\rho}\langle H\mathbf{u}\rangle = 0 \end{aligned} \tag{23}$$

because $\tilde{\rho}(z)$ does not vary in different realizations at each level z. Therefore we can use (20) when the Mach number is small.

The same equations apply to fluidized suspensions of solids in which the dispersed phase is a solid.

Homework 1: Define

$$\boldsymbol{\omega}_v = \varepsilon\mathbf{u}_g + \phi\mathbf{u}_l \quad \text{(Volume averaged velocity)} \tag{24}$$

$$\boldsymbol{\omega}_M = \frac{\rho_g\mathbf{u}_g\varepsilon + \rho_l\mathbf{u}_l\phi}{\rho_g\varepsilon + \rho_l\phi} \quad \text{(mass averaged velocity)} \tag{25}$$

$$\rho_c = \varepsilon\rho_g + \phi\rho_l \quad \text{(composite density)} \tag{26}$$

Assume that the gas is incompressible and show that

$$\textbf{(a)} \quad \text{div}\boldsymbol{\omega}_v = 0 \tag{27}$$

$$\textbf{(b)} \quad \frac{\partial\rho_c}{\partial_t} + \text{div}(\rho_c\boldsymbol{\omega}_M) = 0 \tag{28}$$

Constant State Theories.
If the ensemble average equations are steady then

$$\begin{cases} \text{div}\varepsilon\rho_g\mathbf{u}_g = 0 \\ \text{div}\phi u_l = 0 \end{cases} \tag{29}$$

Suppose that $\rho_g = \tilde{\rho}(z)$ as when gravitational compression is important, and that all ensemble averages depend on the coordinate z of gravity alone. Then

$$\frac{d}{dz}\tilde{\rho}(z)\varepsilon u_g = 0$$
$$\frac{d}{dz}\phi u_l = 0 \tag{30}$$

where u is the z component of $\mathbf{u}$. Obvious $\tilde{\rho}\varepsilon u_g$ and ϕu_l are constants. We leave aside the variation with z and consider true constant states independent of z. Then

$$\begin{cases} \varepsilon u_g = U_g, \\ \phi u_l = U_l, \\ \varepsilon + \phi = 1 \end{cases} \tag{31}$$

where U_g and U_l are the superficial and liquid velocities prescribed at $x = 0$. There are three equations in the four unknowns ε, ϕ, u_g and u_l.

Richardson & Zaki Correlations

RZ studied fluidized suspensions of solid particles with $\varepsilon u_s = U_s$ where s stand for solids and replaces g for gas.) Assuming that $u_l > u_s$ so that the liquid will drag up solids they wrote the slip velocity in the form

$$u_l - u_s = u_0 \Phi(\varepsilon) \tag{32}$$

where u_o is the terminal velocity of a single sphere in the bed when no other particles are there and $\Phi(\varepsilon)$ is an empirical to-be-determined hindered settling function (which needs the explanation in the next section). They found $\Phi(\varepsilon)$ correlations in fluidized beds of particles in batch. In these beds $U_s = 0$ and to a first approximation $u_s = 0$ also; the particles are in equilibrium under weight and drag. Then (32) reduces to

$$u_l = u_1 \Phi(\varepsilon) = u_1 \varepsilon^n \tag{33}$$

where u_1 is the value of u_l then there are no other particles, $\varepsilon = 1$. So u_1 depends on the radius of the sphere, the viscosity η of the fluid, the geometry of the container in which the sphere falls, the Reynolds number, etc. and they assumed that $\Phi(\varepsilon) = \varepsilon^n$ where n depends on the Reynolds number

$\mathbb{R} = u_o d / \nu$. From many experiments they found that

$$n = 4.65 + 19.5 \, d/D \quad \text{when } \mathbb{R} < 0.2$$
$$n = [4.36 + 17.6 \, d/D] \, \mathbb{R}^{-0.03} \quad \text{when } 0.2 < \mathbb{R} < 1$$
$$n = 4.45 \mathbb{R}^{-0.1} \quad \text{when } 1 < \mathbb{R} < 500$$
$$n = 2.39 \quad \text{when } 500 < \mathbb{R} < 7000$$

Some correlations of the Richardson-Zaki type

$$u_g - u_l = u_o \Phi(\varepsilon) \tag{34}$$

for bubbly mixtures are summarized in the paper "How bubbly mixtures foam and foam control using a fluidized bed" by Guitian & Joseph. Many more can be found in José Guitian's (1996) thesis. "Topics on two and three phase flow".

For cuttings transport in aerated muds and for foams we could need to consider RZ type of correlation for three phase flows. Such correlations can be found in the book "Gas-Liquid-Solid Fluidization Engineering" by L.S. Fan (Butterworth, 1994).

Hindered Settling.
The drag on a group of falling particles is larger that the drag on a single one. So the group settles more slowly than a single one. If the drag is greater, the settling velocity of the group is smaller. Equation (33) says that the liquid velocity necessary to hold up one particle is greater than the velocity required to hold up many. If we put the liquid velocity to rest the single particle falls faster. The difference between the drag on one sphere and many is expressed by the "hindered settling function" $\Phi(\varepsilon)$.

Sample calculation of a hindered settling function in Stokes flow.

$$6\pi a \eta u_1 = \text{Drag} = \text{relative weight} = \tfrac{4}{3}\pi a^3 g(\rho_s - \rho_l)$$

Hence $u_1 = \frac{2ga^2}{9\eta}(\rho_s - \rho_l)$.

If there are sidewalls, I would use a side-wall correction for Stokes flow, say (Francis, 1933)

$$u_1 = \frac{2ga^2}{9\eta}(\rho_s - \rho_l)\left[1 - \frac{d}{D}\right]^{2.25} \qquad \begin{aligned} d &= 2a \\ D &= \text{Channel diameter} \end{aligned}$$

For many particles $u > u_1$,

$u_s = 0$

$$u_2 = \frac{A}{A_l}\mathbf{u} = \frac{V}{V_l}\mathbf{u} = \frac{1}{1-\varepsilon}\mathbf{u}$$

u_2

where u_2 is determined by a balance between bouyancy and drag

A_s = Area of solids
$A = A_s + A_l$

$u = (1 - \varepsilon)u_2$ is the velocity required to put the sphere to rest (the sphere falls with velocity u, but there is a backflow of fluids). If I add $-u$ to the whole system, the particles fall with velocity u.

The velocity u_2 is determined by a balance of buoyancy and drag. It's the actual velocity. You must account for

    (i) effective viscosity
    (ii) effective density

The drag on many particles is different because the viscosity of a dispersion is greater

$$\eta_{\text{eff}}(\varepsilon) = \eta_o(1 + \tfrac{5}{2}\varepsilon) \quad \text{Einstein } \varepsilon \ll 1$$

$$\eta_{\text{eff}}(\varepsilon) = \eta[1 + \tfrac{5}{2}\varepsilon + 10.05\varepsilon^2 + A \exp B\varepsilon] \qquad \text{Thomas}$$
$$A = 0.00273 \qquad\qquad\qquad\qquad\qquad\qquad (1965)$$
$$B = 16.6$$

$$\text{Drag} = 6\pi\eta_{\text{eff}}(\varepsilon)u_2 a$$

The weight is calculated relative to the density of the dispersion, rather than the fluid; replace ρ_l with $\rho_{\text{eff}}(\varepsilon) = \rho_s\varepsilon + \rho_l(1 - \varepsilon)$. Hence

$$\rho_s - \rho_{\text{eff}}(\varepsilon) = (\rho_s - \rho_l)(1 - \varepsilon)$$
$$\text{Weight} = \tfrac{4}{3}\pi a^3 g(\rho_s - \rho_l)(1 - \varepsilon)$$
$$\text{Drag} = \text{Weight}$$
$$6\pi a\eta_{\text{eff}}u_2 = \tfrac{4}{3}\pi a^3 g(\rho_s - \rho_l)(1 - \varepsilon)$$
$$u_2 = \frac{2ga^2}{\eta}(\rho_s - \rho_l)(\frac{\eta}{\eta_{\text{eff}}})(1 - \varepsilon) = u_1(\frac{\eta}{\eta_{\text{eff}}(\varepsilon)})(1 - \varepsilon)$$

Hence, the settling velocity of a swarm of spheres is

$$u = (1 - \varepsilon)u_2 = (1 - \varepsilon)^2(\frac{\eta}{\eta_{\text{eff}}(\varepsilon)})u_1 = \Phi(\varepsilon)u_1$$

Homework 2: A fluidized bed 10cm×10cm is filled with 5000, 1mm spheres of density 1.05 gms/cm^3. What is the volume flow of water required to raise the bed height to 1 meter. Use Stokes flow formula and Thomas' viscosity formula.

Solution to Homework

1)

(a)

$$\boldsymbol{\omega}_v = \varepsilon \mathbf{u}_g + \phi \mathbf{u}_l$$

↑

volume average velocity

Show div $\boldsymbol{\omega}_v = 0$

$$\mathrm{div}\boldsymbol{\omega}_v = \mathrm{div}(\varepsilon \mathbf{u}_g + \phi \mathbf{u}_l) = \mathrm{div}(\varepsilon \mathbf{u}_g) + \mathrm{div}(\phi \mathbf{u}_l)$$

use

$$\frac{\partial \phi}{\partial t} + \mathrm{div}(\phi \mathbf{u}_l) = 0$$

$$\frac{\partial \phi}{\partial t} + \mathrm{div}(\varepsilon \mathbf{u}_g) = 0 \quad \text{for incompressible gas}$$

add

$$\frac{\partial \phi}{\partial t} + \frac{\partial \varepsilon}{\partial t} + \mathrm{div}(\phi \mathbf{u}_l) + \mathrm{div}(\varepsilon \mathbf{u}_g) = 0$$

$$\frac{\partial \phi}{\partial t} + \frac{\partial}{\partial t}(1 - \phi) + \mathrm{div}(\phi \mathbf{u}_l + \varepsilon \mathbf{u}_g) = 0$$

since $\qquad \phi + \varepsilon = 1$

but

$$\frac{\partial \phi}{\partial t} + \frac{\partial}{\partial t}(1 - \phi) = \frac{\partial \phi}{\partial t} - \frac{\partial \phi}{\partial t} = 0$$

$$\therefore \mathrm{div}(\phi \mathbf{u}_l + \varepsilon \mathbf{u}_g) = \mathrm{div}(\boldsymbol{\omega}_v) = 0$$

(b)

$$\boldsymbol{\omega}_M = \frac{\rho_g \mathbf{u}_g \varepsilon + \rho_l \mathbf{u}_l \phi}{\rho_g \varepsilon + \rho_l \phi} \quad \text{mass average velocity}$$

$$\rho_c = \rho_g \varepsilon + \rho_l \phi \quad \text{composite density}$$

Show $\frac{\partial}{\partial t}\rho_c + \text{div}(\rho_c \boldsymbol{\omega}_M) = 0$ for incompressible gas $(\rho_g = \text{const})$

$$\frac{\partial}{\partial t}\rho_c = \frac{\partial}{\partial t}(\rho_g \varepsilon + \rho_l \phi)$$

$$= \rho_g \frac{\partial \varepsilon}{\partial t} + \rho_l \frac{\partial \phi}{\partial t} \quad \text{since } \rho_g, \rho_l \text{ are constants}$$

use

$$\frac{\partial \phi}{\partial t} + \text{div}(\phi \mathbf{u}_l) = 0$$

$$\frac{\partial \varepsilon}{\partial t} + \text{div}(\varepsilon \mathbf{u}_l) = 0$$

$$\frac{\partial}{\partial t}\rho_c = -\rho_g \, \text{div}(\varepsilon \mathbf{u}_g) - \rho_l \text{div}(\phi \mathbf{u}_l)$$

$$= -\text{div}(\rho_g \mathbf{u}_g \varepsilon) - \text{div}(\rho_l \mathbf{u}_l \phi)$$

$$= -\text{div}(\rho_g \mathbf{u}_g \varepsilon + \rho_l \mathbf{u}_l \phi)$$

$$= -\text{div}\left\{ \frac{\rho_c}{\rho_c} \cdot (\rho_g \mathbf{u}_g \varepsilon + \rho_l \mathbf{u}_l \phi) \right\}$$

$$= -\text{div}\left\{ \rho_c \cdot \frac{(\rho_g \mathbf{u}_g \varepsilon + \rho_l \mathbf{u}_l \phi)}{\rho_g \varepsilon + \rho_l \phi} \right\}$$

$$= -\text{div}(\rho_c \boldsymbol{\omega}_M)$$

$$\therefore \frac{\partial}{\partial t}\rho_c + \text{div}(\rho_c \boldsymbol{\omega}_M) = 0$$

2) Fluidized bed problem

filled with 5000, 1mm (0.1cm) spheres
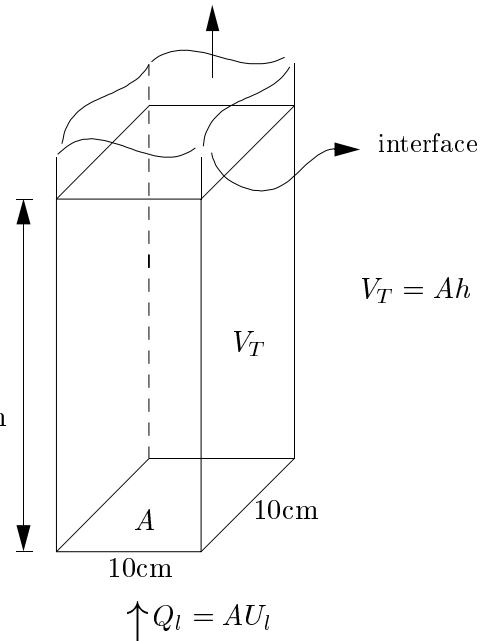
$$\rho_s = 1.05 \text{ gr/cc}$$

$$\rho_l = 1.0 \text{ gr/cc}$$

$$\varepsilon = \frac{5000 \times \frac{4}{3}\pi(0.1/2)^3}{V_T} = 2.62 \times 10^{-4}$$

$$a = \frac{0.1}{2} \text{ cm}$$

$$V_T = 100 \times 10 \times 10 \text{ cm}^3$$

interface

$$V_T = Ah$$

$$V_T$$

$$h = 1\text{m}$$
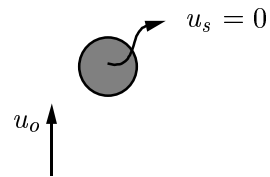
$$A$$

10cm

10cm

$$\uparrow Q_l = AU_l$$

Governing Equations

$$\text{continuity} \begin{cases} \varepsilon u_s = U_s \\ \phi u_l = U_l \\ \varepsilon + \phi = 1 \end{cases}$$

Single Sphere

$$u_s = 0$$

Stokes flow $\qquad u_o = \dfrac{2ga^2}{9\eta}(\rho_s - \rho_l)$

$$u_o$$

$$Re = \frac{2au_o}{\nu} < 1 \qquad \nu = \frac{\eta_l}{\rho_l} \qquad \eta_l = 1\text{cp}$$

Computation of u_l

- **Add wall effect**

$$u_1 = u_o(1 - d/D)^{2.25}$$

$$d = 0.1 \text{cm}$$
$$D = 10 \text{cm}$$

- **Add hindered Settling**

$$u_2 = u_1(1 - \varepsilon)$$

- **Add effective viscosity and effective density**

$$u_l = \frac{2ga^2}{9\eta_{\text{eff}}}(\rho_s - \rho_{l\text{eff}})(1 - \varepsilon)$$

where

$$\eta_{\text{eff}} = \eta(1 + \tfrac{5}{2}\varepsilon + 10.05\varepsilon^2 + Ae^{B\varepsilon})$$
$$A = 0.00273$$
$$B = 16.6$$

$$\rho_{l\text{eff}} = \rho_s\varepsilon + \rho_l(1 - \varepsilon)$$

Hence,

$$u_l = \frac{2ga^2}{9\eta_{\text{eff}}}(\rho_s - \rho_l)(1 - \frac{d}{D})^{2.25}(1 - \varepsilon)^2$$

since $\rho_s - \rho_{l\text{eff}} = (\rho_s - \rho_l)(1 - \varepsilon)$.

Compute Q_l

$$U_l = \frac{Q_l}{A} = (1 - \varepsilon)u_l$$

$$\boxed{Q_l = A(1 - \varepsilon)u_l}$$

$$Q_l = 265 \text{ cm}^3/\text{s}$$