

1 Appendix: A different constitutive equation

For constitutive law we assume

$$\tau\gamma\frac{d\phi}{dt} + \gamma\phi = c - p , \quad (1)$$

which is a statement that the volume fraction relaxes toward the equilibrium value, with time constant τ . In addition we have Darcys law,

$$-\lambda\nabla p = \mathbf{u} , \quad (2)$$

and conservation of mass (in the approximation that gas density is much less than liquid density),

$$\frac{d\log(1 - \phi)}{dt} + \nabla \cdot \mathbf{u} = 0 . \quad (3)$$

The material derivative d/dt is defined by

$$\frac{d}{dt} = \alpha\frac{\partial}{\partial t} + \mathbf{u} \cdot \nabla . \quad (4)$$

Combining (4) and (2), and expanding the material derivative, we obtain

$$\alpha\frac{\partial\phi}{\partial t} - \lambda\nabla p \cdot \nabla\phi + (1 - \phi)\nabla \cdot \lambda\nabla p = 0 \quad (5)$$

as before. Combining (1) and (2), and expanding the material derivative, we obtain

$$\tau\gamma\frac{\partial\phi}{\partial t} - \tau\gamma\lambda\nabla p \cdot \nabla\phi + \gamma\phi = c - p . \quad (6)$$

Equations (5) and (6) are the basic governing equations in this form of the theory. Clearly, when $\tau = 0$, we recovery exactly the same equilibrium theory as was derived above. It is also clear that this version of the theory supports steady uniform states as solution, again exactly as found above. The governing equations for small perturbations around these steady states are however different, as we shall now show. As before, introduce small perturbation quantities, setting

$$\phi = c_1 + \varphi \quad (7)$$

$$p = c_2 + \pi \quad (8)$$

where equilibrium demands $\gamma c_1 + c_2 = 0$. Substituting these quantities into (5) and (6), neglecting quantities of second order in perturbations and treating the mobility as a constant, we obtain

$$\alpha\frac{\partial\varphi}{\partial t} + (1 - c_1)\lambda\nabla^2\pi = 0 \quad (9)$$

$$\tau\gamma\frac{\partial\varphi}{\partial t} + \gamma\varphi = -\pi . \quad (10)$$

If we now take the time derivative of (10), and use (9) to replace $\partial\varphi/\partial t$ by the Laplacian of π , we find

$$\left(\alpha - \tau(1 - c_1)\gamma\lambda\nabla^2\right) \frac{\partial\pi}{\partial t} = (1 - c_1)\gamma\lambda\nabla^2\pi . \quad (11)$$

The perturbation volume fraction, φ , satisfies an identical equation. Initial conditions for φ , $\varphi(\mathbf{x}, 0)$, may be prescribed freely, but once these are given, the initial conditions for π are determined. To see this, we eliminate $\partial\varphi/\partial t$ between (9) and (10), and set $t = 0$, to obtain

$$(1 - c_1)\tau\gamma\lambda\nabla^2\pi(\mathbf{x}, 0) - \alpha\pi(\mathbf{x}, 0) = \alpha\gamma\varphi(\mathbf{x}, 0) . \quad (12)$$

Given the initial values of φ , consistency demands that $\pi(\mathbf{x}, 0)$ satisfies (12). The authors are grateful to Professor G. Barenblatt for bringing this point to their attention. Boundary conditions on the perturbation pressure and volume fraction are the same here as for the model presented above.

The evolution equation for the perturbation pressure, (11), is very different in character from the telegraph equation found for the pressure relaxation form of the constitutive law. It is lower order in time, and, from Laplace transform solutions (obtained with $\pi(\mathbf{x}, 0) = \varphi(\mathbf{x}, 0) = 0$), appears not to exhibit wave propagation; rather, the solutions are diffusive in character although similarity solutions in $x/t^{1/2}$ do not exist (except as an asymptotic state at large t). The rate of advance of the pressure perturbation front is found from these Laplace transform solutions to be faster in early time than would be the case for classical diffusion.

An equation identical in form to (11) governs the pressure within the secondary porosity in the dual porosity theory of slightly compressible flow in naturally fractured porous media [?, WarrenRoot] Equations (6) and (9) of that paper are (re-expressed in notation echoing that used here)

$$\Lambda\nabla^2 p_2 - A_1 \frac{\partial p_1}{\partial t} = A_2 \frac{\partial p_2}{\partial t} , \quad (13)$$

$$A_1 \frac{\partial p_1}{\partial t} = T(p_2 - p_1) . \quad (14)$$

Routine rearrangements then yield

$$\left(\left[A_2 + T\left(1 + \frac{A_2}{A_1}\right)\right] - \Lambda\nabla^2\right) \frac{\partial p_2}{\partial t} = \frac{T}{A_1}\Lambda\nabla^2 p_2 \quad (15)$$

which is identical in form to (11). The pressure within the primary porosity satisfies a different equation. Again, we are grateful to Professor G. Barenblatt for bringing this to our notice.

References

- [1] J.E. Warren & P.J. Root 'The behaviour of naturally fractured reservoirs'
Society of Petroleum Engineers Journal, 245-255, September (1963).